BALASORE COLLEGE OF ENGINEERING & TECHNOLOGY

SERGARH, BALASORE

Lecture Notes

On

DESIGN OF CONCRETE STRUCTURE

3RD Year

5th Semester

Prepared by -:

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Civil Engineering Department

Checked by

MODULE WISE DISTRIBUTION OF LOADS

Module	Chapter with title	Assigned Hour (as per BPUT)	Actual Session Needed	Range of Marksof Questions to be being asked (BPUT)
I	Properties of concrete and reinforcing steel, philosophy, concept and methods of reinforced concrete design, introduction to limit state method, limit state of collapse and limit state of serviceability, application of limit state method to rectangular beams for flexure, shear bond and torsion.	10	16	35-45
II	Design of doubly reinforced beams, design of T and L beams, design of one way and two way slabs, design of staircase.	8	15	15-20
III	Design of short and long columns with axial and eccentric loadings, Design of isolated and combined column footings	8	16	20-30
IV	Retaining walls, various forces acting on retaining wall, stability requirement, design of cantilever and counterfort retaining walls.	8	11	15-20
V	Design of water tanks, design requirements, design of tanks on ground, under -ground and elevated water tanks	6	11	15-20
TOTAL		40	69	100 marks

SYLLABUS

Module I: **(10 hours)**

Properties of concrete and reinforcing steel, philosophy, concept and methods of reinforced concrete design, introduction to limit state method, limit state of collapse and limit state of serviceability, application of limit state method to rectangular beams for flexure, shear bond and torsion.

Module II: **(8 hours)**

Design of doubly reinforced beams, design of T and L beams, design of one way and two way slabs, design of staircase

Module III: **(8 hours)**

Design of short and long columns with axial and eccentric loadings, Design of isolated and combined column footings

Module IV: **(8 hours)**

Retaining walls, various forces acting on retaining wall, stability requirement, design of cantilever and counterfort retaining walls.

Module V: **(6 hours)**

Design of water tanks, design requirements, design of tanks on ground, under -ground and elevated water tanks

BOOKS

1. Design of reinforced concrete structure by N. Subramanian, Oxford university Press
2. Limit state Design by A.K. Jain, Neemchand & Bros
3. Reinforced Concrete Design by S U pillai & D.Menon, Tata Mc Graw Hill.

Digital Learning Resources:

Course Name	Design of concrete structures
Course Link	https://nptel.ac.in/courses/105/105/105105109/#
Course Instructor	Prof. N. Dhang, Department of Civil Engineering,IIT Khragpur

MODULE-1 **Chapter -1**
Session - 1

Learning Objectives
1.1 Introduction
1.2 Concrete and steel with their properties
1.2.1 Advantages Disadvantages of RC members
1.2.2 Materials required for RC member

1.1 *Introduction*

A structure refers to a system of connected parts used to support forces (loads). Buildings, bridges and towers are examples for structures in civil engineering. In buildings, structure consists of walls floors, roofs and foundation. In bridges, the structure consists of deck, supporting systems and foundations. In towers the structure consists of vertical, horizontal and diagonal members along with foundation.

A structure can be broadly classified as (i) sub structure and (ii) super structure. The portion of building below ground level is known as sub-structure and portion above the ground is called as super structure. Foundation is sub structure and plinth, walls, columns, floor slabs with or without beams, stairs, roof slabs with or without beams etc are super-structure. Many naturally occurring substances, such as clay, sand, wood, rocks natural fibers are used to construct buildings. Apart from this many manmade products are in use for building construction. Bricks, tiles, cement concrete, concrete blocks, plastic, steel & glass etc are manmade building materials.

1.2 *Concrete and steel with their properties*

1.2.1 Advantages Disadvantages of RC members

• It has high tensile and compressive strength.

• It is more durable and may long up to 100 years.

• It imparts ductility.

• Raw materials used for construction of RC buildings are easily available and can be transported.

• Overall cost for constructing a building using RC proves to be economical compared to steel and pre-stressed structures.

• RC components can be moulded to any desired shape, if formwork is designed properly.

• If RC structures are properly designed then it can resist the earthquake forces.

Disadvantage

• Tensile strength of RC member is about $1/10^{th}$ of its compressive strength **1.4.**

1.2.2 Materials required for RC member

a. Concrete

Concrete is a product obtained artificially by hardening of the mixture of cement, sand, gravel and water in predetermined proportions. Depending on the quality and proportions of the ingredients used in the mix the properties of concrete vary almost as widely as different kinds of stones. Concrete has enough strength in compression, but has little strength in tension. Due to this, concrete is weak in bending, shear and torsion. Hence the use of plain concrete is limited applications where great compressive strength and weight are the principal requirements and where tensile stresses are either totally absent or are extremely low.

Properties of Concrete

1. **Grade of concrete**

Mild	M20
Moderate	M25
Severe	M30
Very Severe	M35
Extreme	M40

2. Tensile strength $F_{cr} = 0.7 * \sqrt{f_{ck}}$

3. Modulus of elasticity $E_c = 5000 * \sqrt{f_{ck}}$

4. Shrinkage of concrete: Depends on

- Constituents of concrete
- Size of the member
- Environmental conditions

5. Creep of concrete: Depends on

 Strength of the concrete

 Stress in concrete

 Duration of loading

6. Durability: Mainly depends on

 Type of Environment

 Cement content

 Water cement ratio

 Workmanship

 Cover to the reinforcement

7. Cover to the reinforcement

Nominal cover is essential

 Resist corrosion

 Bonding between steel and concrete

b) Reinforcements

- Bamboo, natural fibers (jute, coir etc) and steel are some of the types of reinforcements

Roles of reinforcement in RCC

• To resist Bending moment in case of flexural members

• To reduce the shrinkage of concrete

• To improve the load carrying capacity of the compression member

• To resist the effect of secondary stresses like temperature etc.

• To prevent the development of wider cracks formed due to tensile stress

Advantages of Steel Reinforcement

• It has high tensile and compressive stress

• It is ductile in nature

• It has longer life

• It allows easy fabrication (easy to cut, bend or weld)

• It is easily available

• It has low co-efficient of thermal expansion same as that of concrete

Disadvantages of Steel Reinforcement

• More prone to corrosion

• Loses its strength when exposed to high temperature.

Classification of Steel bars

1. Mild Steel plain bars

 Cold worked steel bars

 Hot rolled mild steel bars

Eg: Fe250

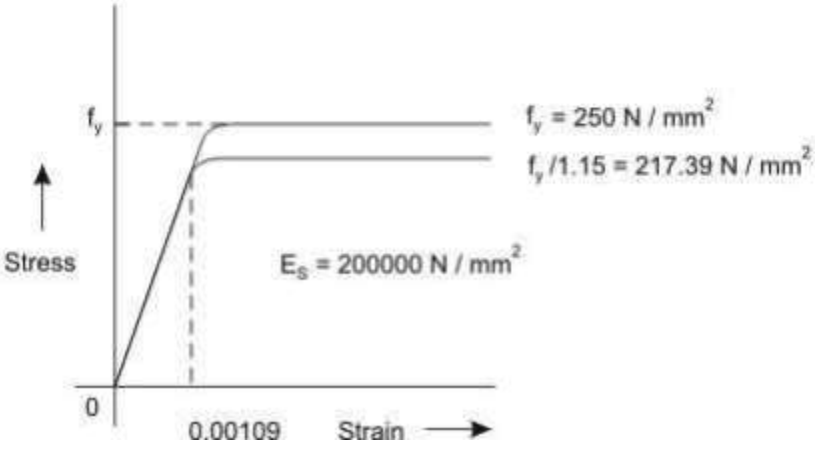
2. High Yield Strength Deformed (HYSD) Bars

Eg: Fe415 & Fe500

3. Steel wire Fabric4. Structural Steel 5. CRS and TMT

- **8mm-10mm** size bars are used in Slabs and Stair ups, which serves as a load bearing member in slab homes.
- **12mm-25mm** size bars are used in Beams & Columns, to make them withstand external loads. **32mm-36mm** size bars are used in the construction of very complex projects like dams, bridges.

- Based on the designs also, we go for the sizes. Sometimes, we use different sizes according to the project specifications.



Stress-strain curves for reinforcement

Figure1.2: Stress-strain curve for Mild steel (idealized) (Fe 250) with definite yield point Figure1.3: Stress-strain curve for cold worked deform bar Figures 1.2 and 1.3 show the representative stress-strain curves for steel having definite yield point and not having definite yield point, respectively. The characteristic yield strength fy steel is assumed as the minimum yield stress or 0.2 per cent of proof stress for steel having no or cold-worked bars (Fig. 1.3), the stress is proportional to the strain up to a stress of 0.8 *f* .

MODULE-1 **Chapter -1**

Session - 2

Learning Objectives

1.3 Types of load on RCC structures and

1.4 Introduce IS: 456-2000 code provision

1.3 Types of Loads on RCC Structures

1. Dead Load IS 875 (Part 1)1987
2. Live Load IS 875 (Part 2)1987
3. Wind Load IS 875 (Part 3)1987
4. Snow Load IS 875 (Part 4)1987
5. Earthquake Load IS 1893 2002

- *Low intensity Zone (IV or less) – Zone II*
- *Moderate intensity Zone (VII) – Zone III*
- *Severe intensity Zone (VIII) – Zone IV*
- *Very Severe Intensity Zone (IX and above) – Zone V*

1.Dead load

Dead loads include the structure's weight or other fixed elements before any live loads are considered. The total weight of the structure is the sum of the dead weight and the live weights applied to it. The volume of each section is multiplied by the unit material weight to determine the total dead load of each structure.

2. Live Load:

Imposed loads, also known as live loads, are another type of vertical load that must be considered when a structure is designed. Non-decelerating or non-impacting loads are known as live loads. For example, the weight of movable partitions or furniture is assumed to produce these loads as part of the building's intended use or occupancy. It's impossible to predict what the current live load will be.

The designer must take into account these additional stresses. It's one of the most significant weights in the design. IS 875 (part 2)–1987 specifies the minimum live load values that must be assumed in new construction projects. On the other hand, it depends on the purpose of the building. The code provides the live load values for the following accommodation class:

- Homes, hotels, hostels, and other types of residential buildings such as boiler room, plant room, garage and storage facilities.
- Institutional structures.
- Buildings for the education of students.
- Buildings for commercial and office use.
- Assemblage facilities.
- Commercial buildings.
- Buildings of the industrial sector.
- Rooms for storing things.

3. Wind Load:

As the structure must withstand its weight, it must also withstand wind, earthquakes, and other external forces. There should be no exceptions to this rule regarding structures, components, and cladding, all of which must withstand wind loads. It is known as a wind load when the wind exerts pressure on a structure. This weight is dispersed over the entire structure's surface area. As a building's height increases, so do the magnitude of this load, i.e., due to the wind, taller structures have a greater impact than shorter ones.

4. Snow Load:

When snow accumulates, it creates a vertical force on the ground. Large amounts of snow can accumulate, burdening the building, especially in areas prone to heavy and frequent winter snowfalls. Code of Practice for Design Loads (Other Than Earthquake) for Buildings and Structures – Snow Loads recommends snow load on roofs of buildings in India.

5. Earthquake Load:

As a result of seismic excitations, the structure experiences an earthquake load. Inertia force varies in mass. Earthquake loading will be much higher because of a structure's greater mass. The structure will fail or be damaged if the earthquake load exceeds the element's moment of resistance.

1.5 Introduce IS: 456-2000 code provision

Certainly! **IS 456:2000** is the Indian Standard code of practice for general structural use of plain and reinforced concrete. It provides guidelines and provisions for designing reinforced concrete structures, including beams, columns, slabs, and foundations.

MODULE-1 **Chapter -1**
Session - 3

Learning Objectives

1.6 Design Philosophies on WSM and LSM
1.7 Methods of Design
1.8 Working Stress Method – Based on Elastic theory
1.9 Ultimate load method or Load factor method.

1.6 Design Philosophies on WSM and LSM

Design of RC member involves

1. Deciding the size or dimension of the structural element and amount of reinforcement required.

2. To check whether the adopted size perform satisfactorily during its life span.

Methods of Design or Design philosophy

1. Working stress method

2. Ultimate or load factor method

3. Limit state method.

Working Stress Method – Based on Elastic theory

Assumptions: -

- Plane section remains plane before and after deformation takes place
- Stress –strain relation under working load, is linear for both steel and concrete
- Tensile stress is taken care by reinforcement and none of them by concrete □□Modular Ratio between steel and concrete remains constant.

Modular ratio

M = 280/3 σ_{cbc}

Where = σ_{cbc} **is permissible stress**

Advantages: -

1. Method is simple

2. Method is reliable

3. Stress is very low under working condition, therefore serviceability is automatically satisfied

Limitations: -

1. Stress strain relation for concrete is not linear for concrete

2. It gives an idea that failure load = working load * factor of safety, but it is not true

3. This method gives uneconomical section.

<u>Ultimate load method or Load factor method</u>

This method uses design load = ultimate load * load factor

Load factor = Design load / Working load

- This method gives slender and thin section which results in excessive deflection and cracks
- This method does not take care of shrinkage of concrete.
- This method does not take of serviceability

Limit State Method

Limit state is an acceptable limit for both safety and serviceability before which failure occurs

1. Limit state of collapse

2. Limit state of serviceability

3. Other limit state

Limit state of Collapse The structure may get collapse because of

- Rupture at one or more cross-sections
- Buckling
- Overturning

While designing the structure following ultimate stresses should be considered

1. Flexure

2. Shear

3. Torsion

4. Tension

5. Compression

Limit state of Serviceability

a) Limit state of deflection

- Lack of safety
- Appearance
- Ponding of water
- Misalignment in machines
- Door, window frames, flooring materials undergoes crack

Methods for controlling deflecting

- Empirical formula – span/depth
- Theoretical - dimension

b) Limit state of cracking

- Appearance
- Lack of safety
- Leakage
- Creation of maintenance problem
- Reduction in stiffness with increase in deflection

- corrosion

Other Limit states

a) Vibration

b) Fire resistance

c) Chemical and environmental actions

d) Accidental loads

Characteristic load

Characteristic load = Mean Load+1.64S

Characteristic Strength

Characteristic Strength = Mean Strength -1.64S

Partial Safety factor

2. For material

3. For load

$\mathbf{F_d = F * \gamma_f}$

Ultimate load method (ULM)

The method is based on the ultimate strength of reinforced concrete at ultimate load is obtained by enhancing the service load by some factor called as load factor for giving a desired margin of safety. Hence the method is also referred to as the load factor method or the ultimate strength method.

In the ULM, stress condition at the state of in pending collapse of the structure is analysed, thus using, the non-linear stress – strain curves of concrete and steel. The safely measure in the design is obtained by the use of proper load factor. The satisfactory strength performance at ultimate loads does not guarantee satisfactory strength performance at ultimate loads does not guarantee satisfactory serviceability performance at normal service loads.

Limit state method (LSM)

Limit states are the acceptable limits for the safety and serviceability requirements of the structure before failure occurs. The design of structures by this method will thus ensure that they will not reach limit states and will not become unfit for the use for which they are intended. It is worth mentioning that structures will not just fail or collapse by violating (exceeding) the limit states. Failure, therefore, implies that clearly defined limit states of structural usefulness has been exceeded.

Limit state are two types

i) Limit state of collapse

ii) Limit state of serviceability.

MODULE-1 **Chapter -1**
Session - 4

Learning Objectives
1.10 LSM method of design on RCC structures.
1.11 Limit State of collapse
1.12 Limit State of serviceability

Limit state is an acceptable limit for both safety and serviceability before which failure occurs
1. Limit state of collapse
2. Limit state of serviceability
3. Other limit state

Limit state of Collapse

The structure may get collapse because of
- Rupture at one or more cross-sections
- Buckling
- Overturning

While designing the structure following ultimate stresses should be considered
1. Flexure
2. Shear
3. Torsion
4. Tension
5. Compression

Limit state of Serviceability a) Limit state of deflection

 Lack of safety
 Appearance
 Ponding of water
 Misalignment in machines
 Door, window frames, flooring materials undergoes crack

Methods for controlling deflecting

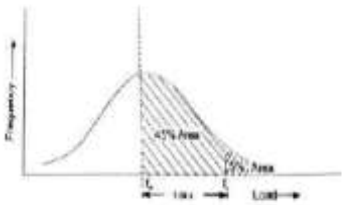
• Empherical formula – span/depth
• Theoretical - dimension

b) Limit state of cracking

 Appearance
 Lack of safety
 Leakage
 Creation of maintenance problem
 Reduction in stiffness with increase in deflection
 corrosion

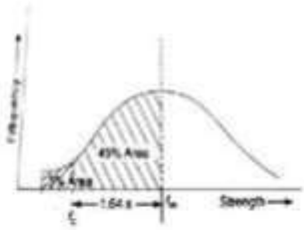
Characteristic load

Characteristic load = Mean Load+1.64S



Characteristic Strength

Characteristic Strength = Mean Strength -1.64S



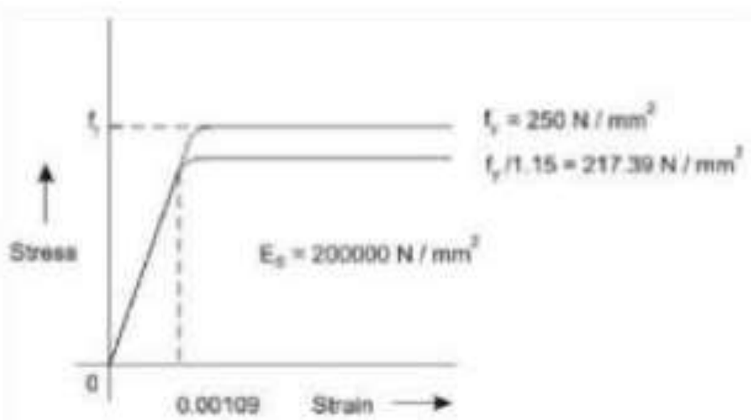
Partial Safety factor

2. For material

3. For load

$$F_d = F * \gamma_f$$

Stress-strain curves for reinforcement



Stress-strain curve for Mild steel (idealized) (Fe 250) with definite yield point

A reinforced concrete structure should be designed to satisfy the following criteria-

i) Adequate safety, in items stiffness and durability

ii) Reasonable economy.

MODULE-1 **Chapter -1**
Session - 5

<u>Learning Objectives</u>

1.13 Limit state of collapse for flexure.

1.14 Assumption/ Stress-strain curve for concrete and steel.

1.13 Limit states of collapse for flexure

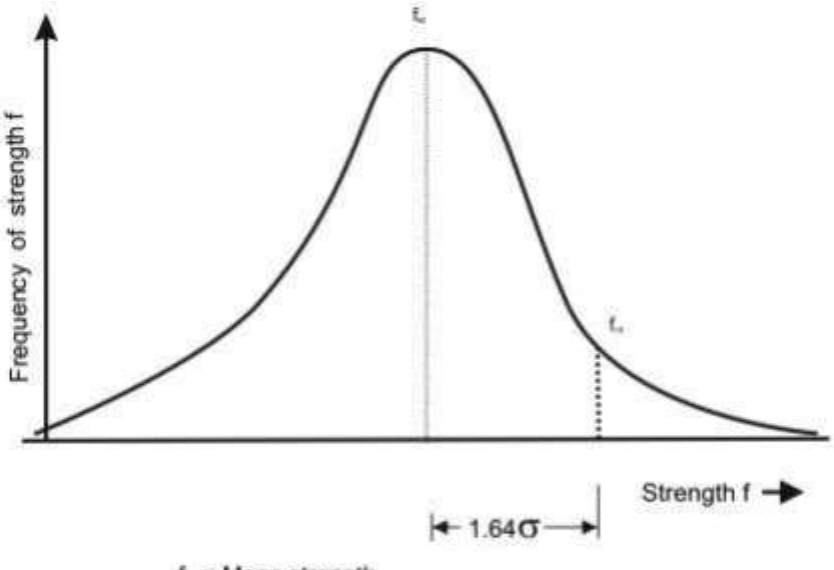
The limit state of collapse of the structure or part of the structure could be assessed from rupture of one or more critical sections and from bucking due to elastic bending, shear, torsion and axial loads at every section shall not be less than the appropriate value at that section produced by the probable most unfavorable combination of loads on the structure using the appropriate factor of safely.

Characteristic and design values and partial safety factor

1. Characteristic strength of materials.

The term _characteristic strength_ means that value of the strength of material below

which not more than minimum acceptable percentage of test results are expected to fall. IS 456:2000 have accepted the minimum acceptable percentage as 5% for reinforced concrete structures. This means that there is 5% for probability or chance of the actual strength being less than the characteristic strength.



Frequency distribution curve for strength

Figure shows frequency distribution curve of strength material (concrete or steel). The value of

K corresponding to 5% area of the curve is 1.65.

The design strength should be lower than the mean strength (f_m)

Characteristic strength = Mean strength –K x standard deviation or

$f_k=f_m-KS_d$

Where, f_k=characteristic strength of the material

f_m=mean strength

K=constant =1.65

S_d=standard deviation for a set of test results.

The value of standard deviation (S_d) is given by

S_d =

Where, δ=deviation of the individual test strength from the average or mean strength of n samples.

n= number of test results.

Characteristic strength of concrete

Characteristic strength of concrete is denoted by f_{ck} (N/mm^2) and its value is different for different grades of concrete e.g. M 15, M25 etc. In the symbol _M_ used for designation of concrete mix, refers to the mix and the number refers to the specified characteristic compressive strength of 150 mm size cube at 28 days expressed in N/mm^2

Characteristic strength of steel

Until the relevant Indian Standard specification for reinforcing steel are modified to include the concept of characteristic strength, the characteristic value shall be assumed as the minimum yield stress or 0.2% proof stress specified in the relevant Indian Standard specification. The characteristic strength of steel designated by symbol f_y (N/mm^2)

Characteristic loads

The term _Characteristic load_ means that values of load which has a 95% probability of not being exceeded during that life of the structure.

The design load should be more than average load obtained from statistic, we have

$F_k=F_m+KS_d$

Where, F_k=characteristic load;

F_m= mean load

K=constant=2.65;

Sd=standard deviation for the load.

Since data are not available to express loads in statistical terms, for the purpose of this standard, dead loads given in IS 875(Part-1), imposed loads given in IS 875(Part-2), wind loads. Given in IS 875 (Part-3), snow load as given in IS 875(Part-4) and seismic forces given in IS 1893 shall be assumed as the characteristic loads.

Design strength of materials

The design strength of materials (f_d) is given by

$$f_d = \frac{f_k}{\gamma_m}$$

Where, f_k=characteristic strength of material.

γ_m = =partial safety factor appropriate to the material and the limit state being

Design loads

The design load (F_d) is given by.

F_d=F_k. □ $_f$

□ $_f$ =partial safety factor appropriate to the nature of loading and the limit state being

considered.

The design load obtained by multi plying the characteristic load by the partial safety factor for load is also known as factored load.

Partial safety factor (□ $_m$) for materials

When assessing the strength of a structure or structural member for the limit state of collapse, the values of partial safety factor, □ $_m$ should be taken as 1.15 for steel.

Thus, in the limit state method , the design stress for steel reinforcement is given by f_y / □ $_{ms}$ =

f_y/1.15=0.87f_y.

According to IS 456:2000 for design purpose the compressive strength of concrete in the structure shall be assumed to be 0.67 times the characteristic strength of concrete in cube and partial safety factor □ $_{mc}$ =1.5 shall be applied in addition to this. Thus, the design stress in

concrete is given by

0.67 f_{ck} / □ $_{mc}$ □ 0.67 f_{ck} /1.5 □ 0.446 f_{ck}

Partial safety factor for loads

The partial safety factors for loads, as per IS 456:2000 are given in table below

Load combination	Limit State of collapse			Limit State of Serviceability		
	DL	LL	WL/EL	DL	LL	WL/EL
DL+IL	1.5	1.5	-	1.0	1.0	-
DL+WL	1.5 or 0.9*	-	1.5	1.0	-	1.0
DL+IL+WL	1.2	1.2	1.2	1.0	0.8	0.8

(* This value is to be considered when stability against overturning or stress reversal is critical)

1.14 Assumption/ Stress-strain curve for concrete and steel.

The behaviour of reinforced concrete beam sections at ultimate loads has been explained in detail in previous section. The basic assumptions involved in the analysis at the ultimate limit state of flexure (Cl. 38.1 of the Code) are listed here.

a) Plane sections normal to the beam axis remain plane after bending, i.e., in an initially straight beam, strain varies linearly over the depth of the section.

b) The maximum compressive strain (at the outer most fibre) *cu* shall be taken as 0.0035 in bending.

c) The relationship between the compressive stress distribution in concrete and the strain in concrete may be assumed to be rectangle, trapezoid, parabola or any other shape which results in prediction of strength in substantial agreement with the results of test. An acceptable stress-strain curve is given below in figure 1.6. For design purposes, the compressive strength of concrete in the structure shall be assumed to be 0.67 times the characteristic strength. The partial safety factor y, = 1.5 shall be applied in addition to this.

d) The tensile strength of the concrete is ignored.

e) The stresses in the reinforcement are derived from representative stress-strain curve for the type of steel used. Typical curves are given in figure 1.3. For design purposes the partial safety factor equal to 1.15 shall be applied.

f) The maximum strain in the tension reinforcement in the section at failure shall not be

less than $\frac{f_y}{1.15E_s} + 0.002$

MODULE-1 **Chapter -1**

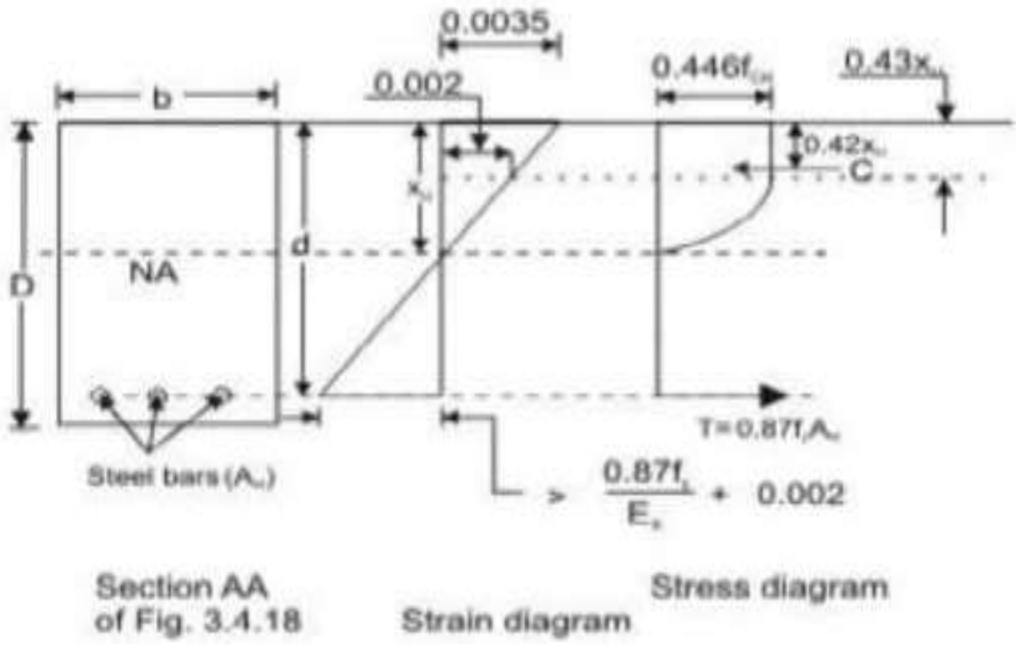
Session - 6

Learning Objectives

1.15 Depth of neutral Axis

1.16 Types of sections (BRS, URS & ORS)

1.15 Depth of neutral Axis



Rectangular beam under flexure *xu* < *xu* max

Limiting depth of neutral axis for different grades of steel

Steel Grade	Fe 250	Fe 415	Fe 500
$x_{u,max} / d$	0.5313	0.4791	0.4791

The limiting depth of neutral axis x_u,max corresponds to the so-called balanced section, i.e., a section that is expected to result in a balanced_ failure at the ultimate limit state in flexure. If the neutral axis depth x_u is less than x_u,max , then the section is under-reinforced (resulting in a tension_ failure); whereas if x_u exceeds x_u,max , it is over-reinforced (resulting in a compression failure).

1.16 Types of sections (BRS, URS & ORS)

A reinforced concrete member is considered to have failed when the strain of concrete in extreme compression fibre reaches its ultimate value of 0.0035. At this stage, the actual strain in steel can have the following values:

(a) Equal to failure strain of steel

(b) More than failure strain, corresponding to under reinforced section.

(c) Less than failure strain corresponding to over reinforced section.

Thus for a given section, the actual value of x_u / d can be determined).

Three cases arise.

Case-1: xu /d equal to the limiting value x_u,max/d : Balanced section.

Case-2: x_u /d less than limiting value: under-reinforced section.

Case-3: x_u /d more than limiting value: over-reinforced section.

Balanced Section

In balanced section, the strain in steel and strain in concrete reach their maximum values simultaneously. The percentage of steel in this section is known as critical or limiting steel percentage. The depth of neutral axis (NA) is $x_u = x_u$,max.

Under-reinforced section

An under-reinforced section is the one in which steel percentage (pt) is less than critical or limiting percentage (p_t,lim). Due to this the actual NA is above the balanced NA and $x_u < x_u$,max .

Over-reinforced section

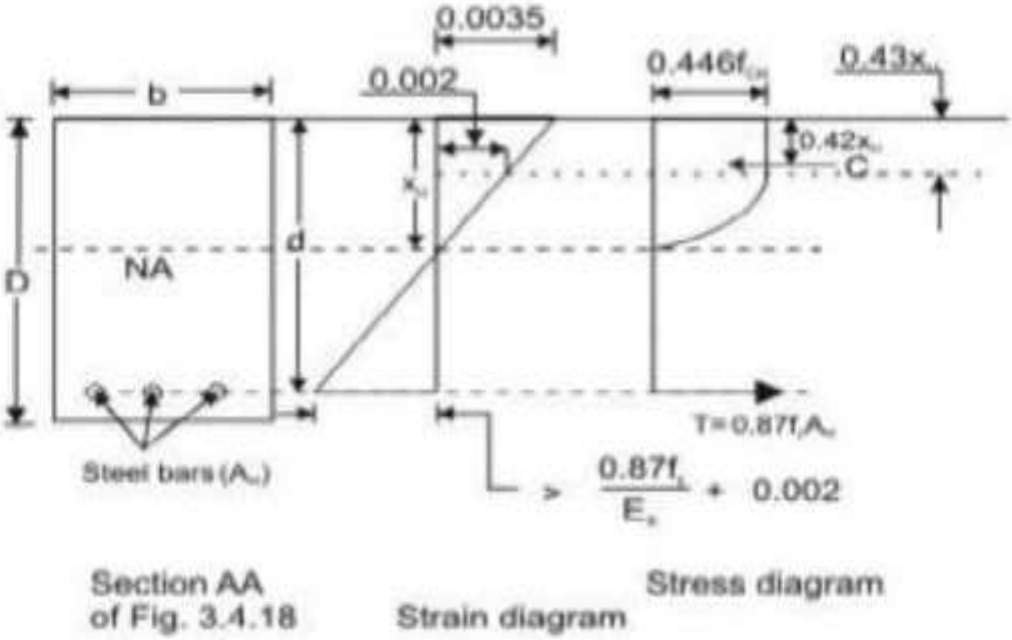
In the over reinforced section the steel percentage is more than limiting percentage due to which NA falls below the balanced NA and $x_u > x_u$,max . Because of higher percentage of steel, yield does not take place in steel and failure occurs when the strain in extreme fibres in concrete reaches its ultimate value.

Learning Objectives

1.17 Beam Notations and analysis of singly reinforced beams.

1.18 Analysis of Singly Reinforced Rectangular Sections

Analysis of a given reinforced concrete section at the ultimate limit state of flexure implies the determination of the ultimate moment MR of resistance of the section. This is easily obtained from the couple resulting from the flexural stresses



Concrete stress-block parameters in compression

*Moment of resistance = C * Z = T * Z*

where *C* and *T* are the resultant (ultimate) forces in compression and tension respectively and *z* is the lever arm.

T = 0.87 fy *Ast*

Concrete Stress Block in Compression

In order to determine the magnitude of Cu and its line of action, it is necessary to analyze the concrete stress block in compression. As ultimate failure of a reinforced concrete beam in flexure occurs by the crushing of concrete, for both under- and over-reinforced beams, the shape of the compressive stress distribution (_stress block_) at failure will be, in both cases, as shown in Fig. The value of Cu can be computed knowing that the compressive stress in concrete is uniform at 0.447 fck for a depth of 3xu/ 7, and below this it varies parabolically over a depth of 4xu/ 7 to zero at the neutral axis.

For a rectangular section of width *b*, Therefore, Cu = 0.36 * fck * b * xu

Also, the line of action of *Cu* is determined by the centroid of the stress block, located at a distance *x* from the concrete fibres subjected to the maximum compressive strain.

Accordingly, considering moments of compressive forces C_u, C_1 and C_2 about the maximum compressive strain location,

Depth of Neutral Axis

For any given section, the depth of the neutral axis should be such that $C_u = T$, satisfying equilibrium of forces.

Equating $C = T$,

Ultimate Moment of Resistance

The ultimate moment of resistance MR of a given beam section is

Modes of failure: Types of section

A reinforced concrete member is considered to have failed when the strain of concrete in extreme compression fibre reaches its ultimate value of 0.0035. At this stage, the actual strain in steel can have the following values:

(a) Equal to failure strain of steel

(b) More than failure strain, corresponding to under reinforced section.

(c) Less than failure strain corresponding to over reinforced section.

Thus for a given section, the actual value of x_u / d can be determined from Eq. (7). Three cases arise.

Case-1: x_u / d equal to the limiting value $x_{u,max}/d$: Balanced section.

Case-2: x_u / d less than limiting value: under-reinforced section.

Case-3: x_u / d more than limiting value: over-reinforced section.

Balanced Section

In balanced section, the strain in steel and strain in concrete reach their maximum values simultaneously. The percentage of steel in this section is known as critical or limiting steel percentage. The depth of neutral axis (NA) is $x_u = x_{u,max}$.

Under-reinforced section

An under-reinforced section is the one in which steel percentage (pt) is less than critical or limiting percentage ($p_{t,lim}$). Due to this the actual NA is above the balanced NA and $x_u < x_{u,max}$.

Over-reinforced section

In the over reinforced section the steel percentage is more than limiting percentage due to which NA falls below the balanced NA and $x_u > x_{u,max}$. Because of higher percentage of steel, yield does not take place in steel and failure occurs when the strain in extreme fibres in concrete reaches its ultimate value.

1.16 General Aspects of Serviceability:

The members are designed to withstand safely all loads liable to act on it throughout its life using the limit state of collapse. These members designed should also satisfy the serviceability limit states. To satisfy the serviceability requirements the deflections and cracking in the member should not be excessive and shall be less than the permissible values. Apart from this the other limit states are that of the durability and vibrations. Excessive values beyond this limit state spoil the appearance of the structure and affect the partition walls, flooring etc. This will cause the user discomfort and the structure is said to be unfit for use.

The different load combinations and the corresponding partial safety factors to be used for the limit state of serviceability are given in Table 18 of IS 456:2000.

Limit state of serviceability for flexural members:

Deflection

The check for deflection is done through the following two methods specified by IS 456:2000 (Refer clause 42.1)

Empirical Method

In this method, the deflection criteria of the member is said to be satisfied when the actual value of span to depth ratio of the member is less than the permissible values. The IS code procedure for calculating the permissible values are as given below

a. Choosing the basic values of span to effective depth ratios (l/d) from the following, depending on the type of beam.

1. Cantilever = 8

2. Simply supported = 20

3. Continuous = 26

b. Modify the value of basic span to depth ratio to get the allowable span to depth ratio.

Allowable l/d = Basic l/d x Mt x Mc x Mf

Where, Mt = Modification factor obtained from fig 4 IS 456:2000. It depends on the area of tension reinforcement provided and the type of steel.

Mc = Modification factor obtained from fig 5 IS 456:2000. This depends on the area of compression steel used.

Mf = Reduction factor got from fig 6 of IS 456:2000

Note: The basic values of l/d mentioned above is valid upto spans of 10m. The basic values are multiplied by 10 / span in meters except for cantilever. For cantilevers whose span exceeds 10 m the theoretical method shall be used.

2 Theoretical method of checking deflection

The actual deflections of the members are calculated as per procedure given in annexure = C ‘

of IS 456:2000. This deflection value shall be limited to the following

i. The final deflection due to all loads including the effects of temperature, creep and shrinkage shall not exceed span / 250.

ii. The deflection including the effects of temperature, creep and shrinkage occurring after erection of partitions and the application of finishes shall not exceed span/350 or 20 mm whichever is less.

Cracking in structural members

Cracking of concrete occurs whenever the tensile stress developed is greater than the tensile strength of concrete. This happens due to large values of the following:

1. Flexural tensile stress because of excessive bending under the applied load

2. Diagonal tension due to shear and torsion.

3. Direct tensile stress under applied loads (for example hoop tension in a circular tank)

4. Lateral tensile strains accompanying high axis compressive strains due to Poisson‘s effect (as in a compression test)

5. Settlement of supports.

In addition to the above reasons, cracking also occurs because of

1. Restraint against volume changes due to shrinkage, temperature creep and chemical effects.

2. Bond and anchorage failures.

Cracking spoils the aesthetics of the structure and also adversely affect the durability of the structure. Presence of wide cracks exposes the reinforcement to the atmosphere due to which the reinforcements get corroded causing the deterioration of concrete. In some cases, such as liquid retaining structures and pressure vessels cracks affects the basic functional requirement itself (such as water tightness in water tank).

Permissible crack width

The permissible crack width in structural concrete members depends on the type of structure and the exposure conditions. The permissible values are prescribed in clause 35.3.2

IS 456:2000 and are shown in table below

Control of cracking

The check for cracking in beams are done through the following 2 methods specified in IS 456:2000 clause 43.1

1. By empirical method:

In this method, the cracking is said to be in control if proper detailing (i.e. spacing) of reinforcements as specified in clause 26.3.2 of IS 456:2000 is followed. These specifications regarding the spacing have been already discussed under heading general specifications. In addition, the following specifications shall also be considered

i. In the beams where the depth of the web exceeds 750 mm, side face reinforcement shall be provided along the two faces. The total area of such reinforcement shall not be less than 0.1% of the web area and shall be distributed equally on two faces at a spacing not exceeding 300 mm or web thickness whichever is less. (Refer clause 25.5.1.3 IS456:2000)

ii. The minimum tension reinforcement in beams to prevent failure in the tension zone by cracking of concrete is given by the following

As = 0.85 fy / 0.87 fy (Refer clause 26.5.1.1 IS 456:2000)

iii. Provide large number of smaller diameter bars rather than large diameter bars of the same area. This will make the bars well distributed in the tension zone and will reduce the width of the cracks.

2. By crack width computations In the case of special structures and in aggressive environmental conditions, it is preferred to compute the width of cracks and compare them with the permissible crack width to ensure the safety of the structure at the limit state of serviceability.

IS 456-2000 has specified an analytical method for the estimation of surface crack width in Annexure-F which is based on the British Code (BS: 8110) specifications where the surface crack width is less than the permissible width, the crack control is said to be satisfied.

Learning Objectives

1.19 Calculation of moment or resistance of singly reinforced beams, check for serviceability and deflection criteria.

Problem-1

A rectangular Beam is to be simply supported on supports of 230 mm width. The clear span of the beam is 6m. The Beam is to have a width of 300 mm. The characteristics superimposed load is 12 kN/m. Using M20 concrete and Fe 415 steel, design the beam.

Solution:

From given:

- **Beam is simply supported on supports of width = 230 mm**
- **clear span of beam = 6 m**
- **width of beam (b) = 300 mm**
- **superimposed load = 12kN/m**
- **fck = 20 N/mm²**
- **fy = 415 N/mm²**

Assumption of depth of beam (d)

Assume, depth of beam (d) = 1/12th to 1/15th of span

= 1/12 X 6000 to 1/15 X 6000

= 500 to 400

Take, d = 400

Provide effective cover of the beam = 500

Then, overall depth of beam = d + eff. Cover

= 400+50

.’. **D = 450 mm**

Effective span calculation:

From clause 22.2 (a) of code IS 456:2000

For supply support Beam:

The **effective span of the beam should be taken as smaller of following two:**

1. Clear span + effective depth of beam

= 6000+400

or,

b) c/c of support

= 6000+ 230/2 + 230/2

= 6230mm

= 6.23m

.’. Effective span (ϑ

$_{eff.}$) = 6.23m

- **Calculation of total loas, total factored load and factored bending moment**

From given,

Imposed load = 12 kM/m

Self-wt. Of beam = γ_{RCC} bD x 1

= 20×0.3×0.45×1

= 3.375

.’. Total load = 12+3.375

= 15.375 kN/m

.’. factored total load = 1.5 x 15.375

.’. W_u = 23.06 kN/m

Now ,

Factored bending moment (mu) is given by

Mu = {Wu (ϑ $_{eff.}$)}/8

= {23.06 x $(6.23)^2$}/8

.’. Mu = 111.878 kM/m

Also, the maximum shear force occurs at the support and is given by

V_u = (wu ϑ $_{eff.}$)/2 = (23.06 x 6.23)/2

.’. V_u = 71.832kN

- Calculation of limiting value of Bending moment ($mu_i lim$)

From ANNEX “G” of Code IS 456:2000;

Also, from clause 38.1 of code 456:2000;

For, Fe 415 steel

$X_{ui\ max}$ = 0.48 x d

= 0.48 x 400

$X_{ui\ max}$ = 192mm

.’. $M_{ui\ lim}$ = 0.36 x 20 x 300 x 192 x (400 – 0.42 x 192)

.’. $M_{ui\ lim}$ = 132.445kN/m

- Comparison of Mu with $M_{ui\ lim}$

Since, Mu (111.878) < $M_{ui\ lim}$ (132.445)

Hence, the section can be designed as singly reinforced section

- Calculation of area of Reinforcement:

From ANNEX "G" of Code IS 456:2000;

Or, 774.671 = Ast – 1.729 x 10^{-4} $(Ast)^2$

By solving, we get Ast = 921.487mm^2

Provider diameter of bars = 20mm

Then number of bars required is

= 2.9346

= 3

.'. Provide 3 bars of 20 mm diameter with Ast,

Provided = 3 x (π/4) x $(20)^2$

= 942mm^2 > 921.487mm^2

- **Design of shear reinforcement**

From clause 40.1 of code IS 456:2000;

Nominal shear stress in beam is = 0.598 N/mm^2

Where, b = total width

d = effective depth

Also, percentage of reinforcement in beam is Pt = 0.785

Then, from table 19 of code IS 456:2000;

Design shear strength of concrete

For M20 concrete and Pt = 0.785

Pt	N/mm^2
At 0.75	0.56
At 1.0	0.62
At 0.785	?

By using linear interpolation

.'. = 0.5684 N/mm^2

Again, from table 20 of code IS 456:2000; maximum shear stress (max) for M20 concrete is = 2.8 N/mm^2

Thus, < <max

Hence, shear force resisted by concrete

= x bd

= 0.5684 x 300 x 400

= 68208 N

.'. Shear force to be resisted by reinforcement is

$V_{us} = V_u$ – x bd

= 71832 – 68208

V_{us} = 3624 N

Now, provide 2 legged 6mm dia vertical stirrups and Fe 250 steel

Then, from clause 40.4 of code 456:2000,

For vertical stirrups

Where, fy = 250 N/mm^2

V_{us} = 3624 N

A_{sv} = 2 x π/4 x (6)^2

= 56.52 mm^2

d = 40 mm (depth of beam)

sv = spacing of stirrups to be calculated then

3624 = (0.87 x 250 x 56.25 x 400)/2

.'. sv = 1356.854 mm

But, maximum spacing is permitted at a minimum of two:

1. 0.75 d = 0.75 x 400 = 300 mm
2. 300 mm

.'. provide S_v = 300 mm

Hence, provide 2 legged 6mm vertical stirrups @300 mm c/c spacing

- **Check for deflection:**

From clause 23.2.1 of code IS 456:2000;

For simply supported beam

Basic value of span of effective depth ratio = 20

Also, factors for no compressive steel (k_1) = 1

for not flanged section (k_2) = 1

And for modification factor (k_3) for tensile steel

From figure 4 of same clause,

fs = 265.4585 N/mm^2

And also , Pt = 0.785

Then, from figure 4 of same clause,

For fs = 235.4585 and

Pt = 0.785

Modification factor (k_3) = 1.18

Then, maximum permitted span to effective depth ratio I.e;

= 1 x 1 x 1.18 x 20 = 23.6

Also, = 15.575

Thus

Hence, deflection control is satisfactory.

- **Check for minimum reinforcement**

The minimum reinforcement is given by

Ast_{min} = (0.85 bd)/fy

= (0.85 x 400 x 300) / 415

= 245.783 mm^2 < 942 mm^2

- **Check for maximum reinforcement**

The maximum reinforcement is given by

Ast_{max} = 4% of bD

= 4/100 x 300 x 450

= 5400mm^2 > 942mm^2

MODULE-1

Chapter -1
Session - 9

<u>Learning Objectives</u>

1.20 Calculation of moment or resistance of singly reinforced beams, check for serviceability and deflection criteria. (continue)

Problem-2

A rectangular reinforced concrete beam is to be simply supported on two walls of 125 mm width with a clear span of 6.0 m. The characteristics live load is 12 kN/m and the grade of concrete is M20. Also, use Fe 415 steel. What is the effective span of the Beam? Design a suitable section for bending and determine the necessary tensile steel.

Solution:

From given,

- Beam is simply supported with width of support = 125 mm
- Clear span = 6.0 m
- Live load = 12 kN/m
- fck = 20 N/mm²
- fy = 415 N/mm²
- Then, effective span (leff) = ?

Design suitable section, also check in deflection:

Assume depth of beam (d)   d = 500 mm

From ANNEX G of code IS 456:2000;

$M_{ui}lim$ =   Or,   [$M_{ui}lim = 0.36$ fck bx_{uimax} X (d-0.42 x_{uimax})] -96-P

also, **from clause 38.1 of code IS 456:2000;**

For Fe 415 steel; ——> (P-70)

$X_{uimax} = 0.48d$

= 0.48 X 500

= 240 mm

Or, $M_{ui}lim$ = 0.36 X 20 X 250 X 240 X (500-0.42 X 240)

- $M_{ui}lim$ = 172.454 kN-m
- Comparison of Mu with $Mu_i lim$

Since, Mu < Mu_ilim

Hence, the section can be designed as singly reinforced section.

Calculation of area of reinforcement

From, Annex 'G' of Code IS 456:2000;

or, $601.5233 = Ast - 1.66 \times 10^1 (Ast)^2$

By solving: Ast = 677.782 mm

Provide, diameter of bar = 20 mm

Then, number of bars required = 2.1585 = 3

Provide 3-20 mm Ø bars with

Ast, provided **= 942 mm^2 > Ast, calculated ok**

.’. Ast,min = 256.024 mm^2 < 942 mm^2 **O.K.**

Maximum reinforcement

(Ast) max = 4% of bD

= 4/100 x 250 x 550

Check: **Minimum reinforcement required is** Ast,min

= 5500mm^2 > 942mm^2

O.K.

Design summary:

b = 250mm

d = 500mm

D = 550mm

3-20 mm Ø bars

MODULE-1 **Chapter - 2**
Session - 10

Learning Objectives
2.1 Introduction on shear reinforcement and types

Shear reinforcement, is to provide the resistance against shear forces to which a beam is subjected to and is usually in the form of stirrups which also serve the purpose of holding the main tensile and compression reinforcement in place.

Why Shear Reinforcement is required:

• Whenever the value of actual shear stress exceeds the permissible shear stress of the concrete used, the shear reinforcement must be provided.

• The purpose of shear reinforcement is to prevent failure in shear, and to increase beam ductility and subsequently the likelihood of sudden failure will be reduced.

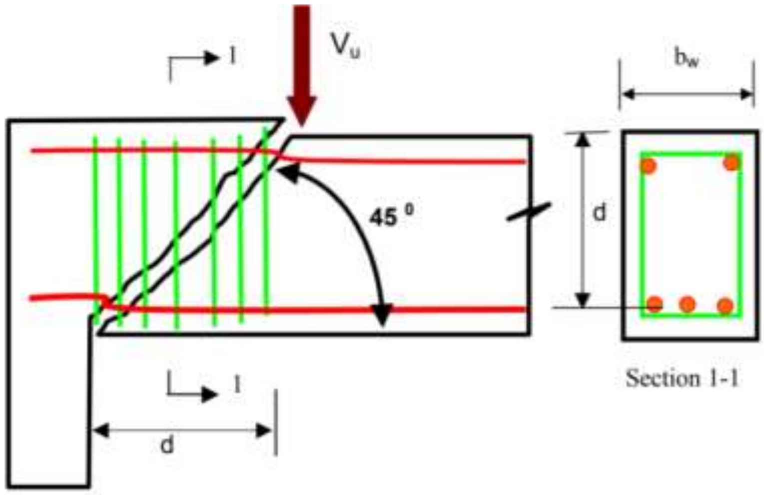
Shear Strength of concrete

• The shear strength (V) of reinforced concrete (RC) beams consists of two parts: Shear resistance of concrete (Vc) and Shear resistance of the transverse reinforcement (Vs).

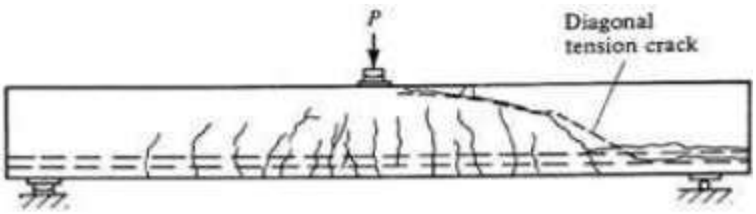
• One of the main objectives of the design of reinforced concrete beams is safety. Sudden failure due to shear low strength is not desirable mode of failure.

• The reinforced concrete beams are designed primarily for flexural strength and shear strength.

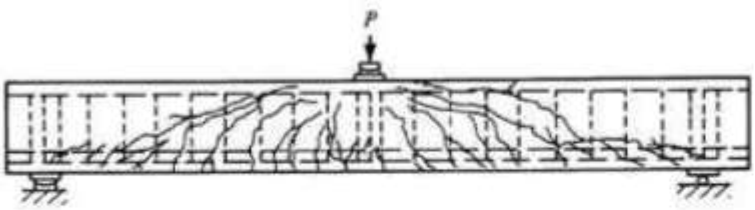
• Beams are structural members used to carry loads primarily by internal moments and shears.



Modes of Cracking in RCC beam



(a)



(b)

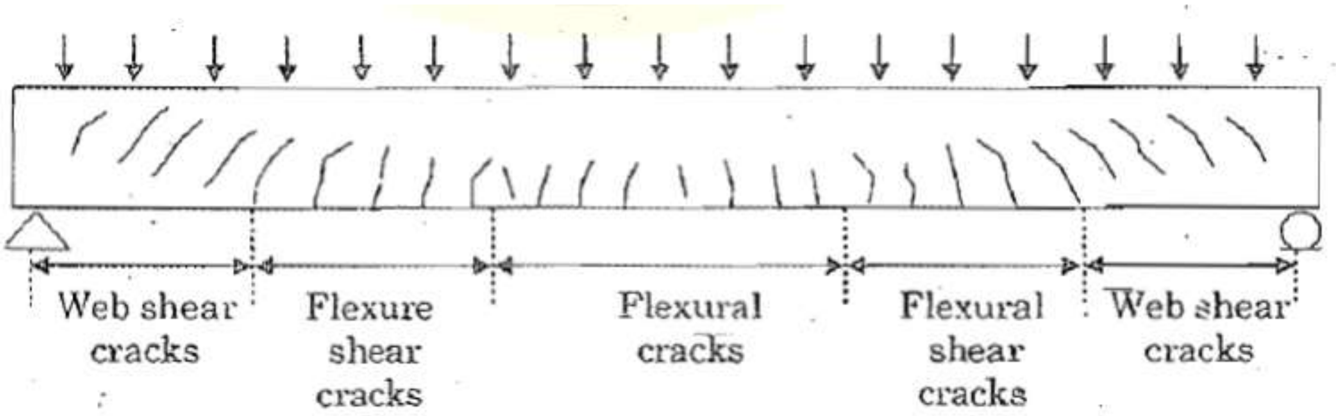
Two types of inclined cracking occur in concrete beams

• Web Shear Cracking

• Flexure – Shear Cracking

- Web Shear cracking starts from near NA location when the PTS due to shear exceed the tensile strength of concrete. Usually occur in I beams (thin webs)

• Flexure – Shear cracking is an extension of vertical flexural cracking and develops due to combined shear and flexural tensile stresses exceed the tensile strength of concrete.



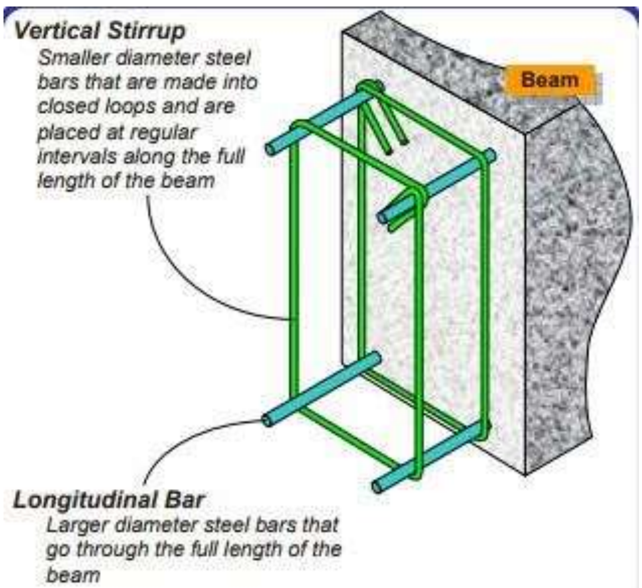
Types of Shear Reinforcement

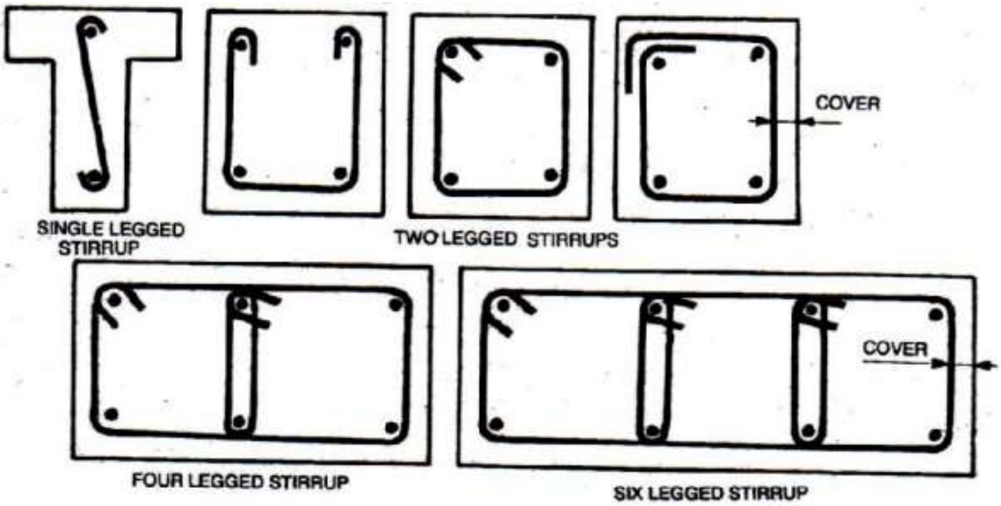
The following three types of shear reinforcement are used

- Vertical stirrups
- Bent up bars along with stirrups.
- Inclined stirrups.

Vertical stirrups

- These are the steel bars vertically placed around the tensile reinforcement at suitable spacing along the length of the beam. Their diameter varies from 6mm to 16mm.
- The free ends of the stirrups are anchored in the compression zone of the beam to the anchor bars (hanger bar) or the compressive reinforcement.
- The spacing of stirrups near the supports is less as compared to spacing near the mid span since shear force is maximum at the supports.





Type of vertical stirrups:

- Single Legged Stirrup
- Two Legged Stirrup
- Four Legged Stirrup
- Six Legged Stirrup

Bent up Bars along with Vertical Stirrups

- These bent up bars resist diagonal tension. These bars are usually bent at 45°.

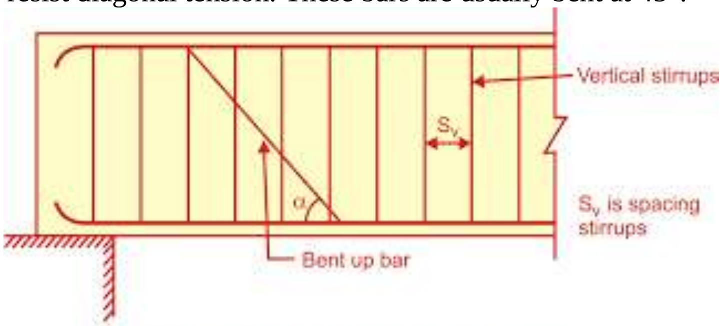


Fig. 5.6. Bent up bars alongwith stirrups.

Inclined Stirrups

- Inclined stirrups are also provided generally at 45° for resisting diagonal tension. They are provided throughout the length of the beam.

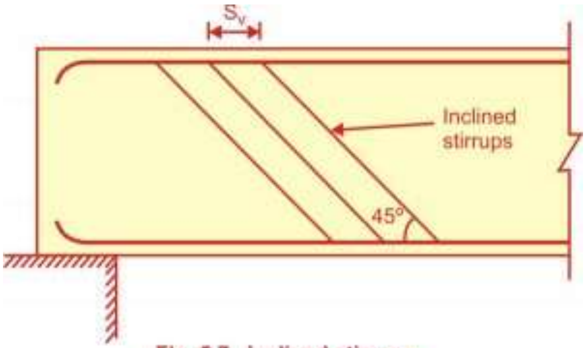


Fig. 5.7. Inclined stirrups.

Learning Objectives

2.2 Design procedure on shear reinforcements (SR) as per IS code

Nominal Shear Stress

The nominal shear stress in beams of uniform depth shall be obtained by the following equation:

$$\tau_v = \frac{V_u}{bd}$$

Where,

V_u=Shear force due to design loads

b=breadth of the member, which for flanged section shall be taken as the breadth of the web, b_w; and

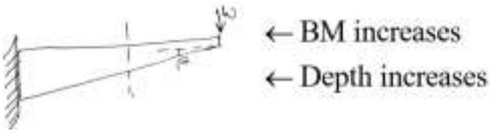
d=effective depth

• For a rectangular concrete beam the shear stress varies parabolically upto neutral axis and after remains constant.

Beams of Varying Depth

In the case of beams of varying depth the equation shall be modified as:

$$\tau_v = \frac{V_u \pm \frac{M_u}{d}\tan\beta}{bd}$$



Where τ_v, V_u, d and b are the same as in Nominal Shear stress M_u=bending moment at the section, and

β=angle between the top and the bottom edges. The negative sign in the formula applies when the bending moment M_u increases numerically in the same direction as the effective depth d increases, and the positive sign when the moment decreases numerically in this direction.

Design Shear Strength of Concrete

The design shear strength of concrete in beams without shear reinforcement is given in Table

For solid slabs, the design shear strength for concrete shall be $\tau_c.k$, where k has the values given below:

Overall Depth of Slab, mm	300 or more	275	250	225	200	175	150 or less
k	1.00	1.05	1.10	1.15	1.20	1.25	1.30

Shear Strength of Members under Axial Compression

For members subjected to axial compression P_u, the design shear strength of concrete, given in Table 19, shall be multiplied by the following factor:

$$\delta = 1 + \frac{3P_u}{A_g f_{ck}} \text{ but not exceeding } 1.5$$

Where

P_u = axial compressive force in Newtons,

A_g = gross area of the concrete section in mm

f_{ck} = characteristic compressive strength of concrete

If $\tau_v < 0.5\tau_c \Rightarrow$ No shear reinforcement

If $0.5\tau_c < \tau_v < \tau_c \Rightarrow$ Minimum shear reinforcement

If $\tau_c < \tau_v < \tau_{c,max} \Rightarrow$ Design shear reinforcement

If $\tau_v > \tau_{c,max} \Rightarrow$ Redesign the section

Design of Shear Reinforcement

When τ_v exceeds τ_c , shear reinforcement shall be provided in any of the following forms:

a. Vertical stirrups,

b. Bent-up bars along with stirrups, and

c. Inclined stirrups

Where bent-up bars are provided, their contribution towards shear resistance shall not be more than half that of the total shear reinforcement.

Shear strength of concrete, $V_c = \tau_c bd$

Shear reinforcement shall be provided to carry a shear, $V_{us} = V_u - V_c$

The strength of shear reinforcement V_{us} shall be calculated as below: a. For vertical stirrups:

$$V_{us} = \frac{0.87 f_y A_{sv} d}{s_v}$$

b. For inclined stirrups or a series of bars bent-up at different cross-sections:

$$V_{us} = \frac{0.87 f_y A_{sv} d}{s_v}(\sin\alpha + \cos\alpha)$$

c. For single bar or single group of parallel bars, all bent-up at the same cross-section:

$$V_{us} = 0.87 f_y A_{sv} d \sin\alpha$$

Where

A_{sv}=total cross-sectional area of stirrup legs or bent-up bars within a distance

s_v=spacing of the stirrups or bent-up bars along the length of the member

τ_v=nominal shear stress

τ_c=design shear strength of the concrete

b=breadth of the member or the breadth of the web b_w

f_y=characteristic strength of the stirrup or bent-up reinforcement which shall not be greater than 415 N/mm^2

α = angle between the inclined stirrup or bent- up bar and the axis of the member, not less than 45°, and

d = effective depth

Maximum spacing of shear reinforcement

The maximum spacing of shear reinforcement measured along the axis of the member shall

not exceed *0.75d* for vertical stirrups

d for inclined stirrups at 45^0

where *d* is the effective depth of the section under consideration.

In no case shall the spacing exceed 300 mm.

Minimum shear reinforcement

Minimum shear reinforcement in the form of stirrups shall be provided such that:

$$\frac{A_{sv}}{bs_v} \geq \frac{0.4}{0.87 f_y}$$

where

A_{sv}=total cross-sectional area of stirrup legs effective in shear,

s_v=stirrup spacing along the length of the member,

b=breadth of the beam or breadth of the web of flanged beam, and

f_y=characteristic strength of the stirrup reinforcement in N/mm^2 which shall not be taken greater than 415 N/mm^2

Equivalent Shear

Equivalent shear,shall be calculated from the formula:

$$V_e = V_u + 1.6\frac{T_u}{b}$$

Where

V_e = equivalent shear,

V_u = shear,

T_u = torsional moment, and

b = breadth of beam.

The equivalent nominal shear stress,

$$\tau_{ve} = \frac{V_e}{bd}$$

The values τ_{ve} of shall not exceed the values of $\tau_{c,max}$

MODULE-1 **Chapter - 2**
Session - 12

Learning Objectives
2.3 Problems on singly reinforced beam.

Problem-3

A reinforced concrete rectangular beam of size 300mm×500mm effective is reinforced with 1500mm^2 on tension face and 500mm^2 on compression face. The characteristic strength of concrete and steel are 25MPa and 500MPa respectively. The shear strength of concrete with the percentage of steel reinforcement is shown below

P%	τ_c, N/mm2
0.75	0.57
1.00	0.65
1.25	0.70

It is subjected to a shear force of 200kN at working loads.

a. The shear resisted by concrete is ………

$$p = \frac{100A_{st}}{bd} = \frac{100 \times 1500}{300 \times 500} = 1\%$$

For $p = 1\%, \tau_c = 0.65\text{N/mm}^2$

$$V_c = \tau_c.bd = 0.65 \times 300 \times 500 = 97.5\text{kN}$$

b. The shear to be resisted by stirrups is ………

$$V_u = 1.5V = 1.5 \times 200 = 300\text{kN}$$

V_{us} : shear to be resisted by stirrups

$$= V_u - V_c = 300 - 97.5 = 202.5\text{kN}$$

c. Spacing of 8 mm diameter 2 legged vertical stirrups is ....

τ_v : Nominal shear stress

$$= \frac{V_u}{bd} = \frac{300\times10^3}{300\times500} = 2\text{N/mm}^2$$

For M25 grade concrete $\tau_{c,\ max}$=3.1N/mm^2

$\tau_c < \tau_v < \tau_{c,\max} \Rightarrow$ provide design shear reinforcement

$$V_{us} = \frac{0.87 f_y . A_{sv} .d}{s_v} \Rightarrow 202.5\times10^3 = \frac{0.87\times500\times2\times50\times500}{s_v} \Rightarrow s_v = 107.4\text{mm}$$

Problem-4

A reinforced concrete rectangular beam of size 300mm×500mm effective is reinforced with 1500mm^2 on tension face and 500mm^2 on compression face. The characteristic strength of concrete and steel are 25MPa and 500MPa respectively. The shear strength of concrete with the percentage of steel reinforcement is shown below

P%	τ_c, N/mm2
0.75	0.57
1.00	0.65
1.25	0.70

It is subjected to a shear force of 50kN at working loads.

a. spacing of 8mm diameter 2 legged stirrups is..........

$$\tau_v = \frac{V_u}{bd} = \frac{1.5\times50\times10^3}{300\times500} = 0.5\text{N/mm}^2$$

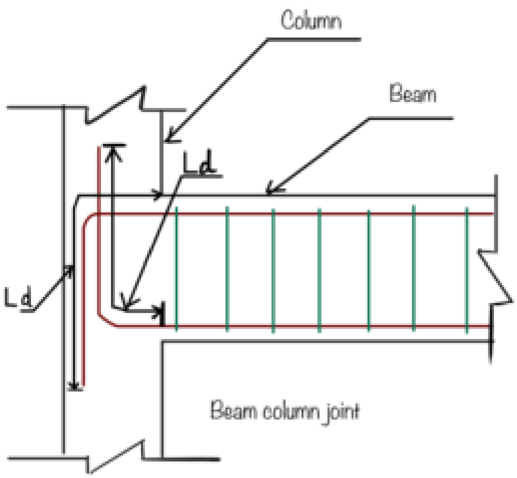
$0.5\tau_c < \tau_v < \tau_c \Rightarrow$ provide minimum shear reinforcement

$$\frac{A_{sv}}{b.s_v} \geq \frac{0.4}{0.87 f_y} \Rightarrow \frac{2\times50}{300\times s_v} \geq \frac{0.4}{0.87\times500} \Rightarrow s_v \leq 362.5\text{mm}$$

Learning Objectives

2.4 Bond and development length (DL)

The development length may be defined as the length of embedment necessary to develop the full tensile strength of the reinforcement, controlled by either pull-out or splitting. In other words, a certain minimum length of the bar, called the development length, has to be provided on either side of a point of maximum steel stress to prevent the bar from pulling out under tension.



Importance of L_d are as follows:

- **To prevent premature bond failure:** No premature bond failure will occur if the actual length L is equal to or greater than Ld.
- **To promote bending failure:** The beam is supposed to fail in bending or shear failure rather than bond failure. We promote bending failure in beams because it is a more predictable and controllable failure mode compared to other types of failure.
- **To control the failure mechanism:** In the case of the bond, we are considering the overall mechanism of failure rather than the limiting stresses to govern the design.

According to Clause 26.2.1 of the Indian code, the calculated tension or compression in any bar at any section shall be developed at each side of the section by an appropriate development length, given by:

Ld=dbfs4τbd

where,

- $\mathbf{d_b}$ is the nominal diameter of the bar,
- $\mathbf{f_s}$ is the stress in the bar at the section considered at design load (for fully stressed bar, $f_s = 0.87\ f_y$)
- $\boldsymbol{\tau_{bd}}$ is the design bond stress as per Table below.

Grade of concrete	M20	M25	M30	M35	M40 and above
Design bond stress , MPa	1.2	1.4	1.5	1.6	1.9
For fully deformed bars	1.18	1.37	1.54	1.71	1.87

Note : As per IS :456, one has to make the following changes in design bond stress value

- **For deformed bar in tension:** τbd values can be increased by 60 %
- **For bars in compression:** τbd values can be increased by 25 %
- **For nominal reinforcement:** τbd is taken as 1.0 MPa

Ld = 0.136dbfy / τbd

MODULE-1

Chapter - 2
Session - 14

Learning Objectives

2.5 Bond and development length (DL) Problems

Determine the anchorage length of 4-20T reinforcing bars going into the support of the simply supported beam shown in Fig. 6.15.5. The factored shear force Vu = 280 kN, width of the column support = 300 mm. Use M 20 concrete and Fe 415 steel.

τ_{bd} for M 20 and Fe 415 (with 60% increased) = 1.6(1.2) = 1.92 N/mm^2

$$L_d = \frac{\phi\,\sigma_s}{4\,\tau_{bd}} = \frac{0.87(415)\phi}{4(1.92)} \quad (\text{when } \sigma_s = 0.87 f_y) = 47.01\phi$$

$$(L_d)_{\text{when } \sigma_s = f_d} \leq \frac{M_1}{V} + L_o$$

Here, to find M_1, we need x_u

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck}\, b} = \frac{0.87(415)(1256)}{0.36(20)(300)} = 209.94 \text{ mm}$$

$$x_{u,max} = 0.48(500) = 240 \text{ mm}$$

Since $x_u < x_{u,max}$; $M_1 = 0.87\, f_y\, A_{st}\, (d - 0.42\, x_u)$

or $M_1 = 0.87(415)\,(1256)\,\{500 - 0.42(209.94)\} = 187.754$ kNm

and $V = 280$ kN

We have from Eq.6.13 above, with the stipulation of 30 per cent increase assuming that the reinforcing bars are confined by a compressive reaction:

$$L_d \le 1.3\,(\frac{M_1}{V}) + L_o$$

$$47.01\phi \le 1.3\,(\frac{M_1}{V}) + L_o$$

or $47.01\phi \le 1.3\,\{\frac{187.754(10^6)}{280(10^3)}\}$; if L_o is assumed as zero.

or $\phi \le 18.54$ mm

Therefore, 20 mm diameter bar does not allow $L_o = 0$.

Determination of L_o:

$$1.3\,(\frac{M_1}{V}) + L_o \ge 47.01\phi$$

Minimum $L_o = 47.01\phi - 1.3\,(\frac{M_1}{V}) = 47.01(20) - 1.3(\frac{187754}{280}) = 68.485$ mm

Learning Objectives
2.6 Design for Torsion Problems

Reinforced concrete sections are also subjected to torsional moments which cause twisting or warping of the section. Some common examples of structures subjected to torsional moments are as follows:

- Ring beam provided at the bottom of the elevated circular water tank.
- L-Beams.
- Beams supporting a cantilever slab.
- Beams curved in plan.

On the basis of simple elastic theory, the shearing stresses induced in a circular shaft subjected to a torsional moment (or torque) is given by the following equation:

$$\frac{T}{J} = \frac{G\theta}{l} = \frac{\tau_{max}}{R}$$

where T is the torsional moment or torque applied
J is the polar moment of inertia
G is the shear modulus or modulus of ridigity
θ is the angle of twist
l is the length of shaft
τ_{max} is the maximum shear stress developed
R is the radius of the shaft

The term GJ is called the torsional rigidity.

As per IS code, the design for torsion is based on the calculation of an equivalent shear force and an equivalent bending moment. The effect of torsional moment (in the form of additional shear force and additional bending moment) is added to the actual bending moment and shear force to get equivalent bending moment and equivalent shear force. But as per IS code, this philosophy gives adequate result only if the torsional moment is not appreciable as compared to the bending moment and shear force values. Cases of RCC sections subjected to pure torsion are very rare and torsion is generally secondary to flexure in RCC beams and hence IS code method for design for torsion can be adequately used for RCC beams. It is easy and simple as the design for torsion is not done separately rather the section is designed for an equivalent bending moment and equivalent shear force and the torsional reinforcement thus calculated, is distributed as per the codal provisions. The equivalent bending moment and equivalent shear force are calculated as follows:

$$M_e = M_u + M_T$$

$$\boxed{M_T = T_u \cdot \frac{1 + D/b}{1.7}}$$

where, M_e is the equivalent bending moment

M_u is the ultimate bending moment

M_T is the additional moment which includes the effect of torsional moment (T_u)

T_u is the torsional moment or torque applied

D is the total depth of beam

b is the breadth of the section

Similarly equivalent shear force (V_e) is given by:

$$V_e = V_u + V_T$$

where V_T is the additional shear force which is caused due to torsional moment T_u

$$\boxed{V_T = 1.6\frac{T_u}{b}}$$

Design Procedure

Design of RCC beams subjected to bending moment, shear force and torsional moment is based upon the equivalent bending moment and equivalent shear force calculated as given above. IS code gives the recommendations for design of such beams which are explained below:

Equivalent Shear Force (V_e)

Design for equivalent shear force is done exactly in the same way as explained in Chapter 5 for flexural shear. Equivalent shear force is calculated as per Clause 41.3 of IS 456.

$$V_e = V_u + \frac{1.6T_u}{b}$$

Equivalent Nominal Shear Stress (τ_{ve})

$$\tau_{ve} = \frac{V_e}{bd}$$

The value of τ_{ve} should be less than $\tau_{c\,max}$

If $\tau_{ve} > \tau_{c\,max}$ then the section is to be redesigned.

- The shear strength of concrete (τ_c) without reinforcement is calculated from Table 5.1 and compare with τ_{ve}.
- If $\tau_{ve} < \tau_c$, then nominal shear reinforcement is provided in the form of stirrups such that

$$\frac{A_{sv}}{b.S_v} \geq \frac{0.4}{0.87f_y}$$

$$\Rightarrow \qquad S_v \leq \frac{0.87f_y.A_{sv}}{0.4b}$$

where A_{sv} is the cross sectional area of stirrup legs

f_y is the characteristic strength of shear reinforcement

b is the breadth of beam

S_v is the spacing of stirrups

- If $\tau_{ve} > \tau_c$ *i.e.*, the equivalent nominal shear stress is greater than the shear strength of concrete, then torsional reinforcement in the form of longitudinal and transverse reinforcement is provided as explained below:

Torsional Reinforcement

The torsional moment induce diagonal tension in the beam and it is seen that such beams show spiral cracking along the periphery of the section. Therefore, torsional reinforcement is provided in the form of longitudinal bars and transverse reinforcement in the form of links or hoops.

(i) Longitudinal reinforcement

As per IS code 456:2000 the longitudinal reinforcement is designed to resist an equivalent bending moment M_e, which includes the effect of torsional bending moment

$$M_e = M_u + M_T$$

$$M_T = \frac{T_u(1 + D/b)}{1.7}$$

(*a*) If $M_T < M_u$, then longitudinal steel required to resist M_e is calculated and distributed as per codal provisions.

(*b*) If $M_T > M_u$, then IS code recommends that the longitudinal reinforcement should also be provided on the compression face of the beam for moment M'_e where

$$M'_e = M_T - M_u$$

In this case, the effect of torsional moment is predominant and more than the ultimate bending moment and is assumed to act opposite to M_u.

(*c*) **Distribution of Longitudinal Reinforcement:** The longitudinal reinforcement to resist torsion is provided as per the codal provisions given in Clause 26.5.1.7 of IS 456:2000.

(*i*) the longitudinal reinforcement should be placed near to the periphery or corners of the beam section.

(*ii*) At least one longitudinal bar should be placed in each corner of the section.

(*iii*) If the depth of the section exceeds 450 mm, additional reinforcement in the form of bars should be provided along the two side faces of the section, the area of side face reinforcement should not be less than 0.1% of the web area. This area should be distributed equally on both the side faces at a spacing not less than 300 mm or breadth of the section.

(*iv*) In T and L beams, where the flanges are in tension, the torsional longitudinal reinforcement must be distributed over the effective flange width or a width equal to one-tenth of the span, whichever is less. If the effective flange width exceeds one tenth of the span, minimum longitudinal reinforcement must be provided in the outer portion of the flange.

(ii) Transverse Reinforcement

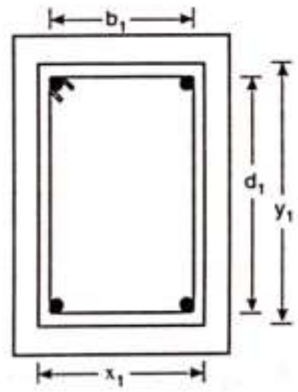


Fig. 9.1

The transverse reinforcement is provided in the form of closed links or hoops which enclose the longitudinal bars provided along the corners or periphery of the beam section. The area of two legged closed hoops is given by:

$$0.87 f_y . \frac{A_{SV}}{x} = \frac{T_u}{bd_1} + \frac{V_u}{2.5 d_1}$$

but $$0.87 f_y . \frac{A_{SV}}{x} \not< (\tau_{ve} - \tau_c) b$$

where x is the spacing of shear stirrups

b_1 and d_1 is the centre to centre distance between corner bars in the direction of width and depth respectively as shown in Fig. 9.1.

Distribution of Transverse Reinforcement (Clause 26.5.1.7 of IS 456)

The transverse reinforcement for torsion is distributed as per the codal provisions explained below:

(*i*) The closed links or stirrups are placed perpendicular to the longitudinal reinforcement and th spacing (x) of such stirrups should not be more than the following:

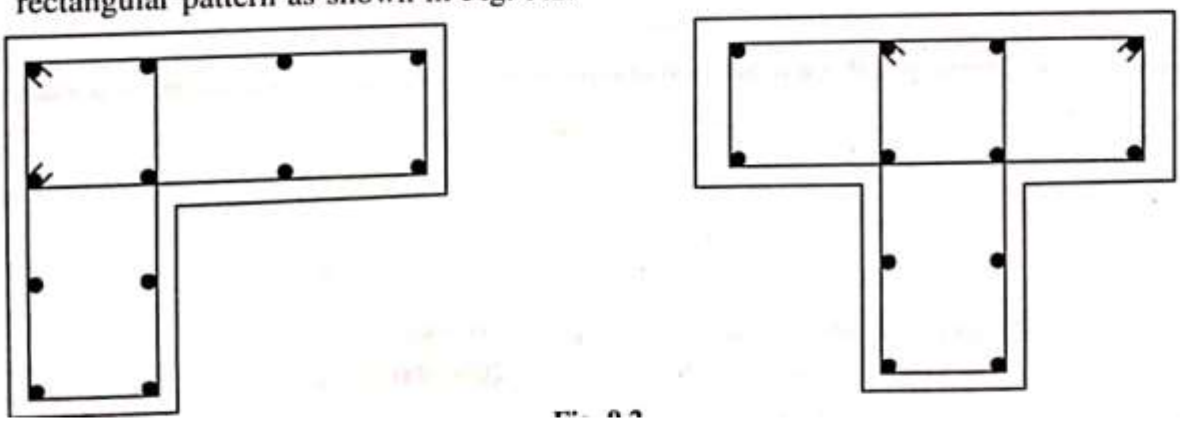
(*a*) short dimension of the stirrup

(*b*) $\frac{x_1 + y_1}{4}$ where x_1 is the short and y_1 is the long dimension of the stirrup

(*c*) 300 mm

(*ii*) In T and L beams, if the main reinforcement is parallel to the beam, transverse reinforcemen should be provided in the flange. Such reinforcement should not be less than 60% of the mai reinforcement at mid span of the slab.

(*iii*) In T and L beams, the closed links or stirrups should interlock and tie the components in rectangular pattern as shown in Fig. 9.2.



BPUT QUESTIONS

Possible Short Type Questions (2 Marks)

1. **Draw idealized stress-strain curve of Fe415 steel.**

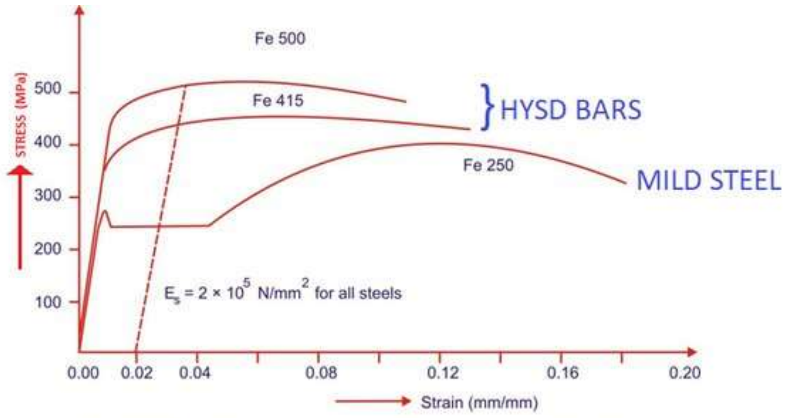
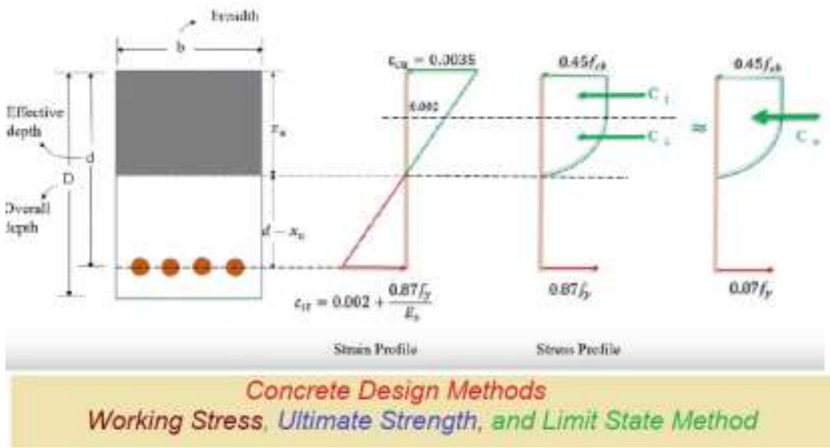


Fig. 1.1 Typical stress strain curves for various types of steel.

2. **List various types of loads acting on a structure as per IS456.**
   1. Dead Load IS 875 (Part 1)1987
   2. Live Load IS 875 (Part 2)1987
   3. Wind Load IS 875 (Part 3)1987
   4. Snow Load IS 875 (Part 4)1987
   5. Earthquake Load IS 1893 2002
3. **State relationship between characteristic compressive strength and flexural strength as per IS456.**
   The theoretical maximum flexural tensile stress occurring in the extreme fibres of RC beam, which causes cracking is referred to as the modulus of rupture (f_{Cr}) The clause 6.2.2 of IS 456 gives the modulus of rupture of flexural tensile strength as f c r = 0.7 √ f c
4. **Write partial safety factor of steel and concrete in the limit state of collapse as per IS456.**

   Partial safety factor of steel and concrete in the limit state of collapse as per IS456

5. **Define stress block.**
   The distribution of compressive stress across the depth of concrete in compression is referred as stress block. stress block may be assumed to be rectangle or parabolic which results in prediction of strength in substantial agreement with the results of test. A commonly used curve is that by various standards which consists of a parabolic for the initial ascending part followed by a horizontal straight line terminating at a prescribed ultimate strain, irrespective of the grade if concrete.



6. **Define anchorage length.**
   Anchorage length is the equivalent length of the reinforcement bar which is considered to be available when a straight bar is bent through some angle. It is provided only at the support. Anchorage length is provided if sufficient development length cannot be provided inside the support.

Possible Long Type Questions (16 Marks)

1. A rectangular beam of 200mm wide and 350mm deep up to the center of the reinforcement has to resist a factored moment of 40 kN-m. Design the section. Use M20 concrete and fe415 grade steel.
2. A rectangular reinforced concrete beam is simply supported on two masonry wall s 230 mm thick and 6 m apart (c/c). The beam is carrying an imposed load of 15 KN/m. Design the beam with all necessary checks. Use M25 concrete and Fe 415 steel.
3. Design a cantilever beam having an effective span of 3m. The beam is carrying a load of 14 KN/m, including its own weight. Use M_{20} concrete and Fe_{415} steel.
4. Design a R.C.C beam 30 cm wide to carry a load of 10 KN/m over a clear span of 5 m. Use M_{20} concrete and Fe_{415} steel.
5. Design an R.C.C simply supported beam of 5 m to carry a super imposed load of 16 KN/m. Use M_{20} Concrete Fe_{415} steel.
6. A simply supported beam R.C.C beam over an effective span of 8 m carrying an imposed load of 30 KN/m. Design the beam using M_{20} concrete and Fe_{415} steel. Show the sketch showing arrangement of reinforcement.
7. A T- Beam has flange dimension 1500 mm X 120 mm. The effective depth of beam is 430 mm and width of web is 250 mm. It is reinforced with 6-20 mm dia. Bars. Determine the ultimate load the beam can carry over a span of 6 m. Use M_{20} concrete and Fe_{415} Steel.
8. A simply supported beam of span 8 m of overall size 250 mm x 400 mm carries a factored moment of 280 kNm at centre. Check, whether the beam satisfies the deflection criteria as per IS codal provisions.

MODULE-2 **Chapter - 3**
Session - 17

Learning Objectives

3.1 Introduction to Doubly Reinforced beam section.

3.1.1 Necessity of Doubly Reinforced Section

3.1.2 Analysis of Doubly Reinforced Beam.

3.1.3 Depth of Neutral Axis(X_u)

3.1.4 Procedure for calculation Moment of Resistance

3.1 Introduction to Doubly Reinforced beam section.

The R.C.C. beams in which the steel reinforcement is placed in the tension as well as compression zone are called as doubly reinforced beams. The moment of resistance of a balanced R.C.C. beam of dimension $b \times d$ is Rbd^2. Sometimes due to head room constraints or architectural considerations the size of the beam is restricted and the same beam ($b \times d$) is required to resist a moment greater than Rbd^2. There are only two ways in which it can be done.

(*i*) By using an over reinforced section.

(*ii*) By using a doubly reinforced section.

The option (*i*) is not a good choice because over reinforced sections are uneconomical and the failure of these beams is sudden without warning. Therefore, it is better to use doubly reinforced beam section in such circumstances. The extra steel provided in the tension and compression zone constitutes, the additional moment of resistance (greater than Rbd^2) required. **This chapter includes the analysis and design of doubly reinforced beams by limit state.**

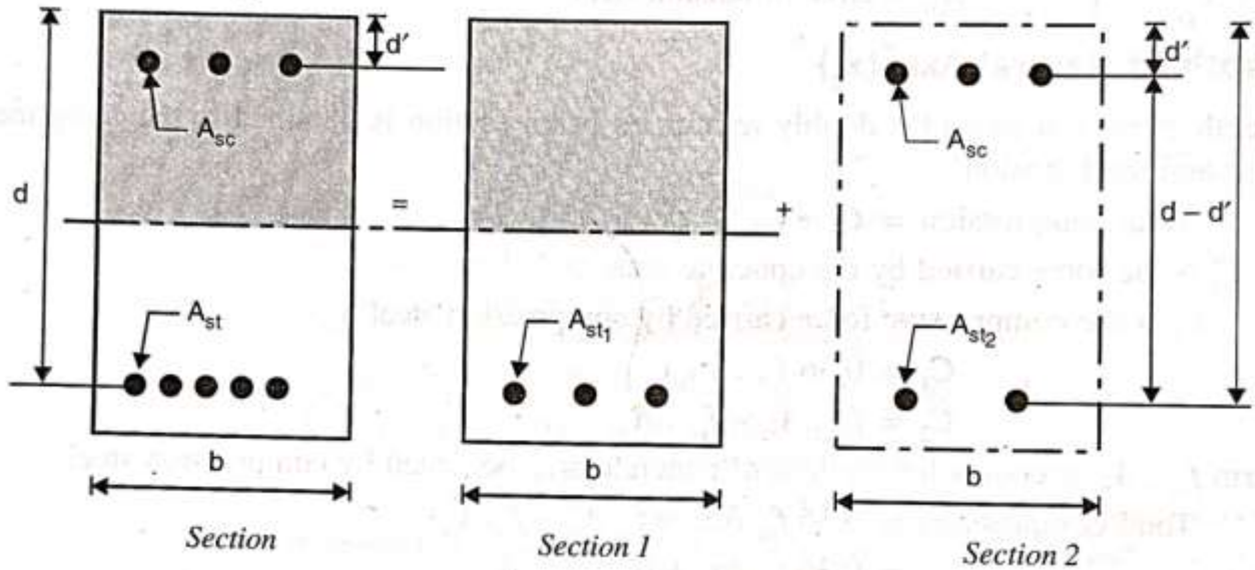
3.1.1 Necessity of Doubly Reinforced Section

Doubly reinforced sections are used in the following conditions:

1. When the dimensions ($b \times d$) of the beam are restricted due to any constraints like availability of head room, architectural or space considerations and the moment of resistance of singly reinforced section is less than the external moment.
2. When the external loads may occur on either face of the member *i.e.*, the loads are alternating or reversing and may cause tension on both faces of the member.
3. When the loads are eccentric.
4. When the beam is subjected to accidental or sudden lateral loads.
5. In the case of continuous beams or slab, the sections at supports are generally designed as doubly reinforced sections.

3.1.2 Analysis of Doubly Reinforced Beam.

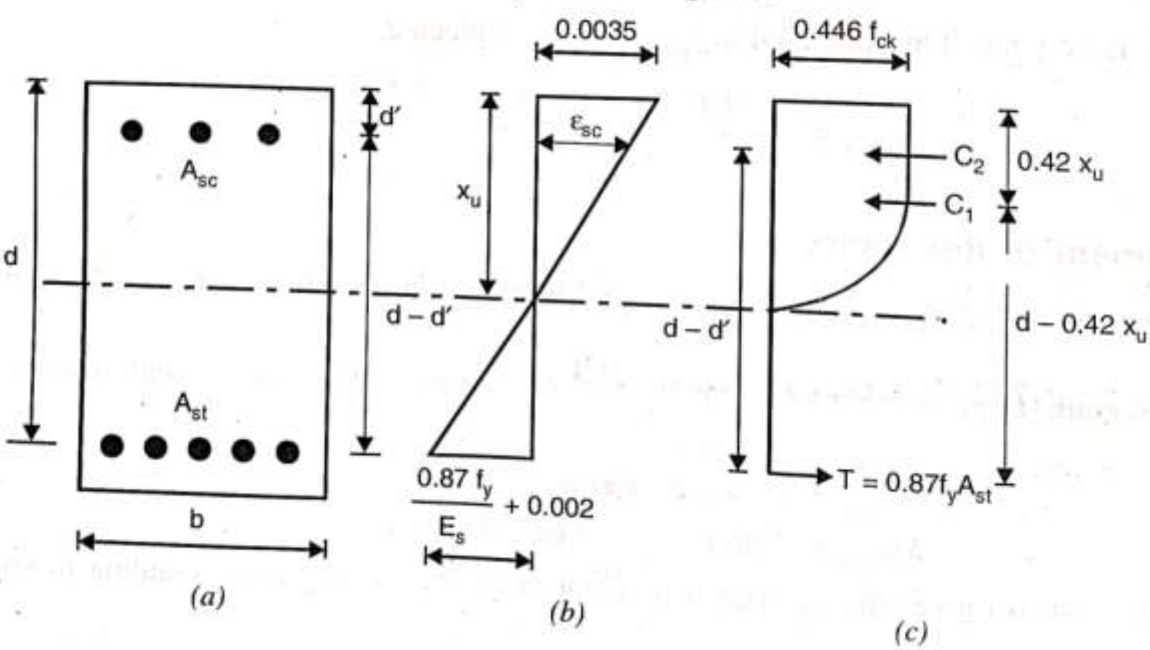
A doubly reinforced beam has moment of resistance greater than that of balanced section. Therefore a doubly reinforced beam subjected to a moment M_u can be analyzed by considering it consist of two section



Section 1: Section 1 consists of a singly reinforced balanced section having area of steel A_{st_1} and moment of resistance $M_{u\,lim}$.

Section 2: Section 2 consists of compression steel A_{sc} and additional tensile steel A_{st_2} corresponding to A_{sc}. The moment of resistance of this section is M_{u_2} such that

$$M_u = M_{u\,lim} + M_{u_2}$$



where b = Width of beam

x_u = Depth of neutral axis

d = Effective depth of beam

f_{sc} = Stress in compression steel

d' = Effective cover to compression steel

f_{cc} = Stress in concrete at the level of steel

A_{sc} = Area of compression steel

A_{st} = Area of tension steel

3.1.3 Depth of Neutral Axis(X_u)

The depth of neutral axis of a doubly reinforced beam section is obtained by equating the total compression and total tension.

$$\text{Total compression} = C_1 + C_2$$

where C_1 is the force carried by the concrete area

C_2 is the compressive force carried by compression steel A_{sc}

$$C_1 = 0.36 f_{ck} \cdot b \cdot x_u$$
$$C_2 = f_{sc} \cdot A_{sc} - f_{cc} \cdot A_{sc}$$

The term $f_{cc} \cdot A_{sc}$ accounts for the loss of concrete area occupied by compression steel.

$$\therefore \quad \text{Total compression} = 0.36 f_{ck}\, b\, x_u + f_{sc}\, A_{sc} - f_{cc}\, A_{sc}$$
$$= 0.36 f_{ck}\, b x_u + (f_{sc} - f_{cc})\, A_{sc}$$
$$\text{Total tension} = T$$
$$T = 0.87 f_y \cdot A_{st}$$

Equating total compression and total tension, we get

$$0.36 f_{ck}\, b\, x_u + (f_{sc} - f_{cc})\, A_{sc} = 0.87 f_y \cdot A_{st}$$
$$x_u = \frac{0.87 f_y A_{st} - (f_{sc} - f_{cc}) A_{sc}}{0.36 f_{ck} . b}$$

Since f_{cc} is very small as compared to f_{sc}, it can be neglected.

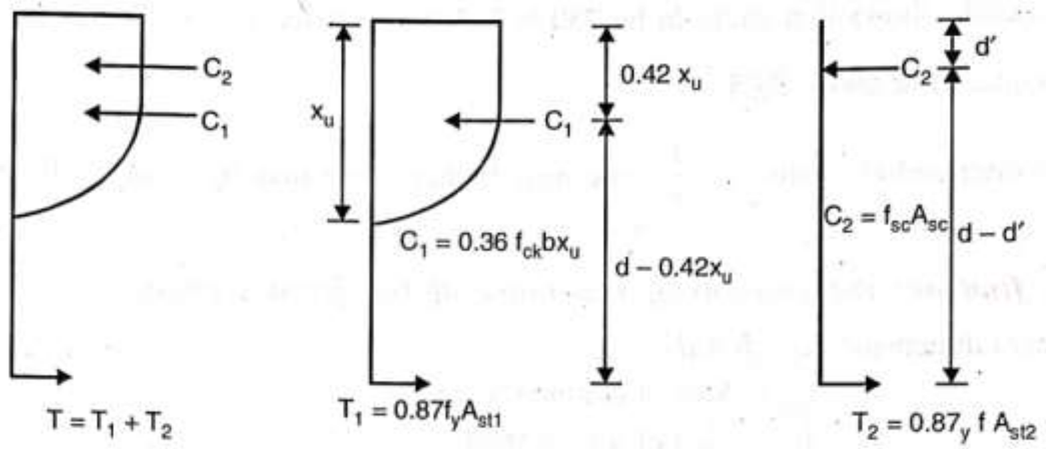
$$\therefore \quad x_u = \frac{0.87 f_y A_{st} - f_{sc} . A_{sc}}{0.36 f_{ck} . b}$$

3.1.4 Procedure for calculation Moment of Resistance

The moment of resistance of doubly reinforced beam is obtained from the stress diagram shown in figure

The limiting moment of resistance M_{ulim}

$$M_{u\,lim} = C_1 \times \text{Lever arm}$$
$$Mu_{lim} = 0.36 f_{ck} \cdot b \cdot x_u\, (d - 0.42\, x_u)$$



(a) Stress diagram *(b) Balanced section* *(c) Additional section having M_{u2}*

$$M_{u2} = C_2 \times \text{Lever arm}$$
$$= (f_{sc} A_{sc} - f_{cc} A_{sc})\ (d - d')$$
$$M_{u2} = (f_{sc} - f_{cc})\ A_{sc}\ (d - d') \quad [f_{cc} \cdot A_{sc} \text{ is due to loss of area of concrete}]$$
$$\boxed{M_u = M_{ulim} + M_{u2}}$$
$$M_u = 0.36 f_{ck}\, b\, x_u\ (d - 0.42\ x_u) + (f_{sc} - f_{cc})\ A_{sc}\ (d - d')$$

If loss of concrete area is neglected then

$$M_u = 0.36 f_{ck}\ b\ x_u\ (d - 0.42\ x_u) + f_{sc}\ A_{sc}\ (d - d')$$

The value of fsc depends upon the amount of strain in compression steel which is obtained from the strain diagram and d' value. If ε_{su} in the strain in concrete at the level of compression steel then

$$\frac{0.0035}{x_u} = \frac{\varepsilon_{sc}}{(x_u - d')} \qquad \text{[From similar triangles in strain diagram]}$$

$$\varepsilon_{sc} = 0.0035 \left(\frac{x_u - d'}{x_u} \right)$$

$$\varepsilon_{sc} = 0.0035 \left(1 - \frac{d'}{x_u} \right)$$

The corresponding values of stress f_{sc} in compression steel can be obtained from the table

Grade of Steel f_y (N/mm^2)	d'/d			
	0.05	0.10	0.15	0.20
250	217	217	217	217
415	355	353	342	329
500	424	412	395	370
550	458	441	419	380

The table clearly shows that stress in Fe 250 is 217 N/mm^2 [$0.87 f_y = 0.87 \times 250 = 217$] for all values of $\frac{d'}{d}$ equal to or above 0.05.

Note: For intermediate value of $\frac{d'}{d}$, the next higher value may be used for finding f_{sc} from Table

3.1.4 Procedure for calculation Moment of Resistance

Procedure to find out the moment of resistance of the given section.

Given: Beam dimensions *i.e.,* $b \times d$

A_{sc} = Area of compression steel

A_{st} = Area of tensile steel

Type of steel and grade of concrete

d' = Cover to compression steel

Procedure:

1. For the given of steel and $\frac{d'}{d}$ ratio, determine f_{sc} from Table 7.1.
2. Determine the depth of neutral axis (x_u)

$$x_u = \frac{0.87\, f_y \,.A_{st} - f_{sc} \,.A_{sc}}{0.36\, f_{ck} \,.b}$$

3. Determine $x_{u\,max}$ and type of beam by comparing x_u and $x_{u\,max}$.
4. The moment of resistance of the section is calculated as:

$$M_u = 0.36\, f_{ck}\; b.x_u\,(d - 0.42\, x_u) + (f_{sc} - f_{cc})A_{sc}\,(d - d')$$

Neglecting f_{cc}, since it is very small.

$$M_u = 0.36\, f_{ck}\; b.x_u\,(d - 0.42\, x_u) + f_{sc}\,.A_{sc}\,(d - d')$$

(*i*) If $x_u < x_{u\,max}$, under-reinforced section and M_u is calculated by above equation.

(*ii*) If $x_u > x_{u\,max}$, over-reinforced section and M_u is calculated by using $x_u = x_{u\,max}$ in the above equation.

Learning Objectives

3.2 Design of Doubly reinforced beams for flexure.

Design of doubly reinforced beam generally comprises of determining area of tension and compression steel as dimensions of the beam are already fixed (or restricted).

Determination of A_{st}

Area of steel corresponding to singly reinforced balanced section (section 1) = A_{st_1}

$$A_{st_1} = \frac{M_{u\lim}}{0.87 f_y (d - 0.42 x_{u\max})}$$

Area of steel corresponding to section 2 = A_{st_2}

Moment of resistance of section 2

$$M_{u_2} = M_u - M_{u\lim}$$

$$M_{u_2} = 0.87 f_y A_{st_2} (d - d') \quad \text{[Considering tensile steel]}$$

or

$$M_{u_2} = [f_s A_{sc} - f_{cc} A_{sc}](d - d') \quad \text{[Considering compression steel]}$$

$$A_{st_2} = \frac{M_{u2}}{0.87 f_y (d - d')}$$

$$A_{st} = A_{st_1} + A_{st_2}$$

Determination of Area of Compression Steel

$$M_{u_2} = (f_{sc} . A_{sc} - f_{cc} . A_{sc})(d - d')$$

Neglecting loss of concrete area

$$M_{u_2} = f_{sc} . A_{sc} (d - d')$$

$$\therefore \quad A_{sc} = \frac{M_{u2}}{f_{sc} (d - d')}$$

The various steps involved is the design of a doubly reinforced beam are as follows:

Given: Dimension of the beam *i.e.,* $b \times D$ or $b \times d$

Grade of concrete and type of steel

Factored bending moment (M_u)

1. Determine the value of f_{sc} for $\frac{d'}{d}$ ratio from Table 7.1.

2. Determine $x_{u\max}$ *i.e* limiting depth of neutral axis and $M_{u\lim}$

$$M_{u\lim} = 0.36 f_{ck} b (d - 0.42 x_{u\max})$$

3. Determine A_{st_1}

$$A_{st_1} = \frac{M_{u\lim}}{0.87 f_y (d - 0.42 x_{u\max})}$$

4. Determine M_{u_2} and A_{st_2}

$$M_{u2} = M_u - M_{u\,\text{lim}}$$

$$A_{st_2} = \frac{M_{u2}}{0.87\, f_y\,(d - d')}$$

5. Determine A_{st} $\quad A_{st} = A_{st_1} + A_{st_2}$, Choose suitable diameter of bar and provide them.

6. Determine area of compression steel (A_{sc})

$$M_{u_2} = M_u - M_{u\,\text{lim}}$$

$$M_{u_2} = 0.87\, f_y\, A_{st_2}\,(d - d')$$

or

$$M_{u_2} = [f_{s'} A_{sc} - f_{cc'}\, A_{sc}](d - d')$$

$$A_{st_2} = \frac{M_{u2}}{0.87\, f_y\,(d - d')}$$

$$A_{st} = A_{st_1} + A_{st_2}$$

7. Check for deflection control

For

$$p_t = \frac{100\, A_{st}}{bd} \text{ and } f_s = 0.58\, f_y \left[\frac{A_{st\,\text{provided}}}{A_{st\,\text{reqd.}}}\right], \text{ find } k_t,$$

$$p_c = \frac{100\, A_{sc}}{bd} \text{ find } k_c$$

$$\left(\frac{l}{d}\right)_{\max} = 20 \times k_t \times k_c \qquad \text{[For simply supported beam]}$$

$$= 7 \times k_t \times k_c \qquad \text{[For cantilever beam]}$$

If $\left(\frac{l}{d}\right)_{\max} > \left(\frac{l}{d}\right)_{\text{provided}}$ then OK.

If $\left(\frac{l}{d}\right)_{\max} < \left(\frac{l}{d}\right)_{\text{provided}}$ then redesign the section.

8. Check for shear is to be done in the same way as explained in chapter 5.

9. Check for development length by satisfying codal provision

$$\frac{M_1}{V_u} + l_0 > L_d$$

10. Design summary and a sketch showing reinforcement detailing.

<u>Problem</u>

An R.C.C beam 230 mm x 500 mm effective is subjected to a factored moment of 200 KNm. Find the reinforcement required. Use M_{20} Concrete and Fe_{415} Steel.

Solution. Given:

$$b = 230 \text{ mm}, \qquad d = 500 \text{ mm}$$

$$M_u = 200 \text{ kNm} = 200 \times 10^6 \text{ Nmm}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

■ $$\frac{d'}{d} = \frac{50}{500}$$

$$= 0.1 \qquad \text{[assuming 50 mm effective cover for compression steel]}$$

$$f_{sc} = 353 \text{ N/mm}^2$$

■ **Limiting moment of resistance ($M_{u\,lim}$)**

$$M_{u\,lim} = 0.36 f_{ck}\, b.\, x_{u\,max}(d - 0.42\, x_{u\,max})$$

$$x_u = 0.48\, d$$

$$= 0.48 \times 500 = 240 \text{ mm}$$

$\therefore$ $$M_{u\,lim} = 0.36 \times 20 \times 230 \times 240\,(500 - 0.42 \times 240)$$

$$= 158658048 \text{ Nmm}$$

■ $$M_{u_2} = M_u - M_{u\,lim}$$

$$= 200 \times 10^6 - 158658048$$

$$M_{u_2} = 41341952 \text{ Nmm}$$

■ **Area of tension steel (A_{st})**

$$A_{st} = A_{st_1} + A_{st_2}$$

$$A_{st_1} = \frac{M_{u\,lim}}{0.87 f_y\,(d - 0.42\, x_{u\,max})}$$

$$= \frac{158658048}{0.87 \times 415\,(500 - 0.42 \times 240)}$$

$$= 1100.7 \text{ mm}^2$$

$$A_{st_2} = \frac{M_{u_2}}{0.87 f_y (d - d')}$$

$$= \frac{41341952}{0.87 \times 415 (500 - 50)}$$

$$= 254.2 \text{ mm}^2$$

$$\text{Total area of tension steel} = A_{st_1} + A_{st_2}$$
$$= 1100.7 + 254.2 = 1354.9 \text{ mm}^2$$

$$\text{Area of one 20 mm bar} = \frac{\pi}{4} \times 20^2 = 314 \text{ mm}^2$$

$$\text{No. of bars reqd.} = \frac{1354.9}{314} = 4.3 \text{ say } 5$$

∴ **Provide 5–20 mm diameter bar as tension steel.**

Learning Objectives

3.2.2 Design of Doubly reinforced beams Problems

Problems

A rectangular beam has a width of 250 mm and effective depth of 500 mm. The beam is provided with tension steel of 5 bars of 28 mm diameter and compression steel of 2 bars of 25 mm diameter. The effective cover to the compression steel being 50 mm. Calculate the ultimate moment capacity of the section, if f_{ck} = 20 N/mm^2 and f_y= 250 N/mm^2

Solution. Given:

$$b = 250 \text{ mm}, d = 500 \text{ mm}$$

$$A_{st} = 5 \times \frac{\pi}{4} \times 28^2 = 3078.7 \text{ mm}^2$$

$$A_{sc} = 2 \times \frac{\pi}{4} \times 25^2 = 981.7 \text{ mm}^2$$

$$d' = 50 \text{ mm}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 250 \text{ N/mm}^2$$

- $$\frac{d'}{d} = \frac{50}{500} = 0.1$$

For $\frac{d'}{d} = 0.1$ and Fe 250

$$f_{sc} = 217 \text{ N/mm}^2$$

Note: For all values of $\frac{d'}{d}$, for Fe 250

$$f_{sc} = 0.87 f_y = 0.87 \times 250 = 217 \text{ N/mm}^2$$

- **Depth of neutral axis (x_u)**

$$x_u = \frac{0.87\, f_y \cdot A_{st} - f_{sc} \cdot A_{sc}}{0.36\, f_{ck} \cdot b}$$

$$= \frac{0.87 \times 250 \times 3078.7 - 217 \times 981.7}{0.36 \times 20 \times 250}$$

$$x_u = 253.66 \text{ mm}$$

- $$x_{u\,max} = 0.53\, d = 0.53 \times 500$$
  $$x_{u\,max} = 265 \text{ mm}$$

- $x_u < x_{u\,max}$, Hence, the section is under reinforced section

- **Moment of resistance (M_u)**

$$M_u = 0.36 f_{ck}\, b\, x_u\, (d - 0.42 x_u) + f_{sc}.A_{sc}\, (d - d')$$

$$= 0.36 \times 20 \times 250 \times 253.66\,(500 - 0.42 \times 253.66) + 217 \times 981.7(500 - 50)$$

$$M_u = 275.51 \times 10^6 \text{ Nmm}$$

$$M_u = 275.51 \text{ kNm}$$

Problem

Determine the moment of resistance of the beam having the following data.

b = 350 mm: d = 900 mm, d' = 50 mm

Tension reinforcement = 5–20 mm dia bars

Compression reinforcement = 2–20 mm dia bars.

Use M15 concrete and Fe 415 steel.

Solution. Given:

$$b = 350 \text{ mm},\ d = 900 \text{ mm}$$

$$A_{st} = 5 \times \frac{\pi}{4} \times 20^2 = 1571 \text{ mm}^2$$

$$A_{sc} = 2 \times \frac{\pi}{4} \times 20^2 = 628 \text{ mm}^2$$

$$d' = 50 \text{ mm}$$

$$f_{ck} = 15 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

- $$\frac{d'}{d} = \frac{50}{900} = 0.05$$

For $$\frac{d'}{d} = 0.05 \text{ and Fe} = 415$$

$$f_{sc} = 351 \text{ N/mm}^2$$

- **Depth of neutral axis (x_u):**

$$x_u = \frac{0.87\, f_y . A_{st} - f_{sc} . A_{sc}}{0.36\, f_{ck}\, b} = \frac{0.87 \times 415 \times 1571 - 351 \times 628}{0.36 \times 15 \times 350}$$

$$x_u = 183.48 \text{ mm}$$

- $x_{u\,max} = 0.48\, d = 0.48 \times 900 = 432$ mm
- $x_u < x_{u\,max}$ Hence, the section is under reinforced

- **Moment of resistance (M_u)**

$$M_u = 0.36 f_{ck}\, b\, x_u\, (d - 0.42\, x_u) + f_{sc}.A_{sc}\, (d - d')$$

$$= 0.36 \times 15 \times 350 \times 183.48\,(900 - 0.42 \times 183.48) + 351 \times 628\,(900 - 50)$$

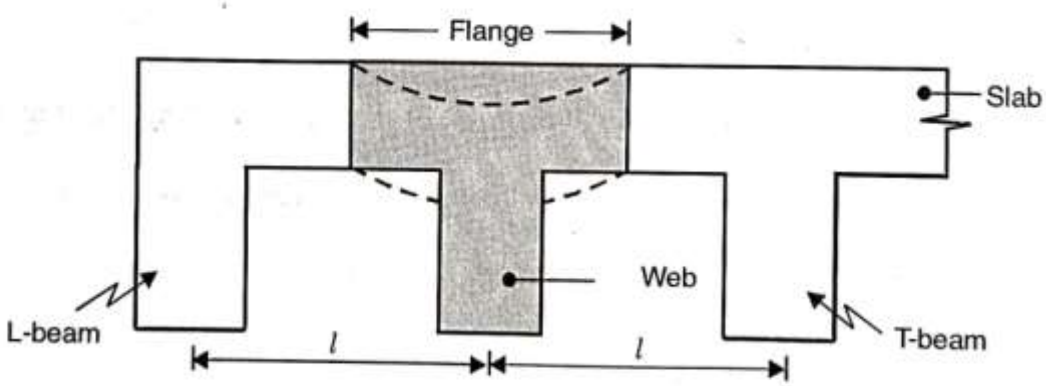
$$= 472740074 \text{ Nmm}$$

$$\boldsymbol{M_u = 472.74 \text{ kNm}}$$

Learning Objectives

4.1 Introduction on T-beams and L-beam

In RCC construction, slabs and beams are cast monolithically. In such construction. A portion of the slab act integrally with the beam and bends along with the beam under the loads. This phenomenon is seen in the beams supported slab system as shown in figure. The portion of the slab which acts integrally with the beam to resist loads is called as flange of the T-beam or L-Beam. The portion of the beam below the flange as web or Rib of the beam. The intermediate beams supporting the slab are called as T-Beam and the end beams called as T-beams and the end beams are called as L-Beams.



The flange of the beam (part of the slab) contributes in resisting compression by adding more area of concrete in compression zone. This results in increasing moment of resistance of the beam section.

However, if the flange is located in tension zone, the concrete area of the flange is to be neglected (cracked) and beam is treated as a rectangular beam.

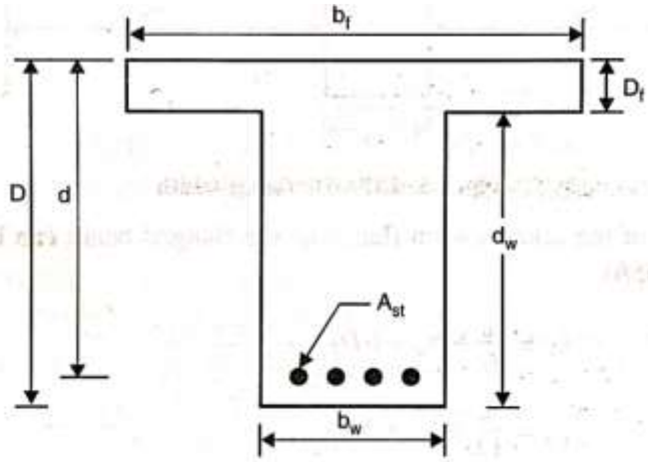
TERMS USED IN T-BEAMS

Breadth of Web (b_w)

Breadth of web is the width of the beam supporting the slab. It should be sufficient enough to accommodate the tensile reinforcement properly. The ratio of width of web to the depth of web is kept as $\frac{1}{3}$ to $\frac{2}{3}$.

Thickness of the Flange (D_f)

The thickness of flange of the T-beam is equal to the thickness or depth of the slab forming the flange of the beam.



Overall Depth of the Beam (D)

Overall depth of a flanged beam is equal to the sum of the depth of flange (D_f) and depth of the web (d_w). It is generally assumed as $\frac{1}{12}$ to $\frac{1}{15}$ of the span, in the case of simply supported beams.

For continuous beams, the overall depth is assumed as follows:

(i) For light loads: $\frac{1}{15}$ to $\frac{1}{20}$ of span

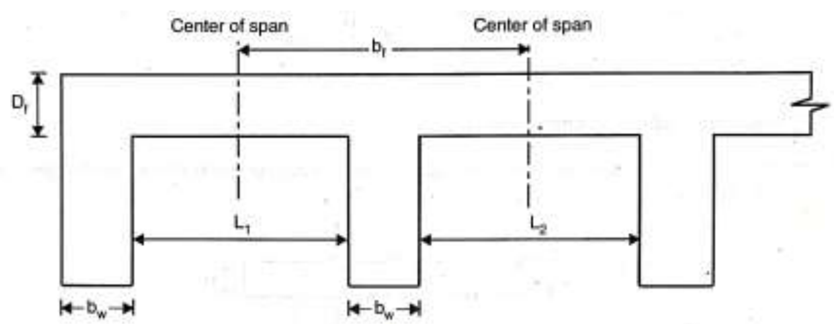
(ii) For medium loads: $\frac{1}{12}$ to $\frac{1}{15}$ of span

(iii) For heavy loads: $\frac{1}{10}$ to $\frac{1}{12}$ of span.

Effective Width of the Flange (b_f)

It is that portion of slab which acts integrally with the beam and extends on either side of the beam forming the compression zone. The effective width of flange mainly depends upon the span of beam, thickness of slab and the breadth of the web. It also depends upon the type of loads and support conditions.

The effective width of flange should not be greater than the breadth of web plus half the sum of clear distances to the adjacent beams on either side as shown in figure



The effective width of the compression flange of the flanged beam can be calculated as follows (Cl. 23.1.2 of Code IS 456).

(*a*) **For T-Beams:** $$b_f = \frac{l_0}{6} + b_w + 6D_f$$

(*b*) **For L-Beams:** $$b_f = \frac{l_0}{12} + b_w + 3D_f$$

(*c*) **For Isolated Beams:** The effective flange width shall be obtained as below but in no case greater than the actual width.

(*i*) T-Beam, $$b_f = \frac{l_0}{\left(\frac{l_0}{b} + 4\right)} + b_w$$

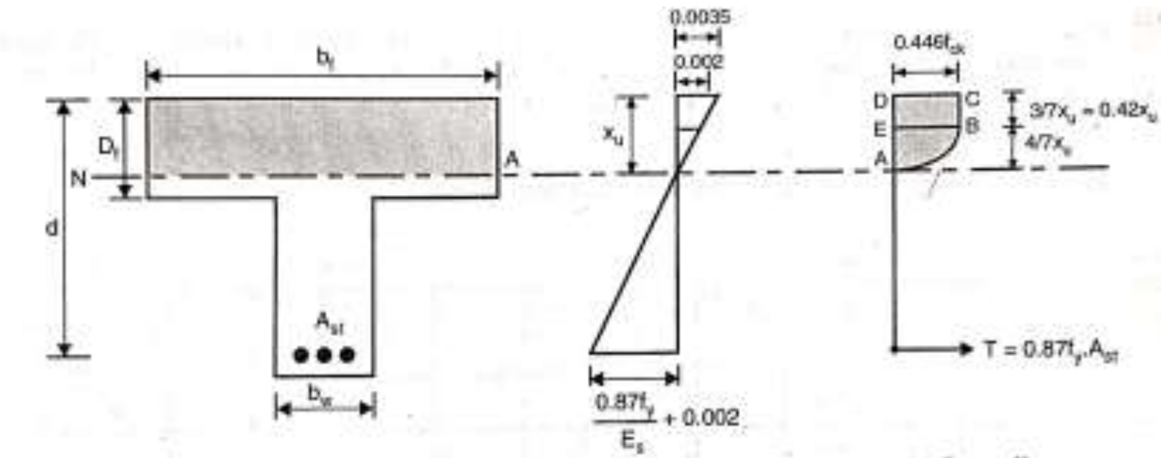
(*ii*) L-Beam, $$b_f = \frac{0.5\,l_0}{\frac{l_0}{b} + 4} + b_w$$

where
b_f = Effective width of flange
b_w = Breadth of web
D_f = Thickness of flange
b = Actual width of flange
l_0 = Distance between the points of zero moments (for continuous beams, l_0 is taken as 0.7 times the effective span)

Learning Objectives

4.2 *Analysis of T-beam and L-beams*

Consider a T-beam having flange width b_f, web width b_w and flange thickness D_f as shown in fig. Them is reinforced with area of steel A_{st} in the tension zone.



(*a*) Section (*b*) Strain diagram (*c*) Stress diagram

Stress Block

The stress block shown in Fig. 8.4 is similar to the one for the rectangular section.

- Maximum compressive stress = $0.446\,f_{ck}$
- Rectangular portion *DCBE* and parabolic portion *EBA*
- *EB* corresponds to strain 0.002 in concrete
- $DE = \frac{3}{7}x_u$ and $EA = \frac{4}{7}x_u$

Depth of Neutral Axis and Moment of Resistance

There are two cases which may arise:

(*i*) Neutral axis falls in the flange *i.e.*, $x_u < D_f$

(*ii*) Neutral axis falls in the web *i.e.*, $x_u > D_f$

Case I: $X_u < D_f$

Depth of Neutral Axis: If $x_u < D_f$ the section will behave as a rectangular section having width equal to b_f. Depth of neutral axis can be found from the equation used for rectangular section after replacing b for b_f.

Depth of neutral axis, $x_u = \dfrac{0.87f_y \,.\, A_{st}}{0.36f_{ck} \,.b_f}$

- **Moment of Resistance (M_u)**

Comparing x_u and $x_{u\,max}$ and determining the type of section as follows:

(*i*) $x_u < x_{u\,max}$, the section is underreinforced and the moment of resistance of the section is given by:

$$M_u = 0.87 f_y . A_{st} (d - 0.42 x_u)$$

(*ii*) $x_u = x_{u\,max}$, the section is balanced and moment of resistance can be calculated as

$$M_{u\,lim} = 0.36 f_{ck} . b_f \, x_{umax} (d - 0.42 x_{umax})$$

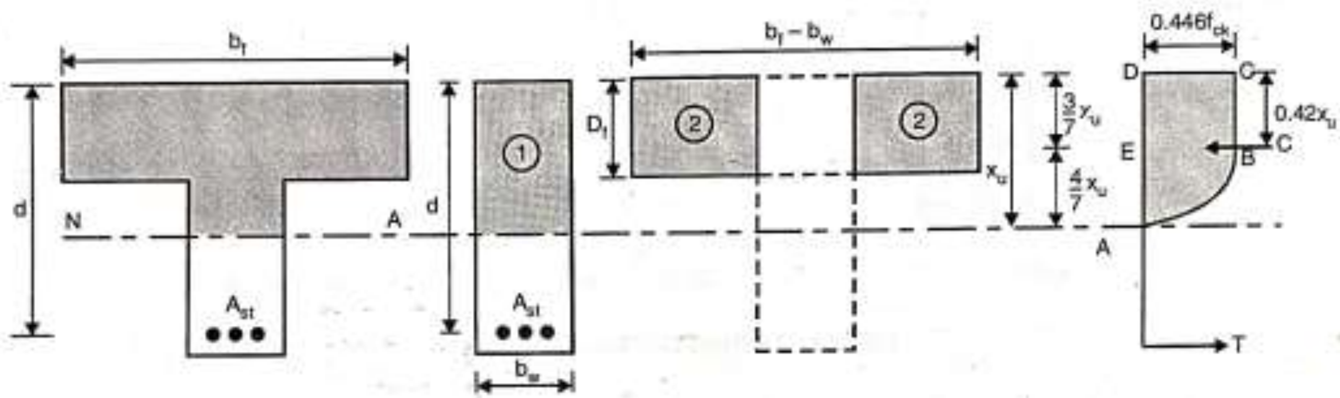
or

$$M_{u\,lim} = 0.87 f_y . A_{st} (d - 0.42 x_{umax})$$

(*iii*) $x_u > x_{max}$, the section is over reinforced and as per IS 456:2000, it should be redesigned (para G-2.1 of IS code)

Case II: $x_u > D_f$

For analysis of Case II, a T-beam section is idealized as shown in Fig.



The idealized T-beam section consists of:

(*i*) A singly reinforced rectangular section (1) of width b_w and effective depth d. The area of steel reinforcement is A_{st}.

(*ii*) A flange section (2) of width $(b_f - b_w)$ and depth D_f.

IS code further classifies this case $x_u > D_f$ *i.e.*, the neutral axis falls in the web, into following two types:

Case 1. $x_{umax} = x_u$ and $x_u > D_f$, balanced section

Case 2. $x_{umax} > x_u > D_f$, under-reinforced section

Case 1. $x_{u\,max} = x_u$ and $x_u > D_f$: Balanced Section

It is the case of balanced section, when x_u is greater than D_f. The moment of resistance of the section (limiting moment of resistance) will depend upon the value of $\frac{D_f}{d}$ ratio (Para G2.2, IS 456). It is because:

- If $D_f \leq \frac{3}{7} x_u$, the depth DE (rectangular portion of stress block) will be greater than D_f and the flange will lie completely in the rectangular portion of stress block.
- If $D_f > \frac{3}{7} x_u$ then $D_f >$ depth DE of the stress block. In this case, the stress distribution will not be rectangular in the flange but some part of parabolic portion will also be there.

As the value of $x_{u\,max}$ depends upon grade of steel. There may be many cases depending upon the type of steel.

The Indian Standard Code IS 456: 2000 has given a simplified approach for analysing such cases. It recommends a single value of $\frac{D_f}{d} = 0.2$*.

Base on the value of $\frac{D_f}{d}$ following cases may arise:

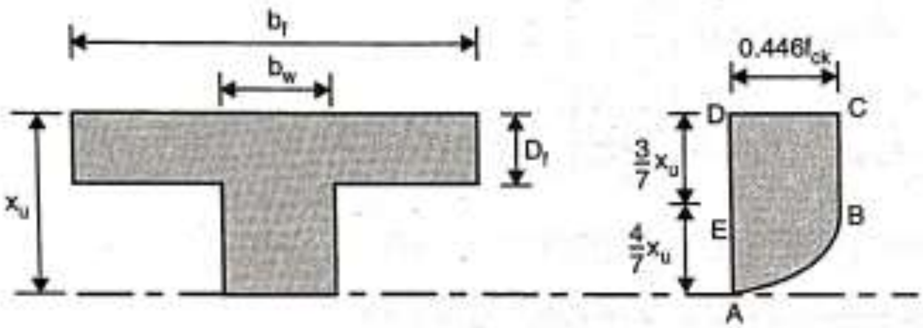
Case 1(*a*): $x_{umax} = x_u$, $x_u > D_f$ and $\frac{D_f}{d} \leq 0.2$

Case 1(*b*): $x_{umax} = x_u$, $x_u > D_f$ and $\frac{D_f}{d} > 0.2$

*[**Note:** For Fe 415, $\frac{3}{7} x_{umax} = \frac{3}{7} \times 0.48d = 0.2d$ = Depth of rectangular portion of stress block]

1. Case 1(*a*): $x_{u\,max} = x_u$, $x_u > D_f$ and $\frac{D_f}{d} \leq 0.2$ or $D_f < \frac{3}{7} x_u$

This case assume that the depth of rectangular portion of stress block is greater than D_f, as shown



$$\frac{D_f}{d} \le 0.2 \left(D_f < \frac{3}{7}x_u\right)$$

Total compression = Compresson in rectangular section $(b_w \times d)$

+ Compression in rectangular section of size $(b_f - b_w) \times D_f$

$= 0.36f_{ck}\, b_w \,.\, x_{u\,max} + 0.446f_{ck}\,(b_f - b_w) \times D_f$

Total tension $= 0.87f_y \,.\, A_{st}$

$$\text{Moment of resistance} = 0.36f_{ck}b_w x_{u\,max}(d - 0.42x_{u\,max}) + 0.446\, f_{ck}\,(b_f - b_w)\,D_f\left(d - \frac{D_f}{2}\right)$$

$$= 0.36\frac{x_{u\max}}{d}\left(1 - \frac{0.42\,x_{u\max}}{d}\right) f_{ck}\, b_w\,.d^2$$

$$+ 0.446\, f_{ck}\,(b_f - b_w)\,D_f \times \left(d - \frac{D_f}{2}\right) \quad ...(i)$$

As per IS code the limiting value or the moment of resistance of such section is given by para G2.2

$$M_u = 0.36\frac{x_{u\max}}{d}\left(1 - \frac{0.42\,x_{u\max}}{d}\right) f_{ck} b_w d^2 + 0.45\, f_{ck}\,(b_f - b_w)\,D_f \times \left(d - \frac{D_f}{2}\right)$$

which is same as the Eq. (*i*) above.

2. Case 1(*b*): $x_u = x_{u\max}$, $x_u > D_f$, $\frac{D_f}{d} > 0.2$ or $D_f > \frac{3}{7}x_u$

The moment of resistance of such sections is calculated by the equation given in *para G2.2.1 of IS code*

$$M_u = 0.36\frac{x_{u\max}}{d}\left(1 - \frac{0.42\,x_{u\max}}{d}\right) f_{ck} b_w d^2 + 0.45\, f_{ck}\,(b_f - b_w)\,Y_f\left(d - \frac{Y_f}{2}\right) \quad ...(ii)$$

where $Y_f = 0.15x_u + 0.65\, D_f$ but not greater than D_f.

The IS code simplified approach for this case recommends the concept of modified thickness of flange equal to Y_f. The average compressive stress over this depth Y_f is assumed to be uniform of intensity $0.446f_{ck} \simeq 0.45\, f_{ck}$.

Case 2: $x_{u\,max} > x_u > D_f$: Under-reinforced section

This is a case of under reinforced section when the neutral axis lies in the web. The moment of resistance of this section depends upon the value of $\frac{D_f}{x_u}$. It can also be divided into following two cases depending upon the value of $\frac{D_f}{x_u}$.

1. **Case 2(*a*): $x_{u\,max} > x_u > D_f$ and $\frac{D_f}{x_u} < 0.43$ $\left(\frac{3}{7} = 0.43\right)$**

$D_f < \frac{3}{7} x_u$. Hence, the entire flange lies in the rectangular stress block portion as shown in Fig. 8.6.

$$\text{Total compression} = 0.36 f_{ck} b_w \,.\, x_u + 0.446 f_{ck} (b_f - b_w) D_f$$

$$\simeq 0.36 f_{ck} b_w x_u + 0.45 f_{ck} (b_f - b_w) D_f$$

$$\text{Total tension} = 0.87 f_y \,.\, A_{st}$$

The depth of neutral axis of such section is calculated by equating total compression and total tension.

$$0.36 f_{ck} b_w \,.\, x_u + 0.45 f_{ck} (b_f - b_w) D_f = 0.87 f_y \,.\, A_{st} \quad ...(iii)$$

From this equation x_u can be calculated. The moment of resistance of such section is given by :

$$M_u = 0.36 f_{ck} b_w \,.\, x_u (d - 0.42 x_u) + 0.45 f_{ck} (b_f - b_w) D_f \left(d - \frac{D_f}{2}\right)$$

$$= 0.36 \frac{x_u}{d} \left(1 - \frac{0.42 x_u}{d}\right) f_{ck} b_w d^2 + 0.45 f_{ck} (b_f - b_w) D_f \left(d - \frac{D_f}{2}\right) \quad ...(iv)$$

which is same as eqn. (*i*) except that $x_{u\,max}$ is replaced by x_u.

2. **Case 2(*b*): $x_{u\,max} > x_u > D_f$, $\frac{D_f}{x_u} > 0.43$ or $D_f > \frac{3}{7} x_u$**

This case corresponds to the case when depth of rectangular portion of stress block is less than depth of flange.

The depth of neutral axis in this case is calculated as follows:

$$0.36 f_{ck}\, b_w \,.x_u + 0.45 f_{ck} \,.Y_f\,(b_f - b_w) = 0.87 f_y \,.A_{st} \quad \text{...(v)}$$

where $Y_f = 0.15 x_u + 0.65 D_f$ but not greater than D_f

The moment of resistance of such section is given by following equation

$$M_u = 0.36 f_{ck}\, b_w \,.x_u\,(d - 0.42 x_u) + 0.45 f_{ck} Y_f\,(b_f - b_w)\left(d - \frac{Y_f}{2}\right)$$

$$= 0.36 \frac{x_u}{d}\left(1 - 0.42 \frac{x_u}{d}\right) f_{ck}\, b_w d^2 + 0.45 f_{ck}\,(b_f - b_w) Y_f \left(d - \frac{Y_f}{2}\right) \quad \text{...(vi)}$$

which is same as Eq. (*ii*), except $x_{u\,max}$ is replace by x_u.

Para G2.3 of IS code also explains the same cases 2(a) and 2(b).

Over-reinforced Section: $x_u > x_{u\,max} > D_f$

When $x_u > x_{u\,max}$, the section is over reinforced. In such sections, the failure occurs by crushing of concrete and it is sudden. The moment of resistnace of such section is limited to $M_{u\,lim}$ (by taking $x_u = x_{u\,max}$). IS code recommends that such sections should be redesigned.

Learning Objectives
4.2 Problems on L-beams and T-beams

Problem: -

Find the moment of resistance of a T-beam having a web width of 240 mm, effective depth of 400 mm, flange width of 740 mm and flange thickness equal to 100 mm. The beam is reinforced with 5-16 mm diameter, Fe_{415} bars. Use M_{20} Concrete.

Solution. Given:

$b_w = 240$ mm, $d = 400$ mm

$b_f = 740$ mm $D_f = 100$ mm

$$A_{st} = 5 \times \frac{\pi}{4} \times 16^2 = 1005.3 \text{ mm}^2$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

- **Assuming the neutral axis to fall in the flange**

$$x_u = \frac{0.87 f_y . A_{st}}{0.36 f_{ck} b_f} = \frac{0.87 \times 415 \times 1005.3}{0.36 \times 20 \times 740}$$

$x_u = 68.1 \text{ mm} < D_f$ Hence N.A. lies in the flange.

$$x_{u\,max} = 0.48d = 0.48 \times 400 = 192 \text{ mm}$$

$x_u < x_{u\,max}$, hence the section is under-reinforced.

- **Moment of resistance (M_u)**

$$M_u = 0.87 f_y . A_{st} . d\left(1 - \frac{A_{st} . f_y}{b_f . d\, f_{ck}}\right)$$

$$= 0.87 \times 415 \times 1005.3 \times 400\left(1 - \frac{1005.3 \times 415}{740 \times 400 \times 20}\right)$$

$$= 134953789.7 \text{ Nmm}$$

$$\mathbf{M_u = 134.95 \text{ kNm}}$$

Problem

The T-beam floor system has 120 mm thick slab supported on beams. The width of beam is 300 mm and effective depth is 580 mm. The bema is reinforced with 8 bars of 20 mm diameter. Use M_{20} grade of concrete and Fe_{415} steel, the beams are spaced 3 m c/c. The effective span of beam is 3.6 m.

Solution. Given: $b_w = 300$ mm $\quad d = 580$ mm

$$A_{st} = 8 \times \frac{\pi}{4} \times 20^2 = 2513 \text{ mm}^2$$

$D_f = 120$ mm $\quad L = 3.6 \text{ m} = 3600$ mm

- **Effective width of flange (b_f)**

$$b_f = \frac{l_0}{6} + b_w + 6D_f$$

$$l_0 \doteq l = 3.6 \text{ m} \qquad \text{[Simply supported beam]}$$

$$b_f = \frac{3600}{6} + 300 + 6 \times 120 = 1620 \text{ mm}$$

Clear span of the slab to the left and right of beam

$$= 3000 - 300 = 2700 \text{ mm}$$

$$b_f \ngtr 0.5\,(L_1 + L_2) + b_w$$

$$b_f \ngtr 0.5\,(2700 + 2700) + 300$$

$$b_f \ngtr 3000 \text{ mm}$$

$$\therefore \quad b_f = 1620 \text{ mm}$$

- **Depth of neutral axis (x_u)**

Assuming neutral axis to fall in the flange

$$x_u = \frac{0.87 f_y \,.\, A_{st}}{0.36 f_{ck}\, b_f} = \frac{0.87 \times 415 \times 2513}{0.36 \times 20 \times 1620}$$

$x_u = 77.8 \text{ mm} < D_f$. Hence N.A. lies in the flange

$$x_{u\,max} = 0.48d = 0.48 \times 580$$

$= 278.4 \text{ mm} > x_u$, hence it is an underreinforced section.

- **Moment of resistance (M_u)**

$$M_u = 0.87 f_y \,.A_{st}\,.d\left(1 - \frac{A_{st}\,.f_y}{b_f d\,.\,f_{ck}}\right)$$

$$= 0.87 \times 415 \times 2513 \times 580\left(1 - \frac{2513 \times 415}{1620 \times 580 \times 20}\right)$$

$$M_u = 496.61 \times 10^6 \text{ Nmm}$$

$$\mathbf{M_u = 496.61 \text{ kNm}}$$

Learning Objectives
5.1 Introduction one-way slab and Two-way slab
5.2 Difference between one way and Two way Slab.

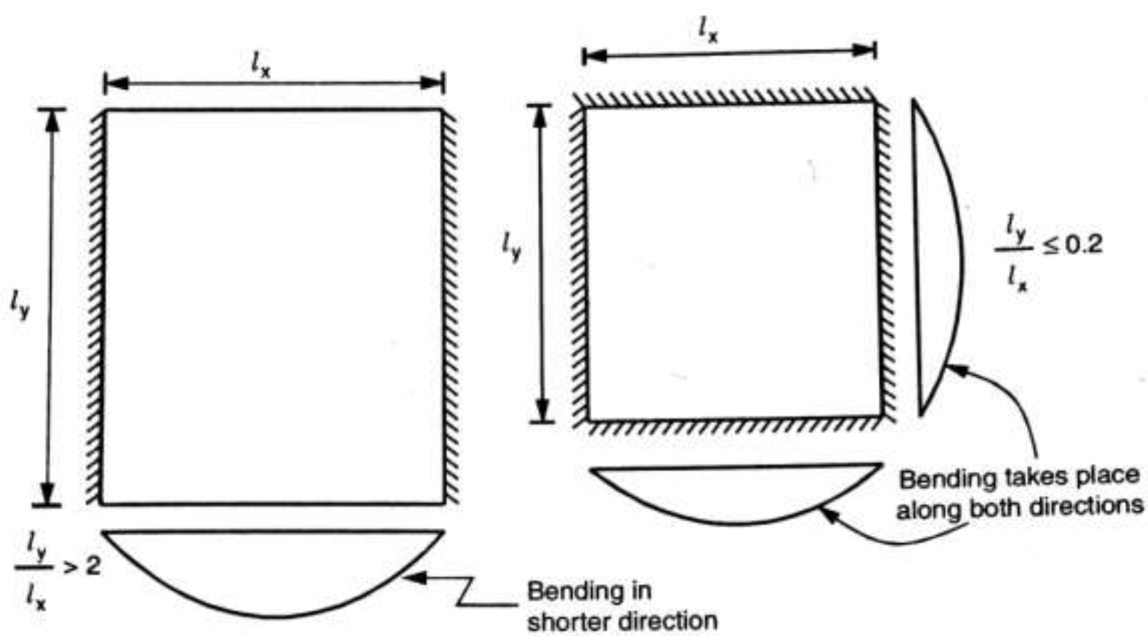
Slab is a two-dimensional or planar element, used in all types of structures such as floors and roof coverings. The thickness of slab is very small as compared to its length and width. Slabs are classified on the basis of $\frac{l_y}{l_x}$ ratio *i.e.*, length of the longer span (l_y)/length of the shorter span (l_x), into following two types:

(*i*) One way slab
(*ii*) Two way slab.

One Way Slab

One way slabs are those slabs in which the $\frac{l_y}{l_x}$ *ratio is greater than 2.* This type of slab is also called as slab spanning in one direction, as the bending takes place only along the shorter span (see Fig. 11.1). Therefore, the main reinforcement is provided along the shorter span.

The one way slab is analysed by assuming it to be a beam of 1 m width.



One way slab

Two Way Slab

The slab which is supported on all the four edges and having $\frac{l_y}{l_x}$ *ratio as less than 2, is called as two way slab.* Two way slab is also called as slab spanning in two directions because bending takes place in both the directions as shown in Fig. 11.1. The main reinforcement is provided along both the directions in a two way slab.

5.2 Difference between one way and Two Way Slab.

S.N	One-Way Slab	Two-Way Slab
1.	The bending moment occurs in only one direction only	The bending moment occurs in only one direction only
2.	Its Longer span to shorter span > 2	Its Longer span to shorter span < 2
3.	Main Reinforcement is provided in one direction only	Reinforcement provided perpendicular to the main reinforcement
4.	Distribution reinforcement provided perpendicular to the main reinforcement	Distribution reinforcement is not provided
5.	It is less expensive compared to two-way slabs	Deflection Shape occurs in Dish-shaped
6.	Support remains of beams on two opposite sides only	Support remains of beams on all four sides
7.	Economic Span Limit remains up to approximately 3.6 meters	Deflection Shape occurs in a parabolic shape
8.	It is designed generally for Low-rise buildings	It is designed generally high-rise buildings
9.	It is Less expensive compared to two-way slabs	It is generally more expensive than one-way slabs

S.N	One-Way Slab	Two-Way Slab
10.	It is more susceptible to deflection	It is lesser susceptibility to deflection
11.	Load-Carrying Capacity is less as compared to two-way slabs	The economic Span Limit remains for Panel sizes up to 6m x 6m

MODULE-2 **Chapter – 5**
Session - 24

Learning Objectives

5.3 Design parameters of one-way slab

5.3.1 Effective Span

5.3.2 Deflection control

5.3.3 Reinforcement in Slabs

5.4 Design procedure of one-way slab

5.3 Design parameters of one-way slab

5.3.1 Effective Span

(a) For Simply Supported Slab

The effective span is taken as smaller of the following:

(*i*) Centre to centre of supports.

(*ii*) Clear distance between the supports plus the effective depth.

(b) For Continuous Slab

In a continous slab, where the width of support is less than $\frac{1}{12}$ of the clear span, the effective span should be taken as given in (*a*) for simply supported slab.

If the supports are wider than $\frac{1}{12}$ of the clear span, or 600 mm whichever is less, the effective span shall be taken as under:

(*i*) For end span, with one end free and the other end continuous or for intermediate spans, the effective span shall be the clear span between supports.

(*ii*) For end span, with one end free and the other end continuous, the effective span shall be equal to clear span plus half the effective depth of slab or clear span plus half the width of discontinuous support whichever is less.

5.3.2 Deflection control

For slabs, the vertical deflection limits are specified by maximum l/d ratio:

(a) For spans upto 10 m

	l/d ratio
Cantilever	7
Singly supported	20
Continuous	26

(*b*) For spans greater than 10 m, the above value may be multiplied by 10/span, except for cantilever, for which exact deflection calculations should be made.

(*c*) Depending on the area and type of tensile steel the above values may be modified as per Fig. 4.1 (Chapter 4).

(*d*) For slabs spanning in two directions, the shorter of the two spans shall be used for calculating span to effective depth ratio.

(*e*) For two way slabs of small spans (upto 3.5 m) with mild steel reinforcement, the shorter span to overall depth (given below) ratios may be assumed to satisfy the deflection limits for loading class upto 3000 N/m^2.

Simply supported : 35

Continuous slab : 40

For high strength deformed bars the values given above should be multiplied by 0.8.

5.3.3 Reinforcement in Slabs

(a) Minimum Reinforcement

The area of reinforcement in either direction in a slab should not be less than 0.15 percent of the total cross-sectional area in case of mild steel reinforcement. In the case of high strength deformed bars, this values can be reduced to 0.12 percent.

(b) Maximum Diameter

The maximum diameter of the reinforcing bar in a slab should not exceed $\frac{1}{8}$th of the total thickness of the slab.

(c) Distribution Reinforcement

Distribution reinforcement is provided in the longer span of one way slab. This steel is as per the minimum reinforcement criteria (*a*) given above. The function of distribution steel are:

(*i*) To distribute the concentrated loads coming on the slab.

(*ii*) To protect against shrinkage and temperature stresses.

(*iii*) To keep the main steel bar in position.

The distribution steel is kept above the main steel and is not provided with hook at the ends.

(d) Spacing of Reinforcement

(1) Minimum Distance bewtween Bars

(i) The minumum horizontal distance between two parralle main bars shallnot be less than

- The diameter of the bar (largest diameter bar is to be considered)
- 5 mm more than the nominal maximum size of coarse aggregate used in concrete.

(*ii*) The vertical distance between two layers of main reinforcement shall be more than:

- 15 mm or
- $\frac{2}{3}$rd the nominal maximum size of aggregate
- Maximum size of the bar

(2) Maximum Distance Between Bars in Tension

(*i*) The spacing of main steel in a slab should not exceed the following:

- 3 times the effective depth of slab.
- 300 mm

(*ii*) The spacing of the bars provided to act as distribution steel (discussed later) or bars provided for preventing temperature and shrinkage stresses shall not exceed the following :

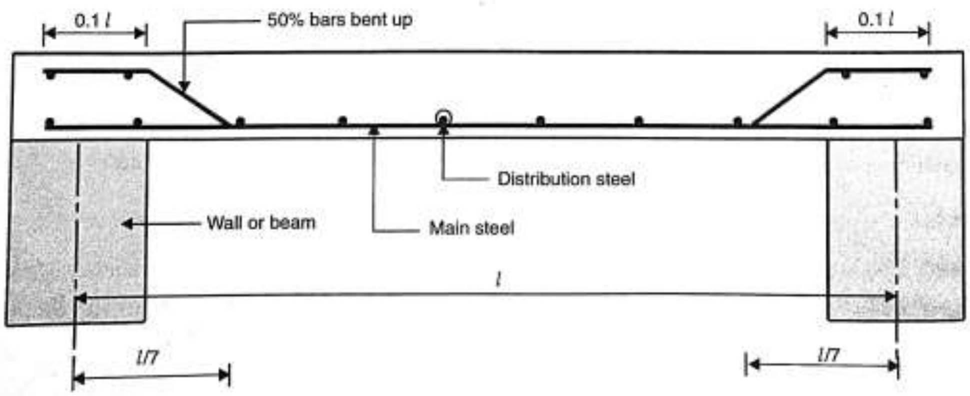
- Five times the effective depth of slab
- 450 mm.

(e) Cover

Nominal cover to be provided in a slab is 20 mm and the other values of cover for different environmental conditions are listed in Is 456.

(f) Bent up Bars

Some of the main reinforcement in slabs are generally bent up near the supports to take up negative moment which may develop due to partial fixity. Generally alternate bars are bent up at a distance of $0.15l \left(\text{or } \frac{l}{7}\right)$ from the centre of supports. The bar available at the upper face should be more than $\frac{l}{10}$ $(0.1l)$ from the centre of support. The reinforcement detailing of one way slab is shown in Fig



(g) Curtailment of Bars

The bars in a slab may be curtailed as per the recommendation given in the chapter-6 about the curtailment of bars in beams. But in practice, the bars are bent and not curtailed in slabs.

(h) Shear Design

Slabs are safe in shear (nominal shear stress is very low since b is large] therefore no shear reinforcement is provided in slabs except that the alternate bars are bent up near the supports.

5.4 Design procedure of one way slab

One way slab is designed exactly as a rectangular beam (given in chapter), the only difference are following:

(*i*) The width of the beam is assumed as one metre.

(*ii*) The depth of slab can be assumed on the basis of control of deflection. Using balanced percentage of steel the $\left(\frac{l}{d}\right)$ ratios are modified. To start with the span to depth ratios are approximated as following for initial depth trial calculations.

(1) For simply supported slabs, 25 to 30

(2) For cantilever slabs 10

(*iii*) In addition to the main tensile reinforcement provided along shorter span, transverse reinforcement or distribution reinforcement is provided.

(*iv*) Some of the main bars in a slab are bent up near the supports $\left(\frac{l}{7}\text{ from centre of the support}\right)$.

(*v*) Shear is to be checked only. No shear reinforcement is provided.

Learning Objectives

5.4.1 Problems on One-way slab

Problem

A simply supported slab of a corridor of a hospital building has a clear span 2.5 m and is supported on beams 230 mm width. Design the slabs, if the beam is carrying a live load of 5 KN/m^2. Use M20 concrete and HYSD Bars.

Solution.

- Assuming total depth $= 120$ mm $\left[d = \frac{l}{25} = \frac{2500}{25} = 100 \text{ mm}\right]$

$$d = 120 - 20 = 100 \text{ mm, assuming effective cover} = 20 \text{ mm}$$

- **Effective span (l)**

It should be least of the following:

(*i*) Centre to centre spacing = 2.5 + 0.23 = 2.73 m

(*ii*) Clear span + effective depth = 2.5 + 0.1 = 2.6 m

- **Design load (w_u), factored moment (M_u) and shear force (V_u)**

For 1 m width of slab

$$\text{Self weight of slab} = 0.12 \times 1 \times 25 = 3.0 \text{ kN/m}^2 \quad \text{[Unit weight of R.C.C.} = 25 \text{ kN/m}^3]$$

$$\text{Live load} = 5 \text{ kN/m}^2$$

$$\text{Total load} = w = 5 + 3 = 8.0 \text{ kN/m}^2$$

$$\text{Design load} = w_u = w \times 1.5 = 8 \times 1.5 \quad \text{[Load factor} = 1.5]$$

$$w_u = 12 \text{ kN/m}^2$$

$$\text{Factored moment} = M_u = \frac{w_u \cdot l^2}{8} = \frac{12 \times 2.6^2}{8}$$

$$M_u = 10.14 \text{ kNm or } 10.14 \times 10^6 \text{ Nmm}$$

$$\text{Factored shear force} = V_u = \frac{w_u \cdot L}{2} = \frac{12 \times 2.5}{2}$$

$$= 15 \text{ kN or } 15000 \text{ N} \quad \text{[where } L \text{ is the clear span]}$$

- **Effective depth required**

$$\frac{x_{u\max}}{d} = 0.48$$

$$R_u = 0.36 f_{ck} \frac{x_{u\max}}{d}\left(1 - \frac{0.42\, x_{u\max}}{d}\right)$$

$$= 0.36 \times 20 \times 0.48\,(1 - 0.42 \times 0.48)$$

$$R_u = 2.76$$

$$b = 1000 \text{ mm}$$

$$d_{reqd.} = \sqrt{\frac{M_u}{R.b}} = \sqrt{\frac{10.14 \times 10^6}{2.76 \times 1000}} = 60 \text{ mm} < 100 \text{ mm, hence OK}$$

$$\text{Adopt } D = 120 \text{ mm and } d = 100 \text{ mm}$$

$d > d_{reqd.}$ Hence the section is under-reinforced.

■ **Area of tension steel (A_{st})**

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{f_{ck} bd}\right)$$

$$10.14 \times 10^6 = 0.87 \times 415 \times A_{st} \times 100 \left(1 - \frac{A_{st} \times 415}{20 \times 1000 \times 100}\right)$$

$$10.14 \times 10^6 = 36105 A_{st} - 7.49 A_{st}^2$$

$$A_{st}^2 - 4820.4 A_{st} + 1.35 \times 10^6 = 0$$

$$A_{st} = \frac{4820.4 \pm \sqrt{(4820.4)^2 - 4 \times 1.35 \times 10^6}}{2}$$

$$A_{st} = 300 \text{ mm}^2$$

Using 8 mm ϕ bar

$$\text{Area of one 8 mm dia bars, } A_\phi = \frac{\pi}{4} \times 8^2 = 50.3 \text{ mm}^2$$

$$\text{Spacing of bars} = \frac{1000.A_\phi}{A_{st}} = \frac{1000 \times 50.3}{300}$$

$$= 167 \text{ mm say } 160 \text{ mm}$$

The spacing should be less than

(*i*) $3d = 3 \times 100 = 300$ mm

(*ii*) 300 mm

$$A_{st \text{ provided}} = \frac{1000 \times 50.3}{160} = 314 \text{ mm}^2$$

As per IS code minimum area of reinforcement to be provided is [For HYSD bars]

$= 0.12\ bD/100$

$= 0.12 \times 1000 \times 120/100$

$= 144 \text{ mm}^2 < 300 \text{ mm}^2$. Hence OK

$\therefore$ Provide 8 mm ϕ bars @ 160 mm c/c in shorter span.

Bending half the bars at 375 mm $\left(\frac{l}{7} = \frac{2600}{7} = 371 \text{ mm}\right)$ from the centre of support or $375 - \frac{230}{2} = 260$ mm from the face of the support. A_{st} at supports is half of that at mid-span.

$$A_{st} \text{ provided at supports} = \frac{314}{2} = 157 \text{ mm}^2 > 120 \text{ mm}^2 \text{, hence OK}$$

■ **Distribution steel**

$$\text{Area of distribution steel} = \frac{0.12bd}{100} = \frac{0.12 \times 1000 \times 120}{100} = 144 \text{ mm}^2$$

$$\text{Using 6 mm } \phi \text{ bars, } A_\phi = \frac{\pi}{4} \times 6^2 = 28.3 \text{ mm}^2$$

$$\text{Spacing} = \frac{1000 . A_\phi}{A_{st}} = \frac{1000 \times 28.3}{144} = 196.5 \text{ mm say 190 mm}$$

$\therefore$ Provide 6 ϕ @ 190 mm c/c in longer direction

(the spacing should be less than (*i*) $5d$ *i.e.*, $5 \times 100 = 500$ mm, (*ii*) 450 mm)

■ **Check for shear**

$$V_u = 15000 \text{ N}$$

$$\text{Nominal shear stress} = \tau_v = \frac{V_u}{bd} = \frac{15000}{100 \times 100} = 0.15 \text{ N/mm}^2$$

At support, $$A_{st} = 157 \text{ mm}^2, P_t = \frac{100 A_{st}}{bd} = \frac{100 \times 157}{100 \times 1000} = 0.15\%$$

For M20 concrete and $P_t = 0.15$ % from Table 5.5.

$$\tau_c = 0.28 \text{ N/mm}^2$$

For 120 mm depth $k = 1.3$, from Table 5.6

$$\tau_c \text{ for slab} = k.\tau_c = 0.28 \times 1.3 = 0.36 \text{ N/mm}^2 > \tau_v. \text{ Hence OK}$$

■ **Check for deflection**

$$P_t = \frac{100 . A_{st}}{bd} = \frac{100 \times 314}{100 \times 1000} = 0.31\%$$

$$f_s = 0.58 f_y \left[\frac{A_{st\text{ reqd}}}{A_{st\text{ provided}}}\right] = 0.58 \times 415 \left[\frac{300}{314}\right] = 230 \text{ N/mm}^2$$

For $P_t = 0.31\%$ $\qquad f_s = 230 \text{ N/mm}^2$ from Fig. 6.1

For $f_s = 190 \text{ N/mm}^2$ $\qquad P_t = 0.31, \quad k_t = 1.89$

For $f_s = 240 \text{ N/mm}^2$ $\qquad P_t = 0.31, \quad k_t = 1.47$

$\therefore$ For $f_s = 230 \text{ N/mm}^2$

$$k_t = 1.89 - \frac{(1.89 - 1.47)}{(240 - 190)} \times (230 - 190)$$

$$k_t = 1.55$$

$$\left(\frac{l}{d}\right)_{max} = 20 \times k_t = 20 \times 1.55 = 31$$

$$\left(\frac{l}{d}\right)_{provided} = \frac{2600}{100} = 26 < 31$$

$$\therefore \quad \left(\frac{l}{d}\right)_{max} > \left(\frac{l}{d}\right)_{provided}. \text{ Hence OK}$$

■ **Check for development length**

At supports

$$M_{u_1} = 0.87 f_y \cdot A_{st} d\left(1 - \frac{A_{st} f_y}{b d f_{ck}}\right)$$

$$= 0.87 \times 415 \times 157 \times 100\left(1 - \frac{157 \times 415}{100 \times 100 \times 20}\right)$$

$$M_{u_1} = 5483819.9 \text{ Nmm}$$

$$V_u = 15000 \text{ N}$$

Using no hook $l_0 = 0$

$$\frac{M_{u_1}}{V_u} + l_0 = \frac{5483819.9}{15000} + 0 = 366 \text{ mm}$$

$$L_d = \frac{\phi(0.87 f_y)}{4\tau_{bd}}$$

$$= \frac{8 \times 0.87 \times 415}{4 \times 1.6 \times 1.2} = 376 \text{ mm}$$

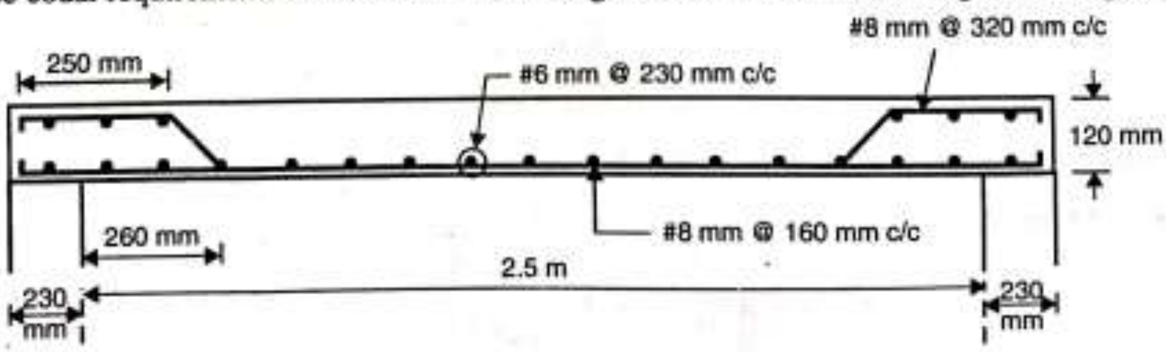
$$\therefore \quad \frac{M_{u_1}}{V} + l_0 \nless L_d$$

Hence codal requirements are not satisfied. Therefore providing a 90° bend at the centre of support

$$l_0 = 8\phi = 8 \times 8 = 64 \text{ mm}$$

$$\therefore \quad \frac{M_1}{V} + l_0 = 366 + 64 = 430 \text{ mm} > L_d$$

Hence codal requirements are satisfied. The arrangement of reinforcement is given in Fig. 11.4.



MODULE-2 **Chapter - 5**
Session – 26,27

Learning Objectives
5.4.2 Design of Two-way slab (Problem)

Problem

Design a reinforced concrete slab for a clear dimension 4m X 5 m. The slab is supported on walls of width 300 mm. The slabs is carrying a live load of 4 KN/m^2 and floor finish 1 KN/m^2. Use M$_{20}$ concrete and Fe$_{415}$ Steel. Use M20 Concrete and Fe$_{415}$ Steel.

Solution. Given:

- $\frac{l_y}{l_x} = \frac{5}{4} = 1.25 < 2$ hence this is a two way slab.

- Assuming $D = 180$ mm $\left[\text{Assuming } \frac{l}{d} = 25 \Rightarrow d = \frac{4000}{25} = 160 \text{ mm}\right]$

  $d = 180 - 15 - 4 = 161$ mm (assuming clear cover as 15 mm and 8 mm as the dia. of main bar)

- **Effective span**

  Effective span in X-direction:

(*i*) Centre to centre $4 + 0.3 = 4.3$ m

(*ii*) Clear span + Effective depth $= 4 + 0.161 = 4.161$ m

$l_x = 4.161$ m and $l_y = 5.161$ m

Similarly effective span in Y direction, $l_y = 5.161$ m

- **Design load (w_u)**

Self wt. of slab $= 0.18 \times 1 \times 25 = 4.5$ kN/m [Unit weight of R.C.C. = 25 kN/m^3]

Finishing load $= 1 \times 1 = 1$ kN/m

Live load $= 4 \times 1 = 4$ kN/m

Total load $= 4.5 + 1 + 4 = 9.5$ kN/m

Factored or design load $= 9.5 \times 1.5$ [load factor = 1.5]

$= 14.25$ kN/m

- **Design moment and shear**

From Table 12.1

$$\text{for } \frac{l_y}{l_x} = 1.25$$

$$\alpha_x = 0.072 + \frac{0.079 - 0.072}{1.3 - 1.2} \times (1.25 - 1.2)$$

$$\alpha_x = 0.075$$

$$\alpha_y = 0.056$$

$$M_{ux} = \alpha_x w_u \,.\, l_x^2$$

$$M_{ux} = 0.075 \times 14.25 \times 4.161^2 = 18.5 \text{ kNm or } 18.5 \times 10^6 \text{ Nmm}$$

$$M_{uy} = \alpha_y w_u l_x^2$$

$$M_{uy} = 0.056 \times 14.25 \times 4.161^2 = 13.81 \text{ kNm or } 13.81 \times 10^6 \text{ Nmm}$$

Maximum shear force $= V_u$

$$V_u = w_u \cdot \frac{l_x}{2}$$

$$= 14.25 \times \frac{4.161}{2}$$

$$= 29.65 \text{ kN}$$

$$V_u = 29.65 \text{ kN}$$

■ **Minimum depth required (d_{reqd})**

$$d_{reqd} = \sqrt{\frac{M_u}{R_u \cdot b}} \qquad [R_u = 2.76 \text{ for M20 concrete and Fe 415, Table 6.1}]$$

$$= \sqrt{\frac{18.5 \times 10^6}{2.76 \times 1000}}$$

$d_{reqd} = 82 \text{ mm} < d_{assumed}$. Hence OK.

■ **Design of main reinforcement**

(*i*) along shorter span in X-direction (middle strip):

$$\text{Width of middle strip} = \frac{3}{4} l_y$$

$$= \frac{3}{4} \times 5.16 = 3.9 \text{ m}$$

$$d = 161 \text{ mm}$$

$$M_u = 0.87 f_y A_{st}\, d\left[1 - \frac{A_{st} f_y}{bd\, f_{ck}}\right]$$

$$18.5 \times 10^6 = 0.87 \times 415 \times A_{st} \times 161 \left[1 - \frac{415 A_{st}}{1000 \times 161 \times 20}\right]$$

$$A_{st}^2 - 7760 \cdot A_{st} + 2463284 = 0$$

$$A_{st} = \frac{7760.5 - \sqrt{(7760.5)^2 - 4 \times 2463284}}{2}$$

$$A_{st} = 332.5 \text{ mm}^2$$

Using 8 mm ϕ bars $\quad A_\phi = \frac{\pi}{4} \times 8^2 = 50.3 \text{ mm}^2$

$$\text{Spacing} = \frac{1000 \times A_\phi}{A_{st}} = \frac{1000 \times 50.3}{332}$$

= 151 mm say 150 mm (spacing is less than $3d$ and 300 mm)

$$A_{st\,min} = \frac{0.12}{100} bD = \frac{0.12 \times 1000 \times 180}{100} = 216 \text{ mm}^2$$

$$A_{st\,provided} = \frac{1000 \times 50.3}{150} = 335 \text{ mm}^2 > 216 \text{ mm}^2. \quad \text{Hence OK}$$

∴ Provide 8 mm ϕ @ 150 mm c/c in the middle strip of width 3.9 m

(*ii*) Along longer span in *Y*-direction (middle strip):

$$\text{Width of middle strip} = \frac{3}{4} l_x$$

$$= \frac{3}{4} \times 4.16 = 3.12 \text{ m}$$

Effective depth along *y* direction,

$$d = 161 - 4 - 4 = 153 \text{ mm}$$

$$M_u = 0.87 f_y A_{st} . d \left(1 - \frac{f_y A_{st}}{b d f_{ck}}\right)$$

$$13.81 \times 10^6 = 0.87 \times 415 \times A_{st} \times 153 \left(1 - \frac{415\, A_{st}}{1000 \times 153 \times 20}\right)$$

$$A_{st}^2 - 7325.1\, A_{st} + 1.83 \times 10^6 = 0$$

$$A_{st} = \frac{7325 - \sqrt{(7325)^2 - 4 \times 1.83 \times 10^6}}{2}$$

$$A_{st} = 261 \text{ mm}^2 > A_{st\,min}$$

$$\text{Spacing of 8 mm dia. bars} = \frac{1000 \times 50.3}{261} = 192 \text{ mm}$$ [Spacing is less than 3*d* and 300 mm]

∴ Provide 8 mm ϕ bars @ 190 mm c/c in middle strip of width 3.12 m

(*iii*) Reinforcement in edge strip

$$A_{s\,min} = 216 \text{ mm}^2$$

Using 8 mm ϕ bars

$$\text{Spacing} = \frac{1000 \times 50.3}{216} = 232 \text{ mm}$$ (Spacing is less than 5*d* or 450 mm)

Using 8 mm ϕ bars @ 230 mm c/c in the edge strip of width $\frac{1}{2}(5.16 - 3.9)$ *i.e.*, 1.04 m along X-direction and edge strip of width $\frac{1}{2}(4.16 - 3.12)$ *i.e.*, 0.52 m along *Y*-direction.

- **Check for shear**

$$\text{Nominal shear stress} = \tau_v = \frac{V_u}{bd}$$

$$\tau_v = \frac{22810}{1000 \times 153} = 0.20 \text{ N/mm}^2$$

$$p_t = \frac{100A_{st}}{bd}$$

$$p_t = \frac{100 \times 335}{1000 \times 161} = 0.21\%$$

For $p_t = 0.21$ and M20 concrete, Table 5.1

$$\tau_c = 0.28 + \frac{0.36 - 0.28}{0.25 - 0.15} \times (0.21 - 0.15)$$

$$= 0.33 \text{ N/mm}^2$$

For 180 mm thickness of slab $K = 1.24$ from Table 5.2 [Cl. 40.2.1.1 of IS 456]

$$\tau_c = 0.33 \times 1.24 = 0.41 \text{ N/mm}^2 > \tau_c$$

∴ Shear reinforcement is not required.

- **Check for deflection**

$$P_t = 0.21\%$$

$$f_s = 0.58 f_y \left[\frac{A_{st\text{ reqd}}}{A_{st\text{ provided}}}\right]$$

$$= 0.58 \times 415 \left[\frac{333}{348}\right] = 240 \text{ N/mm}^2$$

For $P_t = 0.21\%$ $\quad f_s = 240$ N/mm², from Fig. 6.1

$$k_t = 1.6$$

$$\left(\frac{l}{d}\right)_{\max} = 20 \times 1.6 = 32.$$

$$\left(\frac{l}{d}\right)_{\text{provided}} = \frac{4161}{161} = 26$$

∴ $\left(\frac{l}{d}\right)_{\max} > \left(\frac{l}{d}\right)_{\text{provided}}$ Hence OK

- **Torsional reinforcement at corners**

$$\text{Mesh size} = \frac{l_x}{5} = \frac{4.16}{5} = 0.832 \text{ mm say } 840 \text{ mm}$$

Area of torsional reinforcement

$$= \frac{3}{4} \times 335 = 251 \text{ mm}^2$$

Using 8 mm ϕ bars $\quad A_\phi = \frac{\pi}{4} \times 8^2 = 50.3$

$$\text{Spacing} = \frac{1000 \times 50.3}{251}$$

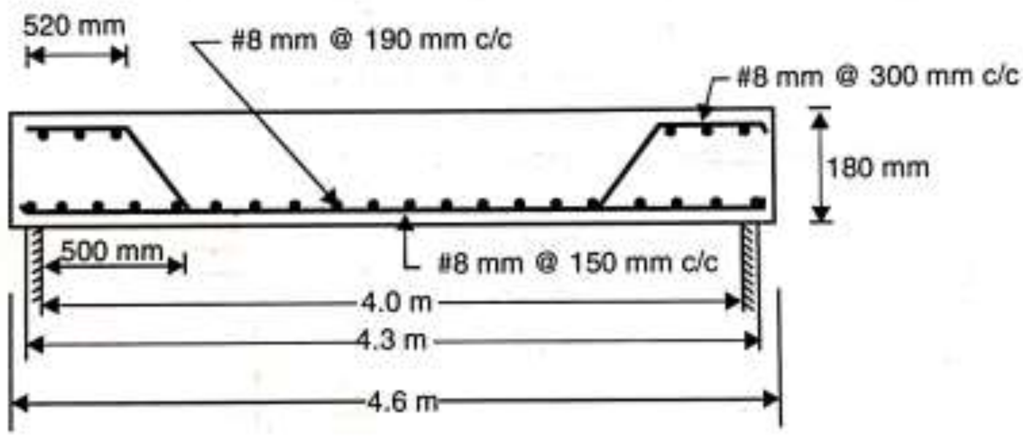
$$= 201 \text{ mm say } 200 \text{ mm}$$

$\therefore$ Provide 8 mm mesh of bars @ 200 mm c/c in a mesh.

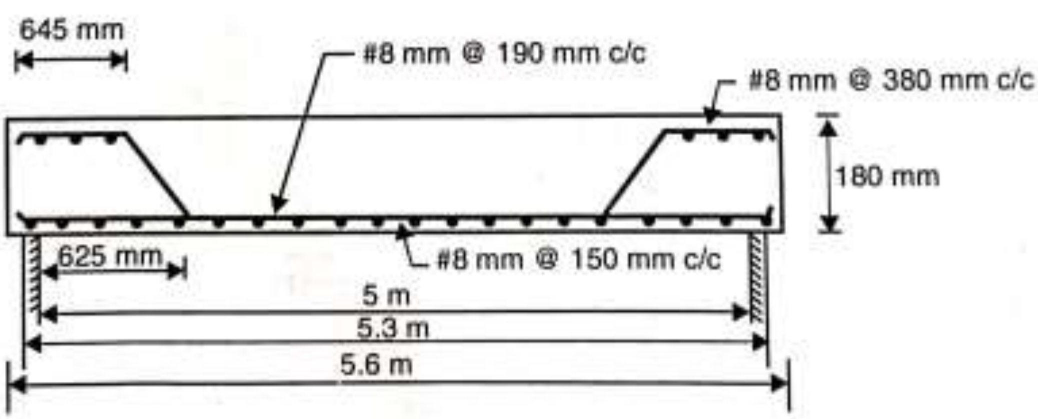
■ **Arrangement of reinforcement**

(*i*) Bending half of the main bars at a distance of $0.15l_x$ *i.e.*, 0.65 m from the centre of support or (650 – 150) 500 mm from edge of support along X direction available length of bars at top = 650 – 130 *i.e.*, 520 mm from centre of support.

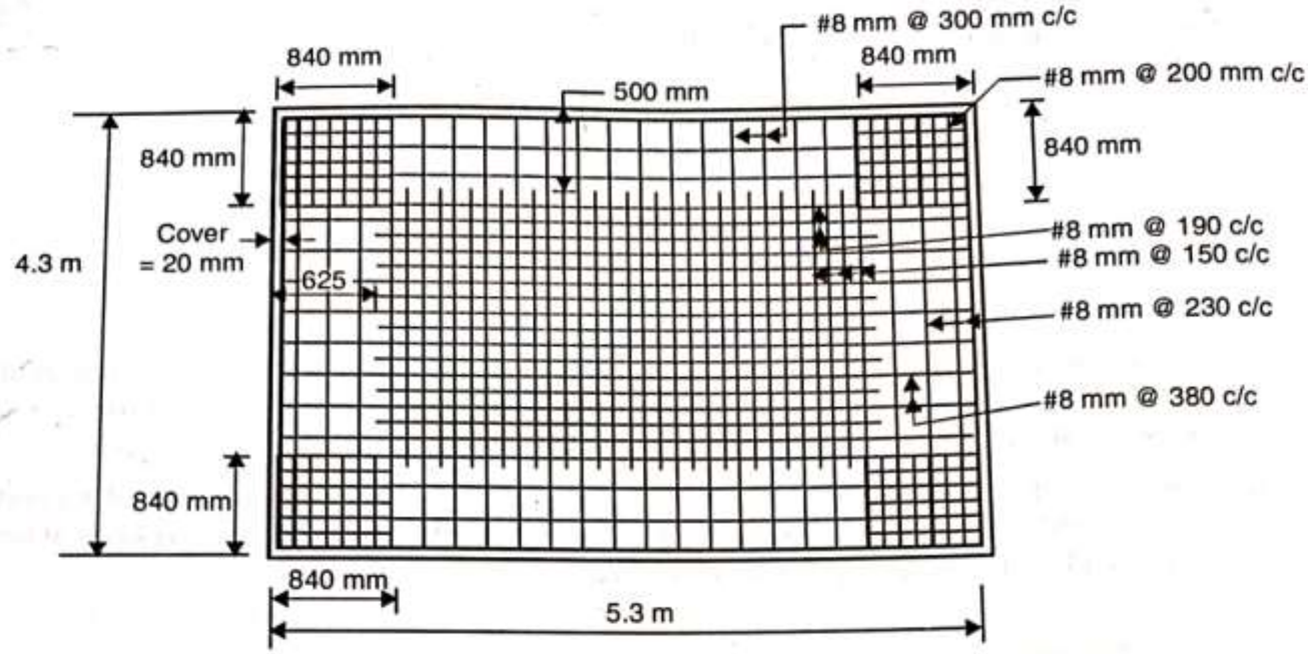
(*ii*) Bending half of the main bars at a distance of $0.15l_y$ *i.e.*, 775 mm from centre of support or 775 – 150 = 625 mm from edge of support along Y direction, available length of bars at top = 775 – 130 = 645 mm from centre of support.

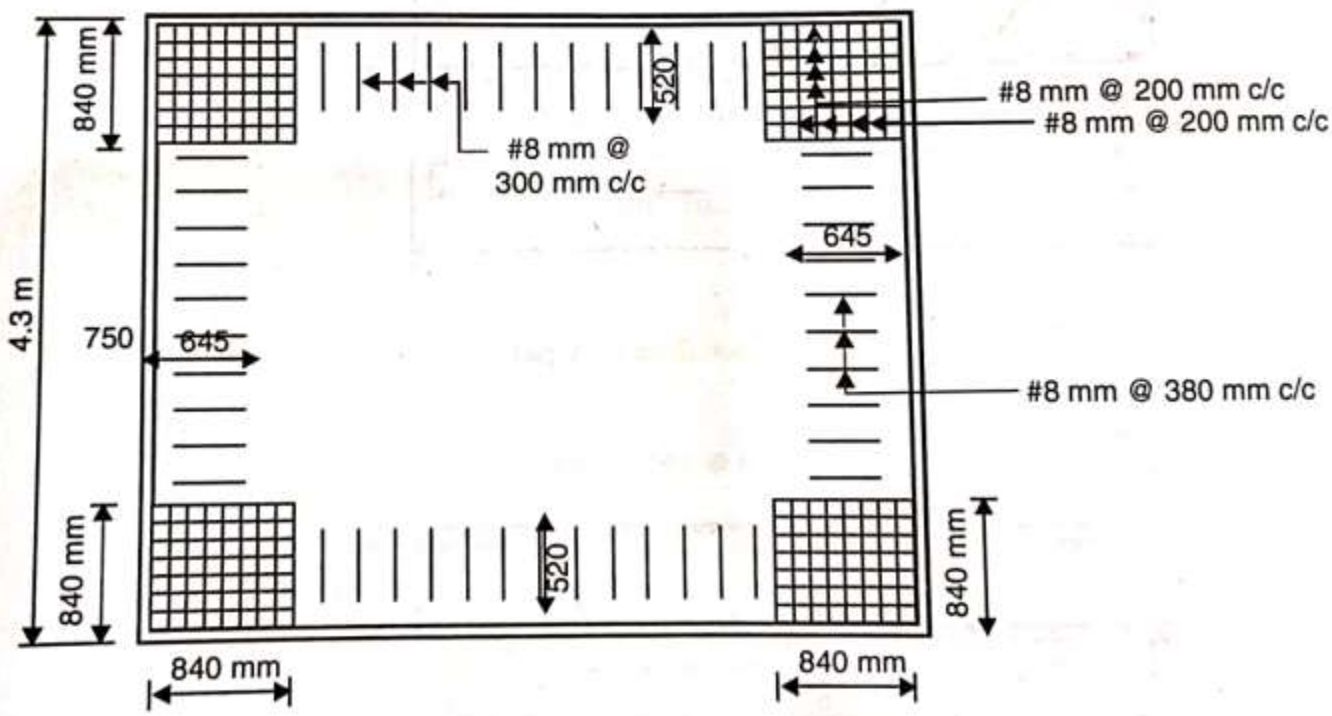


(*a*) Section along short span



(*b*) Section along long span


#8 mm @ 300 mm c/c
840 mm
840 mm
#8 mm @ 200 mm c/c
500 mm
840 mm
840 mm
Cover = 20 mm
4.3 m
625
#8 mm @ 190 c/c
#8 mm @ 150 c/c
#8 mm @ 230 c/c
#8 mm @ 380 c/c
840 mm
840 mm
5.3 m


840 mm
520
#8 mm @ 200 mm c/c
#8 mm @ 200 mm c/c
#8 mm @ 300 mm c/c
645
4.3 m
750
645
#8 mm @ 380 mm c/c
840 mm
520
840 mm
840 mm
840 mm

MODULE-2

Chapter - 6
Session - 28

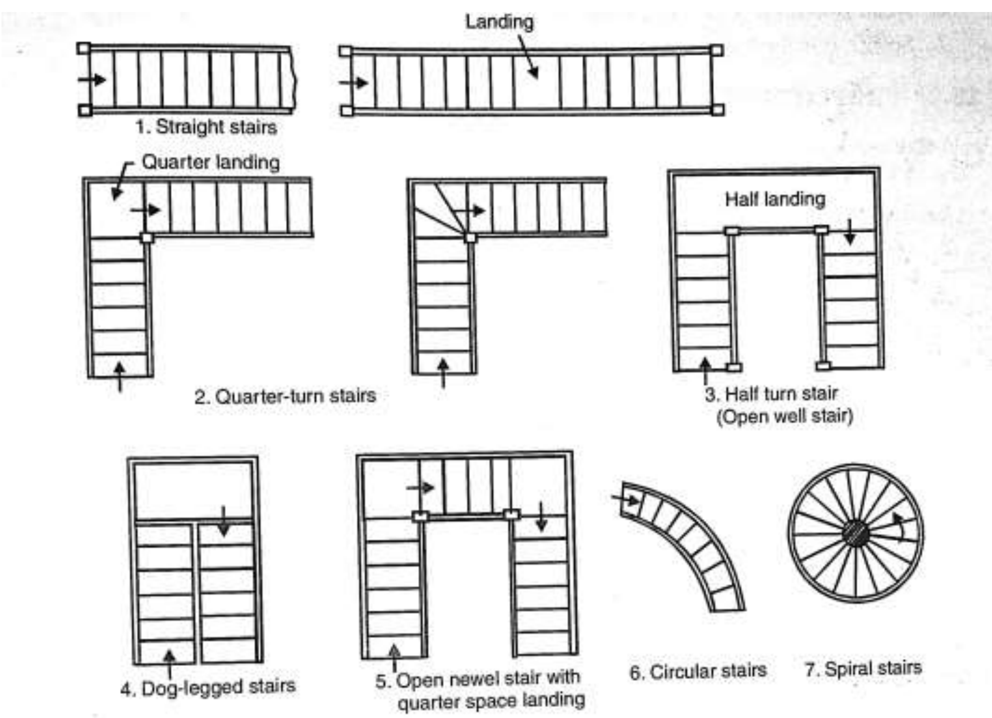
Learning Objectives
6.1 Staircase components and design
6.1.1 Terminology
6.1.2 Staircase design problem

A staircase is a means of giving access to different floors or levels of building. Staircases are used in almost all buildings. It consists of a number of steps arranged in a way that a person can move from one level to another. The arrangement of steps is as per convenience, standards and space available. Commonly used staircase are as follows.

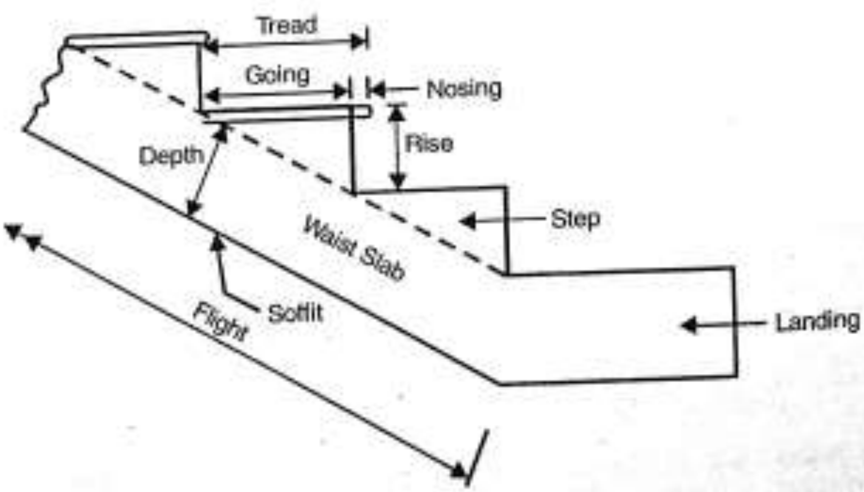
(1) Straight stairs
(2) Quarter turn stairs
(3) Open well type (Half turn stair)
(4) Dog legged stairs
(5) Circular or spiral stairs

6.1.1 Terminology

1. **Flight:** Flight is the length of the staircase between two landings. It is the sloping and portion (slab) of the stairs. The number of steps in a flight varies from 3 to 12.
2. **Landing:** Landing is the intermediate, horizontal portion provided in a staircase. It is provided for relaxing while climbing and entering or existing a staircase.
3. **Rise:** The vertical height of a step is called rise or riser. It varies from 150 mm to 180 mm for residential building and 120 to 150 mm for public building.



4. **Tread:** The horizontal distance between two risers on a step is called as tread. The width of a tread is kept as 200 mm to 250 mm for residential building and 200 to 300 mm for public building.
5. **Going and Nosing:** The horizontal distance between two risers is known as going and the portion projecting out from the riser surface is called as nosing. Nosing is provided when the available horizontal distance for a tread is less.



6. **Head Room:** It is the clear height available between one flight and other above it.
7. **Soffit:** It is the bottom surface of the waist slab.

PROPORTIONING OF STAIRCASE

A staircase is proportional on the basis of space available and some thumb rules mentioned below :

1. 2 × Riser + Tread = 600 to 640 mm
2. Riser × Tread = 40000 to 42000 mm^2
3. Width of private staircase is about 900 mm and that of public staircase is kept about 1800 mm to 2400 mm.
4. Clear height between a flight and the other vertically above it (head room) should not be less than 2.10 m.
5. The angle of flight with the horizontal should be between 25° to 40°.
6. For free flow of users the width of landing should be equal to the width of stairs.

Problem

Design a dog legged staircase for an office building in a room measuring 3.0 m x 6.0 m (clear dimension).Floor to floor height is 3.5 m. The building is a public building liable to over crowding. Stairs are supported on brick walls 230 mm thick at the end of landings. Use M_{20} concrete Fe_{415} Steel.

Solution. ■ **Proportioning of Various Dimensions of Stair case**

Available width of staircase $= 3.0$ m

Considering 2 flights of dog logged staircase, let us assume width of each flight as 1.35 m

Space between the two flights $= 3.0 - 2 \times 1.35 = 0.30$ m

Floor to floor height $= 3.5$ m

As there will be two flights, each flight will have a height of

$$\frac{3.50}{2} = 1.75 \text{ m}$$

Assuming height of risers as 150 mm as it is a public building.

Number of risers $= \dfrac{1750}{150} = 11.66$ say 12

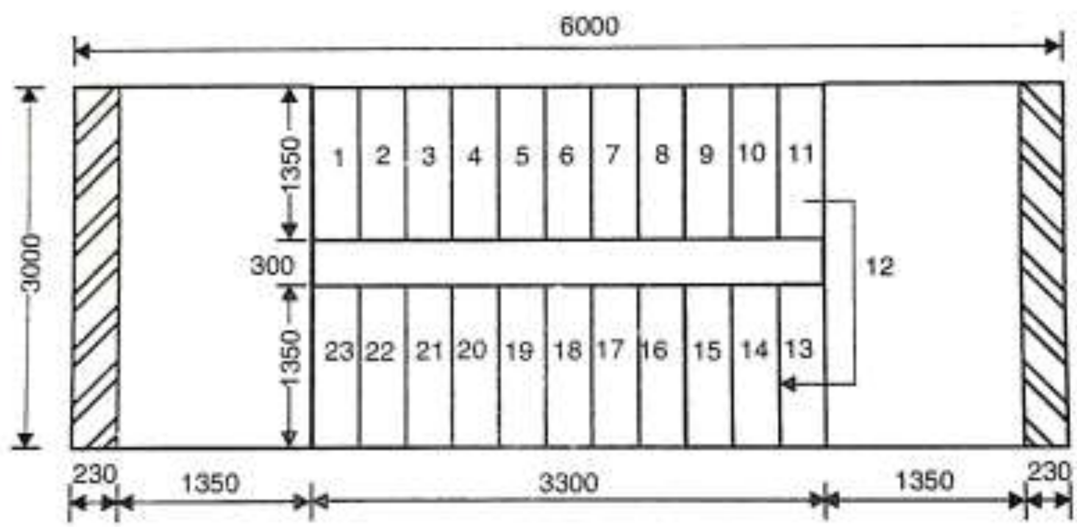
Number of treads required $=$ Number of risers $- 1 = 12 - 1 = 11$

Let width of each tread $= 300$ mm

Total going $= 300 \times 11 = 3300$ mm $= 3.30$ m

Total length available $= 6.0$ m

$\therefore$ Width of each landing $= \dfrac{6.00 - 3.30}{2} = 1.35$ m $= 1350$ mm.

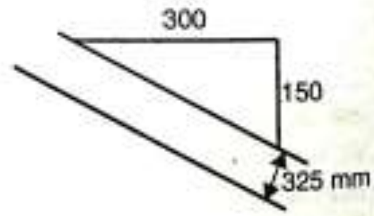


Design of staircase

■ Effective span of flight $=$ Centre to centre distance of walls

$$= 6.00 + \frac{0.23}{2} + \frac{0.23}{2} = 6.23 \text{ m} = 6230 \text{ mm}$$

■ Thickness of waist slab $= \frac{1}{20}$ of span (approx.)



$$= \frac{6.23}{20} \times 1000 = 311.5 \text{ mm}$$

Let us take $d = 300$ mm and $D = 325$ mm.

■ Loads

(*i*) Weight of waist slab in plan (per m width of flight)

$$= D\sqrt{1+\frac{R^2}{T^2}} \times 25 = 0.325\sqrt{1+\frac{150^2}{300^2}} \times 25 = 9.1 \text{ kN/m}$$

(*ii*) Weight of steps (per m width of flight)

$$= \frac{25RT}{2T} = \frac{1}{2} \times \frac{0.15 \times 0.30}{0.30} \times 25 = 1.875 \text{ kN/m}$$

Total dead load $= 9.1 + 1.875 = 10.975$ kN/m

Live load $= 5 \text{ kN/m}^2 = 5$ kN/m per m width of staircase

Total DL + LL $= w = 10.975 + 5 = 15.975$ kN/m say 16 kN/m

Factored load, $w_u = 16 \times 1.5 = 24$ kN/m

For landing

$DL = 0.325 \times 25 \times 1.0 = 8.125$ kN/m

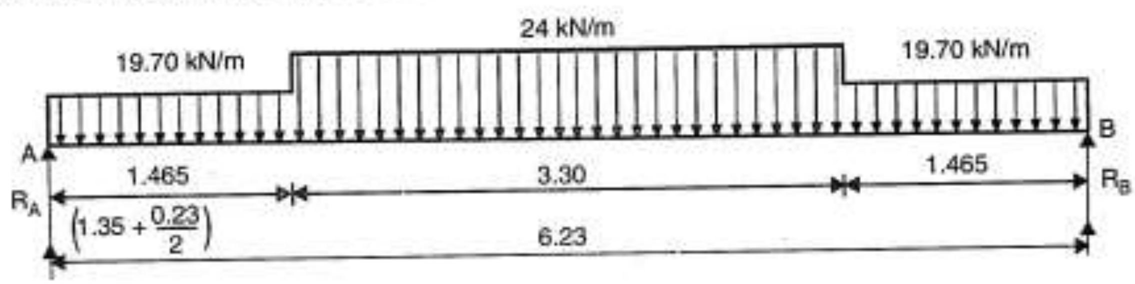
$LL = 1 \times 5.0 \text{ kN/m}^2 = 5.0$ kN/m per m width of landing

Total, $DL + LL = 13.125$ kN/m

Factored load $= 1.5 \times 13.125$ kN/m

$= 19.70$ kN/m

Load diagram of the stairs shall be as follows:



■ **Design moment**

$$\text{Reaction at supports, } R_A = R_B = \frac{(2 \times 19.70 \times 1.465) + (24 \times 3.30)}{2} = 68.5 \text{ kN}$$

$$\text{BM at mid span, } M_u = \left(68.5 \times \frac{6.23}{2}\right) - \left[19.70 \times 1.465 \times \left(\frac{1.465 + 3.30}{2}\right)\right] - \left(24 \times \frac{3.30}{2} \times \frac{3.30}{4}\right)$$

$$M_u = 112 \text{ kNm}$$

Max. BM allowed for a singly reinforced section with Fe 415 bars [Table 6.2]

$$M_{u\,lim} = 0.138\, f_{ck}\, bd^2 = 0.138 \times 20 \times 1000 \times 300^2 \times \frac{1}{10^6}$$

$$= 2484 \text{ kNm} > 112 \text{ kNm.}$$

Hence section can be designed as singly reinforced.

■ **Area of reinforcement**

$$M_u = 0.87\, f_y\, A_{st}\, d\left(1 - \frac{A_{st}}{bd} \times \frac{f_y}{f_{ck}}\right)$$

$$112 \times 10^6 = 0.87 \times 415 \times A_{st} \times 300\left(1 - \frac{A_{st}}{1000 \times 300} \times \frac{415}{20}\right)$$

$$= 108315\, A_{st} - 7.49\, A_{st}^2$$

$$A_{st}^2 - 14461.28\, A_{st} + 14.9 \times 10^6 = 0$$

$$A_{st} = \frac{14461.28 \pm \sqrt{(14461.28)^2 - 4 \times 14.9 \times 10^6}}{2} = 1117 \text{ mm}^2$$

Using 16 mm bars, $A_\phi = \frac{\pi}{4} \times 16^2 = 201 \text{ mm}^2$ $\boxed{\text{Spacing} = \frac{A_\phi \times 1000}{A_{st}}}$

$$\text{Spacing} = \frac{201}{1117} \times 1000 = 179 \text{ mm}$$

provide 16 mm φ @ 170 c/c

$$\text{Distribution steel} = 0.12\% \text{ of area} = \frac{0.12}{100} \times 1000 \times 325 = 390 \text{ mm}^2$$

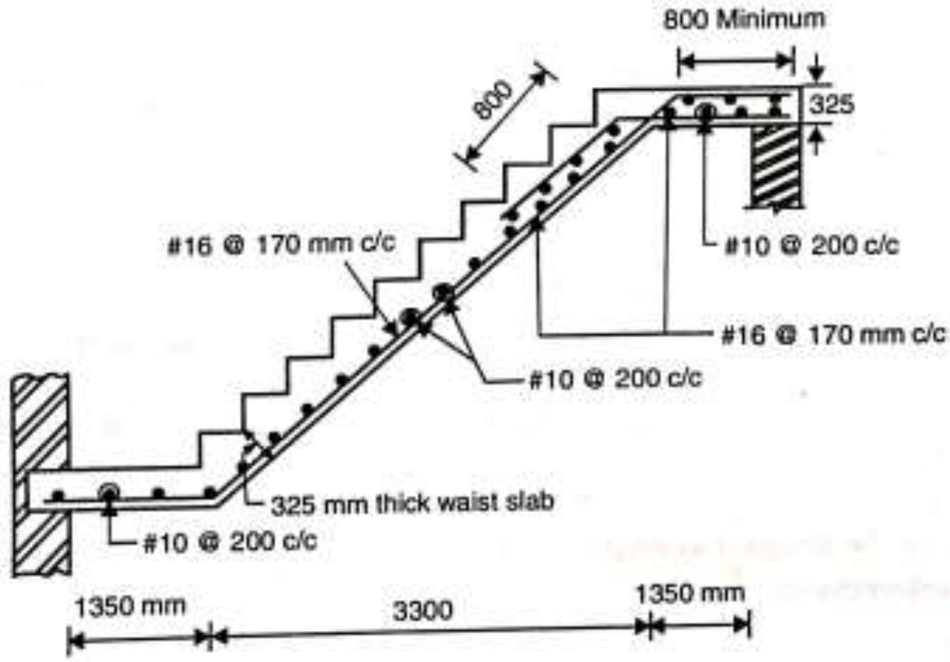
$$\text{Spacing of 10 φ bars} = \frac{78.5}{390} \times 1000 = 201 \text{ mm}$$

provide 10 φ @ 200 mm c/c

■ **Development length**

$$L_d = \frac{\phi.(0.87\, f_y)}{4\,\tau_{bd}} = \frac{16 \times 0.87 \times 415}{4 \times 1.6 \times 1.2} = 752 \text{ mm}$$

Providing 800 mm length of bars at points where L_d is required as shown in Fig. 15.13.



Possible Short Type Questions & Answers

1. **What is doubly reinforced beam?**
   As steel is the material which is very strong in comp and tension, add steel in compression zone in compression zone, Such reinforced concrete sections having steel reinforcement both on tensile and compressive faces are known as doubly reinforced section.
2. **What are the conditions in which we design doubly reinforced beam?**
   The doubly reinforced concrete beam design may be required when a beam's cross-section is limited because of architectural or other considerations. As a result, the concrete cannot develop the compression force required to resist the given bending moment. In that case, steel bars are added to the beam's compression zone to improve it at compression.
3. **What do you mean by flange beam?**
   A flange beam is a type of structural beam used in construction and engineering, characterized by its shape, which includes flanges and a web. The flanges are the top and bottom horizontal elements of the beam, and the web is the vertical element that connects the flanges.
   T-Beams: Shaped like the letter "T," with a single flange at the top.
   Flange beams are commonly used in construction for support in buildings, bridges, and other structures due to their ability to bear heavy loads and provide structural stability.
4. **Define one-way slab?**
   One way slabs are those slabs in which the l_y/l_x ratio is greater than 2. This type of slab is also called as slab spanning in one direction, as the bending takes place only along shorted span.
5. **Define two-way slab?**
   The slab which is supported on all four edges and having l_y/l_x ratio as less than 2, called as two-way slab. Two-way slab is called as slab spanning in two directions because bending takes place in both the directions.

Possible Long Type Questions

1. Determine the ultimate moment of resistance of a rectangular beam 300 mm X 600 mm(effective) reinforced with 5-25 mm diameter bars in tension zone and 2-25 mm dia. Bars in compression zone. Use M_{20} concrete and Fe_{415} Steel. Take $d' = 60$ mm.
2. Determine the ultimate moment of resistance of a beam 300 mm X 500 mm(effective) reinforced with 7 bars of 20 mm in tension zone 2-20 mm bars in compression zone. Use M_{20} concrete and Fe_{415} steel. Take $d' = 50$ mm.
3. Find the ultimate moment of resistance of doubly reinforced beam with following data: Beam size = 230 mm X 600 mm (effective)
   Ast= 1520 mm^2, Asc= 554 mm^2 , d'= 40 mm Use M_{20} concrete and Fe_{415} Steel
4. A simply supported slab of a corridor of a hospital building has a clear span 2.5 m and is supported on beams 230 mm width. Design the slab,if the beam is carrying a live load of 5 KN/m^2. Use M_{20} concrete and HYSD Fe_{415} bars.
5. Design a straight staircase supported on one side on wall and on the other side on a stinger beam. The horizontal span of stairs is 1.5 m. The rise is 150 mm and the tread of stairs is 280 mm. The live load acting on the stairs is 3 KN/m^2. Use M_{20} concrete and Fe_{415} Steel.

Learning Objectives
7.1 Introduction on Columns
7.1.1 Classification of columns
7.1.2 Materials of construction

7.1 Introduction on Columns

A compression member *i.e.,* column is an important element of every reinforced concrete structure. These are used to transfer the load of superstructure to the foundation safely. Mainly columns, struts and pedestals are used as compression members in buildings, bridges, supporting system of tanks, factories and many more such structures.

A column is defined as a vertical compression member which is mainly subjected to axial loads and the effective length of which exceeds three times its least lateral dimension. The compression member whose effective length is less than three times its least lateral dimension is called as **Pedestal.** The compression member which is inclined or horizontal and is subjected to axial loads is called as **Strut.** Struts are used in trusses.

7.1.1 Classification of columns

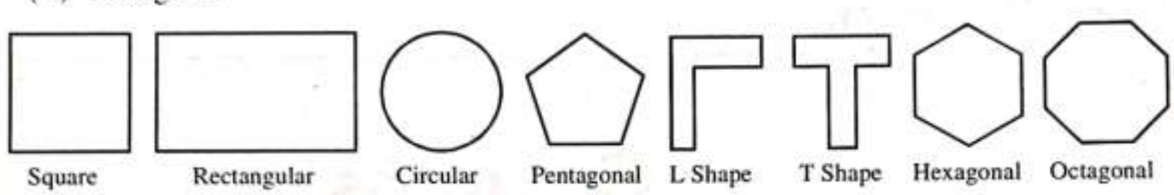
Columns are classified based on different criteria such as:

1. Shapes of cross-section.
2. Material of construction.
3. Type of loading.
4. Slenderness ratio.
5. Type of lateral reinforcement.

Shapes of Cross-section

On the basis of shape of the cross-section of the column, the column may be classified as following:

(*i*) Square
(*ii*) Rectangular
(*iii*) Circular
(*iv*) Pentagonal
(*v*) Hexagonal
(*vi*) Octagonal
(*vii*) T-shape or L-shape etc.



7.1.2 Materials of construction

Columns may be classified as following, as per the material used for construction.

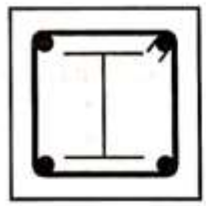
(*i*) **Timber Columns:** Timber columns are generally used for light loads. They are used in small trusses and wooden houses. These are called as posts.

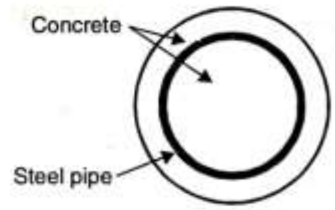
(*ii*) **Masonry Columns.** These are used for light loads.

(*iii*) **R.C.C. Columns:** R.C.C. columns are used for mostly all types of buildings and other R.C.C. structures like tanks, bridges etc.

(*iv*) **Steel Columns:** Steel columns are used for heavy loads.

(*v*) **Composite Columns:** Composite columns are used for very heavy loads. They consist of steel sections like joists (I or H sections) embedded in R.C.C. section as shown in Fig





MODULE-3 **Chapter - 7**
Session - 31

Learning Objectives
7.1.3 Types of loading,
7.1.4 Slenderness ratio
7.1.5 Effective length of column.

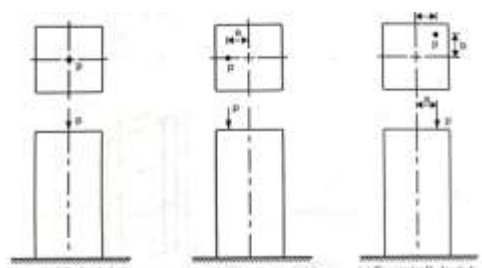
7.1.3 Types of loading,

A column may be classified as follows, based on type of loading:

(*i*) Axially loaded columns.
(*ii*) Eccentrically loaded columns.

(*i*) **Axially loaded column:** The columns which are subjected to loads acting along the longitudinal axis or centroid of the column section are called as axially loaded columns. Axially loaded column is subjected to direct compressive stress only and no bending stress develops anywhere in the column section.

(*ii*) **Eccentrically loaded columns:** Eccentrically loaded columns are those columns in which the loads do not act on the longitudinal axis of the column. They are subjected to direct compressive stress and bending stress both. Eccentrically loaded columns may be subjected to uniaxial Bending.



(a) Axially loaded column (b) Eccentrically loaded column (uniaxial bending) (c) Eccentrically loaded column (biaxial bending)

7.1.4 Slenderness ratio

The slenderness ratio of a compression member is defined as the ratio of effective length to the least lateral dimension. The columns are classified as following two types depending upon the slenderness ratio:

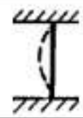
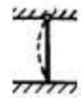
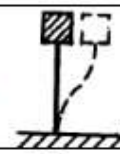
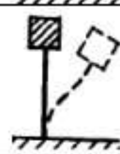
(*i*) Short columns
(*ii*) Long columns

(*i*) **Short Columns:** The column is considered as short when the slenderness ratio of column *i.e.*, ratio of effective length to its least lateral dimension $\left(\frac{l_{ef}}{b}\right)$ is less than or equal to 12.

(*ii*) **Long Column:** If the slenderness ratio of the column is greater than 12, it is called as long or slender column.

7.1.5 Effective length of column.

Table.Effective length of compression member

Sl. No.	Degree of End Restraint of Compression Members	Figure	Theo. Value of Effective Length	Reco. Value of Effective Length
1	Effectively held in position and restrained against rotation in both ends		0.50 l	0.65l
2	Effectively held in position at both ends, restrained against rotation at one end		0.70 l	0.80l
3	Effectively held in position at both ends, but not restrained against rotation		1.0 l	1.0l
4	Effectively held in position and restrained against rotation at one end, and at the other restrained against rotation but not held in position		1.0 l	1.20l
5	Effectively held in position and restrained against rotation in one end, and at the other partially restrained against rotation but not held in position		-	1.5l
6	Effectively held in position at one end but not restrained against rotation, and at the other end restrained against rotation but not held in position		2.0 l	2.0l
7	Effectively held in position and restrained against rotation at one end but not held in position nor restrained against rotation at the other end		2.0 l	2.0l

Learning Objectives

7.1.6 Types of lateral reinforcement

7.1.7 IS:456-2000 specification on column

Concrete is strong in compression. Thus, a column can be made up of plain concrete but it is always advisable to use R.C.C. columns instead of plain concrete columns because of following reasons :

1. A plain concrete column requires very large area as compared to R.C.C. columns. We have seen earlier that steel can take load m-times that of concrete of the same area. For a particular load, the section of an R.C.C. column will be much thinner than that of plain concrete. Thus by using R.C.C. columns a lot of space can be saved as the size of the column will be less.
2. A minimum area of steel is always provided in the column whether it is required for carrying load or not. It is done to resist tensile stresses which may be caused due to eccentricity of loads. (It is practically impossible to have a perfectly axially loaded structure).

There are two types of reinforcements provided in a R.C.C. column:

(*a*) Longitudinal reinforcement

(*b*) Transverse reinforcement.

(a) longitudinal Reinforcement

The longitudinal reinforcement consist of steel bars placed longitudinally in a column. It is also called as main steel. The functions of longitudinal reinforcement are as follows:

(*i*) To share the compressive loads along with concrete, thus reducing the overall size of the column and leaving more usable area.

(*ii*) To resist tensile stresses developed due to any moment or accidental eccentricity.

(*iii*) To impart ductility to the column.

(*iv*) To reduce the effect of creep and strinkage due to continuous constant loading applied for a long time.

(b) Transverse Reinforcement

The transverse reinforcement is provided along the lateral direction of the column in the form of ties or spirals enclosing the main steel. The function of transverse steel are as following

(*i*) To hold the longitudinal bars in position.

(*ii*) To prevent buckling of the main longitudinal bars.

(*iii*) To resist diagonal tension caused due to transverse shear developed because of any moment or load.

(*iv*) To impart ductility to the column.

(*v*) To prevent longitudinal splitting or bulging out of concrete by confining it in the core.

7.1.7 IS:456-2000 specification on column

7.1.7.1 Cover

The nominal cover for a longitudinal reinforcing bar in a column shall not be less than any of the following:

(*a*) 40 mm

(*b*) the diameter of the bar.

In the case of small sized columns of minimum dimensions of 200 mm or under whose reinforcing bar do not exceed 12 mm, a nominal cover of 25 mm may be used.

7.1.7.2 Slenderness Limit

The unsupported length between the end supports shall not exceed 60 times the least lateral dimensions.

$$l_{\text{unsupported}} \ngtr 60\, b$$

If in any given plane, one end of the column is restrained then unsupported length (l) shall not exceed 100 b^2/D.

$$l \ngtr \frac{100b^2}{D}$$

where b is the width of the section

D is the depth of the section.

Learning Objectives

7.1.7.3 Minimum eccentricity and arrangements of reinforcements

7.1.7.3 minimum Eccentricity

All columns shall be designed for minimum eccentricity given by:

$$e_{\min} = \frac{\text{Unsupported length}}{500} + \frac{\text{Lateral dimension}}{30}$$

$$e_{\min} \geq 20 \text{ mm}$$

It is not possible to have a perfectly axially loaded column. Some eccentricity will always be there due to non-homogeneity of materials and imperfections of the construction or due to some other reasons. Therefore IS 456:2000 recommends that all types of columns shall be designed for minimum eccentricity as mentioned above and given in Clause 25.4 of the code.

7.1.7.4 Longitudinal Reinforcement

(*a*) The cross-sectional area of longitudinal reinforcent in a column should not be less than 0.8 percent and not more than 6 percent of the gross cross-sectional area of the column.
The limit of minimum 0.8 to reinforcement is there to prevent buckling of column due to any accidental eccentricity causing moment. The use of 6 percent steel may involve practical difficulties because of congestion thus causing problem in placing and compacting concrete. Therefore, the code recommends a lower value of 4% instead of 6% for practical purposes.

(*b*) The minimum number of longitudinal bars in a column shall be four in rectangular column and 6 in circular column.

(*c*) The bars shall not be less than 12 mm in diameter.

(*d*) A reinforced concrete column having helical reinforcement shall have at least six longitudinal bars within the helical reinforcement.

(*e*) In a helically reinforced column, the longitudinal bars shall be in contact with the helical reinforcement and equidistant around its inner circumference.

(*f*) Spacing of longitudinal bars measured along the periphery of the column shall not exceed 300 mm.

(*g*) In any column that has a larger cross-sectional area than that required to support the load the percentage of steel shall be based upon the area of concrete required to resist the direct stress and not upon the actual area of the column.

(*h*) In case of pedestals, in which the steel reinforcement is not taken into account in strength consideration, nominal reinforcement, not less than 0.15% of the gross cross-sectional area, shall be provided.

7.1.7.5 Lateral or transverse Reinforcement

The lateral or transverse reinforcement in a column may be provided in the form of lateral ties or spirals or helical reinforcement. The IS code recommendations for lateral reinforcement are as follows :

(*a*) **Diameter of lateral ties:** The diameter of the lateral ties (links) should be greater than

(*i*) $\frac{1}{4}$th of the diameter of the largest longitudinal bar.

(*ii*) 6 mm.

(*b*) **Pitch of lateral ties:** The pitch or spacing of the lateral ties (links) should not be greater than the following:

(*i*) least lateral dimension of the column.

(*ii*) 16 times the diameter of the smallest longitudinal bar.

(*iii*) 300 mm.

(*c*) **Diameter of spiral or helical reinforcement:** The diameter of helical or spiral reinforcement bar should be greater than

(*i*) $\frac{1}{4}$th of the diameter of the largest longitudinal bar.

(*ii*) 6 mm.

(*d*) **Pitch of helical reinforcement:** Helical reinforcement should be spaced evenly and its ends should be anchored properly by providing one and a half turn extra. The pitch of the helical turns should not be more than:

(*i*) 75 mm

(*ii*) $\frac{1}{6}$th of the diameter of core of concrete.

The pitch of the helical turns should not be less than:

(*i*) 25 mm

(*ii*) 3 times the diameter of steel bar forming the helix.

7.1.7.6 Arrangement of Transverse Reinforcement

The arrangement of transverse reinforcement as per IS 456:2000 (Clause 26.5.3.2(*b*)) is as follows :

(*i*) If the longitudinal bars are not spaced more than 75 mm or either side, the transverse reinforcement need only to go round corner and alternate bars for the purpose of providing effective lateral supports.

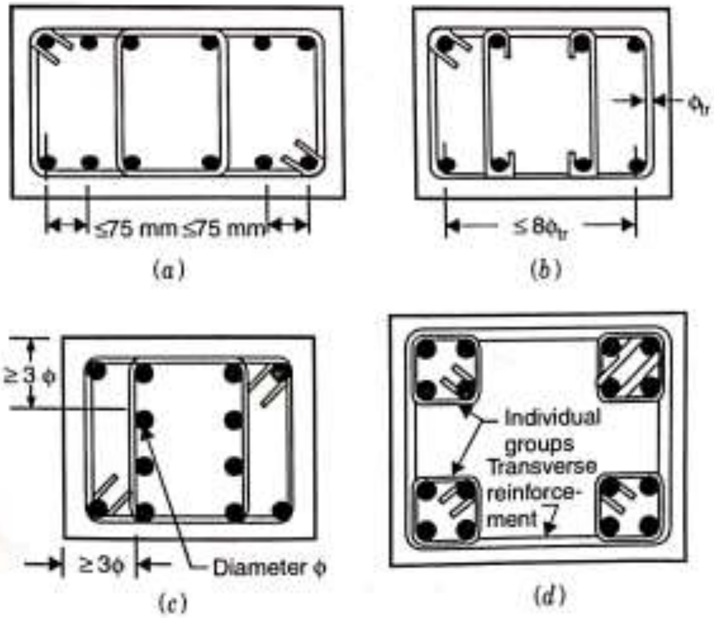
(*ii*) If the longitudinal bars are spaced at a distance not exceeding 48 times the diameter of the bar, the ties are effectively tied in two directions, additional longitudinal bars in between these bars need to be tied in one direction by open ties.

(*iii*) Where the longitudinal reinforcing bars in a compression member are placed in more than one row, effective lateral support to the longitudinal bars in the inner rows may be assumed to have been provided if:

(*a*) Transverse reinforcement is provided for the outer most row in accordance with (*ii*)

(*b*) No bar of the inner row is closer to the nearest compression face than three times the diameter of the largest bar in the inner row.

(*iv*) Where the longitudinal bars in a compression member are grouped (not in contact) and each group adequately tied with transverse reinforcement in accordance with the codal provisions, the transverse reinforcement for the compression member as a whole may be provided on the assumption that each group is a single longitudinal bar for the purpose of determining the pitch and diameter of the transverse reinforcement in accordance with the codal provisions. The diameter of such transverse reinforcement should not exceed **20 mm.**



Learning Objectives

7.2.1 Ultimate load formulation and calculations

7.2.1.1 Short column with rectangular ties

7.2.1.2 Short column with helical reinforcement

7.2.1.1 Short column with rectangular ties

The short column or lateral ties shall be designed by the following equation (*i*) if the minimum eccentricity does not exceed 0.05 times the lateral dimension *i.e.*,

$$e_{min} = \frac{l}{500} + \frac{D}{30} \geq 20 \text{ mm}$$

$$e_{min} \not> 0.05\, D$$

$$\boldsymbol{P_u = 0.4\, f_{ck} \,.A_c + 0.67\, f_y\, A_{sc}} \qquad \ldots(i)$$

where P_u = factored axial load

A_c = area of concrete

A_{sc} = area of longitudinal reinforcement of column

f_{ck} = characteristic strength of the compression longitudinal reinforcement.

If the minimum eccentricity is greater than $0.05D$, the section is designed for combined, axial load and bending.

Note. Minimum eccentricity as per code = 20 mm

$\therefore \quad 0.05D \geq 20$ mm

or $\quad D \geq \frac{20}{0.05}$ or 400 mm

Also $\quad \frac{l}{500} + \frac{D}{30} \leq 0.05\, D$ or $\frac{D}{20}$

$\frac{l}{500} \leq \frac{D}{20} - \frac{D}{30}$ or $0.016D$

or $\quad D \geq 0.12\, l$

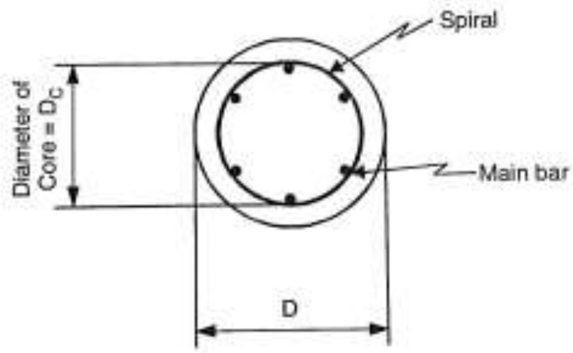
Hence all column which have D less than 400 mm or 0.12 l, the minimum eccentricity will be greater than 0.05 D and hence equation (*i*) will not be applicable.

7.2.1.2 Short column with helical reinforcement

The strength of a column with helical reinforcement is calculated as following:

$$P_u = 1.05\left[0.4\, f_{ck}\, A_c + 0.67\, f_y\, A_{sc}\right] \quad \ldots(ii)$$

Columns with helical reinforcement (spirals) are more ductile and have improved strength. Thus, their load carrying capacity is more than that of the columns with lateral ties. *As per IS 456: 2000 the safe load for columns with helical reinforcement is 1.05 times the safe load carried by a similar column with lateral ties, provided the requirements given below are fulfilled :*



The ratio of the volume of helical reinforcement to the volume of the core shall not be less than $0.36\left(\dfrac{A_g}{A_k}-1\right)\dfrac{f_{ck}}{f_y}$

$$\frac{\text{Volume of helical reinforcement}}{\text{Volume of core}} > 0.36\left(\frac{A_g}{A_k}-1\right)\frac{f_{ck}}{f_y}$$

where
A_g = gross cross sectional area of column say $\frac{\pi}{4}D^2$ for a circular column.
A_k = area of the core of helically reinforced column measured to the outside diameter of helix.
f_{ck} = characteristic compressive strength of concrete.
f_y = characteristic strength of the helical reinforcement but not exceeding 415 N/mm^2
ϕ_s = diameter of spiral
p = pitch of spiral

$$A_g = \frac{\pi}{4}D^2,$$

$$A_k = \frac{\pi}{4}D_c^2$$

$$D_c = D - 2\,(\text{end cover})$$

$$D' = D - 2\,(\text{end cover}) - \phi_s$$

Volume of helical reinforcement per mm height of column

$$= \frac{\text{Circumference of spiral} \times \text{Area of spiral}}{\text{Pitch of spiral}} = \frac{\pi \times D' \times \frac{\pi}{4} \times \phi_s^2}{p}$$

Volume of core per mm height of column $= \frac{\pi}{4} \times D_c^2 \times 1.$

Learning Objectives

7.3 Design of axially loaded short columns

Steps for design of short axially loaded column

Given: Factored load (Pu)

Material – Grade of concrete and steel.

1. Assume suitable percentage of A_{sc} (say 0.8% to 4%), $A_{sc} = pA_g$.
2. Determine A_c in terms of A_g.

$$A_c = A_g - A_{sc}$$

3. Calculate A_g as follows:

$$P_u = 0.4f_{ck}(A_g - A_{sc}) + 0.67f_y \cdot A_{sc}$$

Putting $A_{sc} = p.A_g$, calculate A_g as all other terms are known.

4. Calculate dimensions of column as follows:
   For square column, $B^2 = A_g$
   For rectangular column, $B \times D = A_g$ [assume B and calculate D]
5. Provide area of reinforcement (A_{sc})
6. Design lateral ties as per IS specifications.

Note: If length of column is given, then calculate effective length (l_{eff}) on the basis of end conditions given in Table 13.1 then check for slenderness ratio and minimum eccentricity as follows:

$$\frac{l_{eff}}{b} < 12$$

Calculate minimum eccentricity e_{min} as follows:

$$e_{min} = \frac{l}{500} + \frac{D}{30} \geq 20 \text{ mm, and } \frac{e_{min}}{D} < 0.05 \text{ for an axially loaded column.}$$

Problem

A reinforced concrete short column is 400 mm x 400 mm and has 4 bars of 20 mm diameter. Determine the ultimate load carrying capacity of column if M_{20} concrete and Fe_{415} steel is used Assume e_{mim} < 0.05D.

Solution. Given: $b = 400$ mm, $d = 400$ mm

$$A_{sc} = 4 \times \frac{\pi}{4} \times 20^2 = 1256.6 \text{ mm}^2$$

$$A_g = 400 \times 400 = 160000 \text{ mm}^2$$

$$f_{ck} = 20 \text{ N/mm}^2 \quad \text{[For M 20 concrete]}$$

$$f_y = 415 \text{ N/mm}^2 \quad \text{[For Fe 415 steel]}$$

$$A_c = A_g - A_{sc}$$

$$= 160000 - 1256.6 = 158743.4 \text{ mm}^2$$

Load on column, $P_u = 0.4 f_{ck} A_c + 0.67 f_y . A_{sc}$

$$= 0.4 \times 20 \times 158743.4 + 0.67 \times 415 \times 1256.6$$

$$P_u = 1619344.8 \text{ N or } 1619.3 \text{ kN}$$

∴ Ultimate load carrying capacity of column = **1619.3 kN**

Problem

Find the ultimate load carrying capacity and allowable load for a short column of size 500 mm X 500 mm . The column is reinforced with 4-25 mm diameter bars. Use M_{20} concrete and HYSD grade Fe_{415} Steel. Assume e_{mim} < 0.05D.

Solution. Given: $b = 500$ mm, $d = 500$ mm

$$A_{sc} = 4 \times \frac{\pi}{4} \times 25^2 = 1964 \text{ mm}^2$$

$$f_{ck} = 20 \text{ N/mm}^2 \quad \text{[For M20 concrete]}$$

$$f_y = 415 \text{ N/mm}^2 \quad \text{[For Fe 415 steel]}$$

$$A_g = 500 \times 500 = 250000 \text{ mm}^2$$

$$A_c = A_g - A_{sc} = 250000 - 1964 = 248036 \text{ mm}^2$$

Since $e_{min} < 0.05\,D$

$$P_u = 0.4 f_{ck} . A_c + 0.67 f_y . A_{sc}$$

$$= 0.4 \times 20 \times 248036 + 0.67 \times 415 \times 1964$$

$$P_u = 2530378.2 \text{ N}$$

Ultimate load $= 2530.3$ kN

Allowable service load $= \dfrac{2530.3}{1.5}$

$= $ **1686.92 kN**

MODULE-3 **Chapter - 7**
Session - 35

Learning Objectives
7.4 Design of axially loaded short columns (Problems)

Problem
An R.C.C short column of 400 mm X 500 mm is carrying a factored load of 3000 KN. Design the column assuming e_{min} <0. 05D.Use M_{25} concrete and Fe_{415} steel.

Solution. Given:

$$b = 400 \text{ mm},$$
$$D = 500 \text{ mm}$$
$$P_u = 3000 \text{ kN} = 3000 \times 10^3 \text{ N}$$

$$e_{min} < 0.05 \text{ D}$$
$$f_{ck} = 25 \text{ N/mm}^2 \quad \text{[For M25 concrete]}$$
$$f_y = 415 \text{ N/mm}^2 \quad \text{[For Fe 415 steel]}$$

■ **Area of steel (A_{sc})**

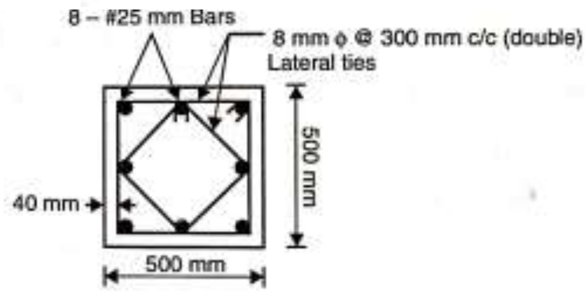
$e_{min} < 0.05\ D$, So factored load is written as

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$
$$A_c = A_g - A_{sc}$$
$$= 400 \times 500 - A_{sc} = 200000 - A_{sc}$$
$$3000 \times 10^3 = 0.4 \times 25(2000000 - A_{sc}) + 0.67 \times 415 \times A_{sc}$$
$$268.05\ A_{sc} = 1000 \times 10^3$$
$$A_{sc} = 3730.6 \text{ mm}^2$$

Using 25 mm ϕ bar, $A_\phi = \frac{\pi}{4} \times 25^2 = 490 \text{ mm}^2$

No. of bars reqd $= \frac{3730.6}{490} = 7.6$ says $= 8$ nos.

$\therefore$ Provide 8–25 mm ϕ bars as shown is Fig. 13.9.



■ **Lateral ties**

The diameter of ties should not be less than

(*i*) $\frac{1}{4} \times 25 = 6.25$ m

(*ii*) 6 mm

$\therefore$ Using 8 mm dia ties.

The pitch of ties should not be less than following:

(*i*) Least lateral dimension = 400 mm

(*ii*) $16 \times 25 = 400$ mm

(*iii*) 300 mm

$\therefore$ Provide 8 mm ϕ @ 300 mm c/c as double ties. The arrangement of reinforcement is shown is **fig.**

Problem

A short R.C.C square column is required to carry a factored load of 1900 KN. Design the column. Assume $e_{min} < 0.05D$ and use M_{20} concrete and mild steel.

Solution. Given:

$$P_u = 1900 \text{ kN} = 1900 \times 10^3 \text{ N}$$

$$e_{min} < 0.05\,D$$

$$f_{ck} = 20 \text{ N/mm}^2 \qquad \text{[For M20 concrete]}$$

$$f_y = 250 \text{ N/mm}^2 \qquad \text{[For mild steel bar]}$$

$$e_{min} < 0.05\,D$$

$$\therefore \quad D > \frac{20}{0.05} \text{ or } 400 \text{ mm} \qquad [\because e_{min} = 20 \text{ mm}]$$

- Assuming 0.8 percent steel,

$$A_{sc} = \frac{0.8}{100} \times A_g = 0.008\,A_g$$

$$A_c = A_g - A_{sc}$$

$$= A_g - 0.008\,A_g = 0.992\,A_g$$

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y . A_{sc}$$

$$1900 \times 10^3 = 0.4 \times 20 \times 0.992 A_g + 0.67 \times 250 \times 0.008 A_g$$

$$9.276\,A_g = 1900 \times 10^3$$

$$A_g = 204962.2 \text{ mm}^2$$

$$\text{Side of column} = 452.7 \text{ mm}$$

$\therefore$ Adopting 460 mm × 460 mm as size of column

$$A_{sc} = 0.008 \times 460 \times 460$$

$$= 1693 \text{ mm}^2$$

$\therefore$ Providing 20 mm dia bars

$$A_\phi = \frac{\pi}{4} \times 20^2 = 314 \text{ mm}^2$$

$$\text{Number of bars reqd} = \frac{1693}{314} \simeq 6$$

∴ Provide 6–20 mm φ bars,

$$A_{sc} = 6 \times 314 = 1884 \text{ mm}^2$$

■ **Lateral ties**

The diameter of lateral ties should not be less than

(*i*) $\frac{1}{4} \times 20 = 5$ mm

(*ii*) 6 mm

Using 6 mm φ ties

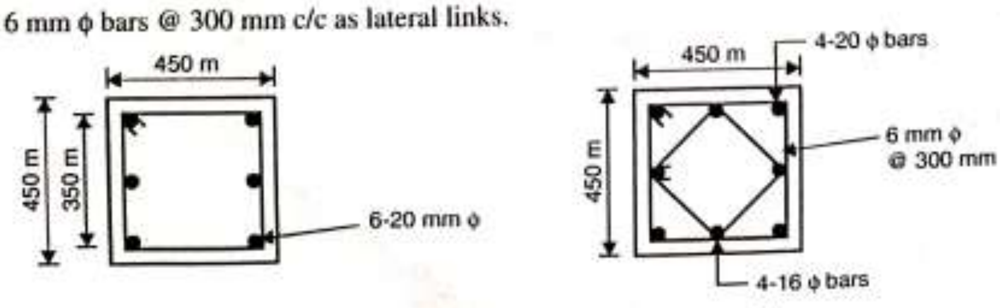
The pitch of the ties should not exceed

(*i*) Least lateral dimension *i.e.*, 460 mm

(*ii*) $16 \times 20 = 320$ mm

(*iii*) 300 mm

Provide 6 mm φ bars @ 300 mm c/c as lateral links.



(a) Wrong arrangement *(b)*

The distance between the corner bars is greater than 300 mm ($450 - 2 \times 40 - 20 = 350$ mm). Hence the arrangement of reinforcement is to be changed. Let us use 4–20 mm dia bars and 4–16 mm dia bars as shown in Fig. 13.10.

$$A_{sc \text{ provided}} = 4 \times 314 + 4 \times 201 = 2060 \text{ mm}^2$$

MODULE-3 **Chapter - 7**
Session - 35

Learning Objectives

7.4 Design of Short columns with helical reinforcement

Problem

Design the reinforcement for a circular column of diameter 500 mm subjected to an ultimate load of 1600 KN and an Ultimate moment of 125 KNm about the major axis. Use M_{20} Concrete Fe_{250}.

Solution. Given

$$b = D = 500 \text{ mm}$$
$$Pu = 1600 \text{ kN}$$
$$Mu = 125 \text{ kNm}$$
$$f_{ck} = 20 \text{ N/mm}^2$$
$$f_y = 250 \text{ N/mm}^2 \text{ for bars upto 20 mm dia}$$
$$= 240 \text{ N/mm}^2 \text{ for bars over 20 mm dia}$$

- Assuming effective cover to be 50 mm, $d' = 50$ mm

$$\frac{d'}{D} = \frac{50}{500} = 0.1$$

- Pitch of the ties should not be less than:

$$D = 500 \text{ mm}$$
$$16 \times 25 = 400$$
$$= 300 \text{ mm}$$

Hence provide 8 mm ties @ 300 mm c/c.

For $\dfrac{P_u}{f_{ck} D^2} = 0.32$ and $\dfrac{M_u}{f_{ck} bD^3} = 0.05$

from Chart 52, we get

$$\frac{p}{f_{ck}} = 0.087 \Rightarrow p = 0.08 \times 20 = 1.74$$

- $$p = \frac{A_{sc}}{A_g} \times 100$$

$$A_{sc} = 1.74 \times \frac{\pi}{4} \times \frac{(500)^2}{100} = 3416 \text{ mm}^2$$

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Session - 36

Learning Objectives
7.5 Introduction on footings .
7.5.1 Classification

7.5 Introduction on footings.

For 25 mm diaemter bars,

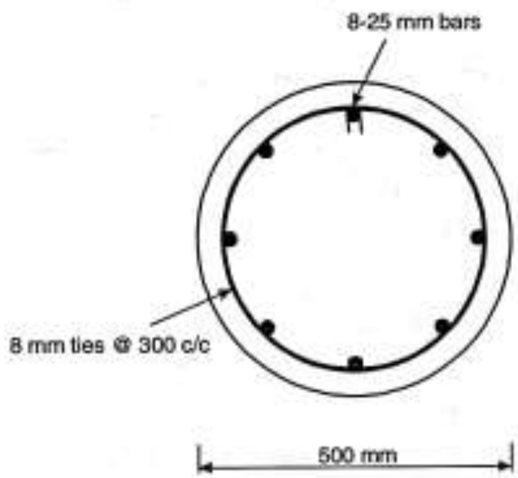
$$A_{sc} = \frac{3416}{240} \times 250 = 3559 \text{ mm}^2$$

Hence provide 8–25 mm diameter bars (A_{sc} = 3926 mm^2)

Design of Lateral Ties

- Diameter of ties should not be less than
  - $\frac{1}{4} \times 25 = 6.25$ mm
  - 6 mm

Hence provide 8 mm diameter lateral ties



Every R.C.C. structure can be divided into two parts:

(*i*) Portion which is above the ground, called super structure.

(*ii*) Portion below the ground level, called sub-structure or foundation.

The purpose of foundation in a structure is as follows:

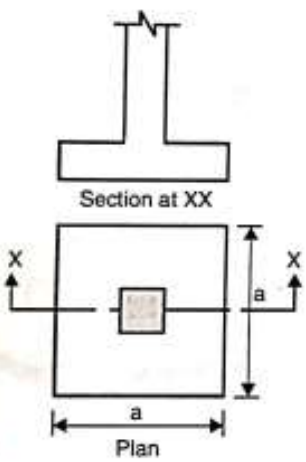
(*i*) To safely transmit the loads and moments from the super-structure to the soil, so that the pressure on the soil does not exceed bearing capacity at any point.

(*ii*) To ensure safety with respect to permissible settlement, tilting in one direction, overturning, uplift pressure etc.

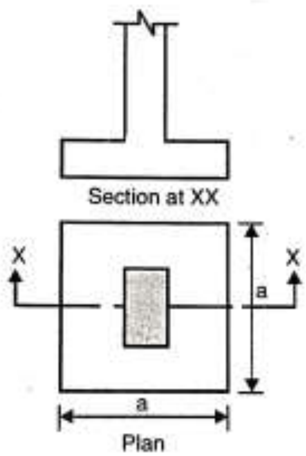
7.5.1 Classification

Different types of foundations are provided depending upon the type of structure, distribution of loads, type and capacity of sub-soil, presence and level of water table etc. Following are some of the common types of foundations:

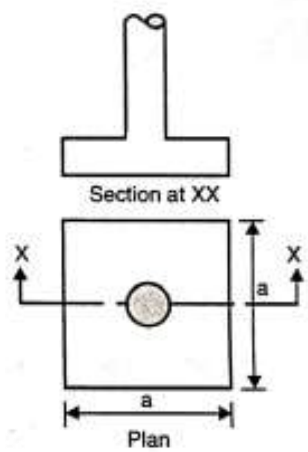
- *(i)* ***Isolated footing***
- *(ii)* ***Rectangular footing***
- *(iii)* ***Circular footing***
- *(iv)* ***Circular footings.***
- *(v)* ***Combined footings***
- *(vi)* ***Continuous footing***
- *(vii)* ***Strap footing***
- *(viii)* ***Raft footing***
- *(ix)* ***Pile footing***
- *(x)* ***Well foundation.***



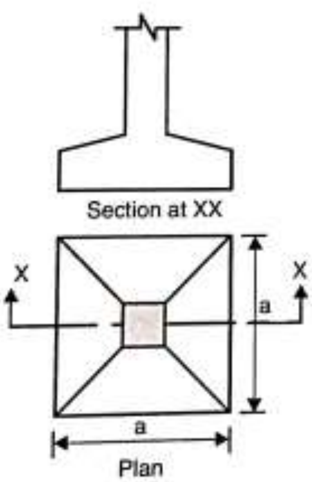
Square footing



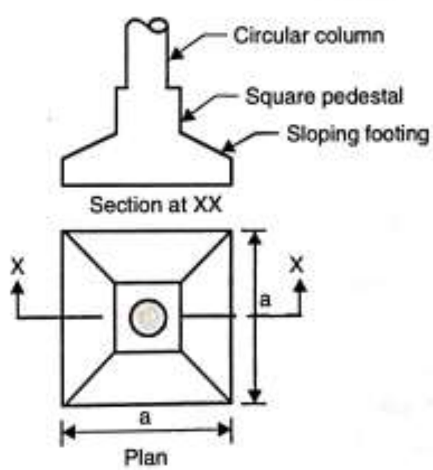
Rectangular footing



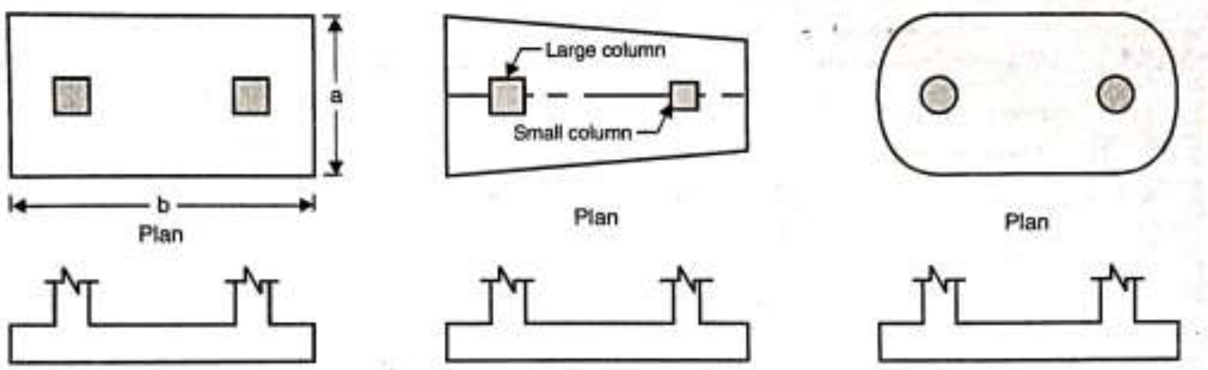
Square footing for a circular or polygonal column



Square columns with sloping footing

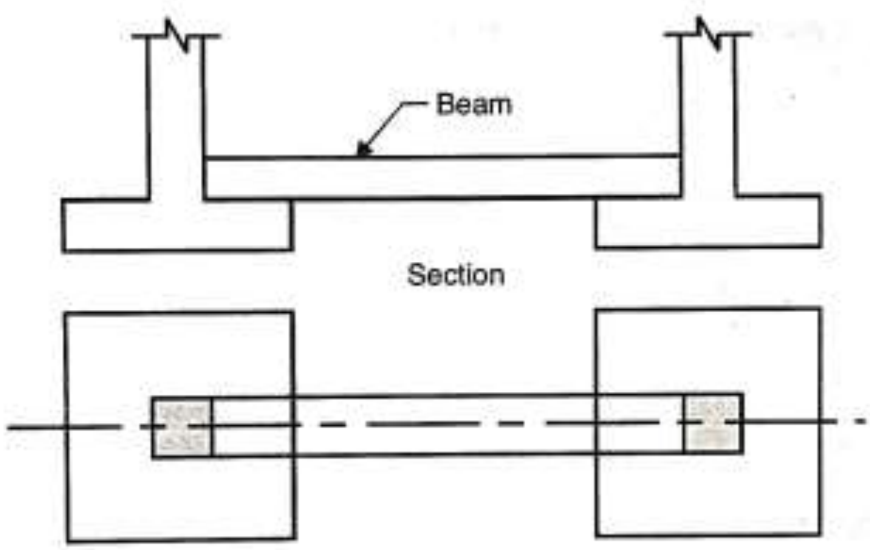


Circular column with square pestal and footing

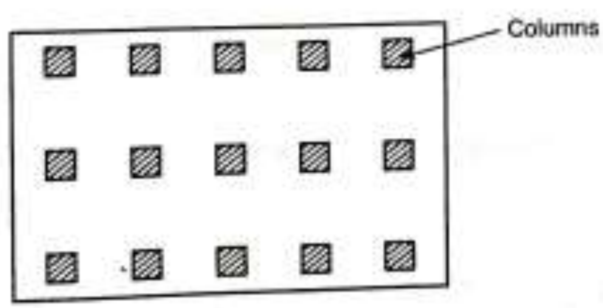


(a) Rectangular (b) Trapezoidal (c) Elliptical

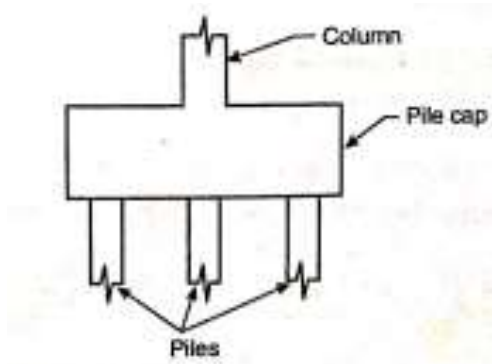
Combined Footings



Strap foundation



Raft Foundation



Pile foundation

MODULE-3

Chapter - 7
Session - 37

Learning Objectives
7.5.2 IS Codal provisions for design of footings

The footings which are provided under single columns are called as isolated footings. These are usually square or rectangular and rarely circular. Even for columns of circular, hexagonal, octagonal or any other shape, it is preferable to provide rectangular or square foundations. Isolated footings are ideally provided when loads are small and the soil is not very poor, *i.e.*, its bearing capacity is sufficient. Isolated footings are of two types:

(*a*) Uniform thickness footings
(*b*) Tapered thickness footings (sloped footing).

Codal Provision for design of isolated footing

(1) Thickness at the Edge of Footings

The thickness at the edge of footings shall not be less than 150 mm.

(2) Moments and Forces

The greatest bending moment to be used in the design of an isolated footing shall be at sections located as follows :

(*a*) At the face of the column, pedestal or wall, for footings supported a concrete, pedestal or wall, Fig. 14.6 (*a*).
(*b*) Halfway between the centre-line and the edge of the wall for footings under a masonry wall (Fig. 14.6 (*b*)).
(*c*) For isolated footing, shear stress at critical section should not exceed $k_s \tau_c$ (Cl. 31.6.3 of IS code)

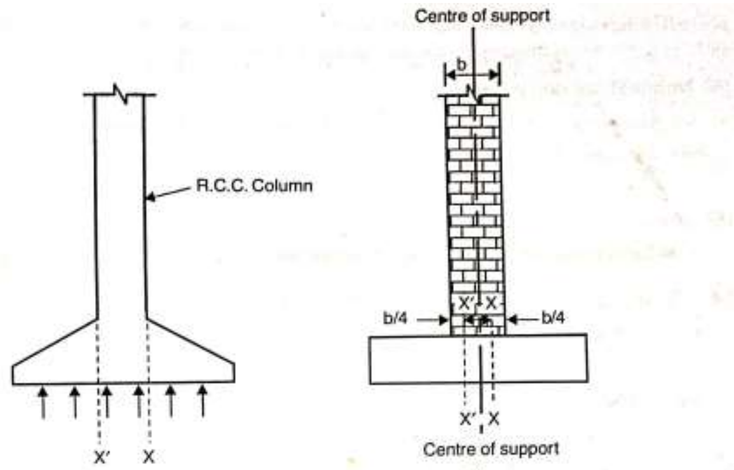
$$\tau_v \leq k_s \tau_c$$

where

$$k_s = (0.5 + \beta_c) \text{ but } k_s \ngtr 1$$

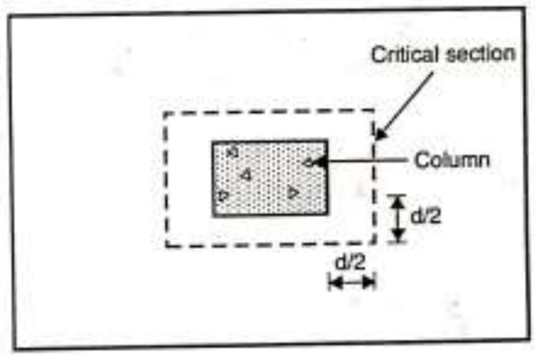
$$\beta = \frac{\text{Short dimension of column}}{\text{Long dimension of column}}$$

$$\tau_c = 0.16\sqrt{f_{ck}}$$



(a) (b)

(3) Shear and Bond



Shear strength of footings is governed by more severe of the following two conditions:

(a) One way shear: The footing acts as a wide beam *i.e.*, one way action; the critical section for this shear shall be assumed at a distance '*d*' equal to the effective depth of footing from the face of the column, pedestal or wall.

(b) Two way action or punching shear : The Crtical section for shear shall be at a distance of d/2 from the face of the column. Where d is the effective depth of the section. The critical section for checking the development length in a footing shall be assumed at the same plane as described for the bending moment.

(4) Tensile Reinforcement

(a) In a square footing, the reinforcement in each direction shall be distributed uniformly across the full width of the footing.

(b) In rectangular footing, the reinforcement in the long direction shall be distributed across the full width of the footing. For reinforcement in short direction, a central band equal to the width of the footing is taken length of the footing and reinforcement determined as per the equation given below shall be uniformly distributed across the central band:

$$\frac{\text{Reinforcement in central bandwidth}}{\text{Total reinforcement in short direction}} = \frac{2}{\beta + 1}$$

where β is the ratio of the long side to the short side of the footing. The remainder of the reinforcement shall be uniformly distributed in the outer portion of the footing.

(5) Nominal Reinforcement

(*i*) Minimum reinforcement and spacing shall be as per requirement of solid slab.

(*ii*) The nominal reinforcement for concrete sections greater than 1 m thick shall be 360 mm^2 per m length in each direction, if it works out lesser as per requirements of solid slab.

(6) Cover to Reinforcement

A minimum clear cover of 50 mm shall be provided to the reinforcement in foundations under soil.

MODULE-3 **Chapter - 7**
Session - 38

Learning Objectives
7.5.3 Design procedure of isolated footings

Design Steps for rectangular Isolated footing

Given: Load on column, safe bearing capacity of soil, grade of concrete and steel.

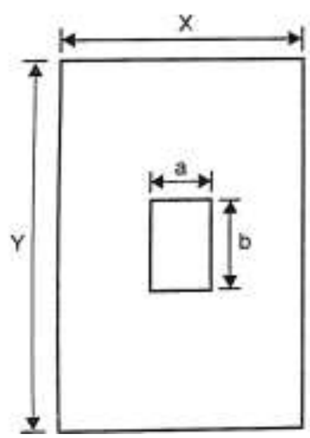
1. Find design constants $\frac{x_{umax}}{d}$ and R_u for given steel and concrete grades from Table 4.2 and Table 4.3.
2. Calculate area of footing as follows:

$$A = \frac{w_c + w_f}{q_0}$$

where w_c = load on column

w_f = self wt of footing + pedestal if provided (usually considered as 10% of w_c)

q_0 = safe bearing capacity of soil



3. Calculate the size of footing:

(*a*) For square footing, side of footing $S = \sqrt{A}$, round off

(*b*) For rectangular footing, assume one dimension (say X) and calculate the other dimensions (say Y) as follows:

$$Y = \frac{A}{X} \text{ round off to nearest 5 or 10 cms.}$$

X

Alternatively, if ratio of width to length of column and footing are assumed to be similar (say a/b), then bending moment is same in both directions.

4. Calculate the soil pressure due to factored column load only, as follows:

$$P_u = \frac{1.5\, w_c}{X.Y}$$

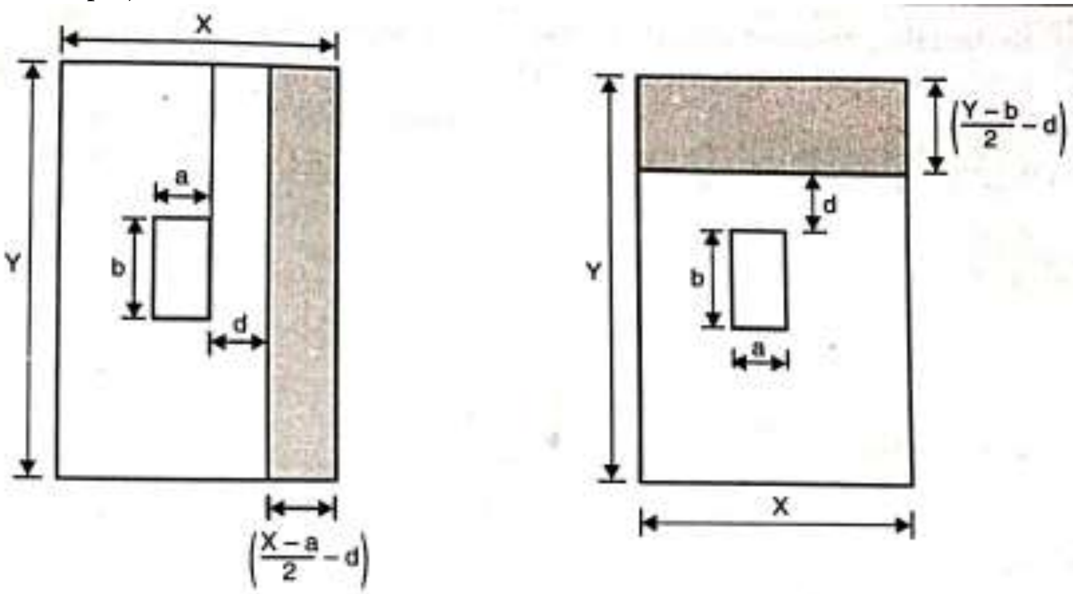
where w_c = column load

X = shorter dimension of footing

Y = longer dimension of footing

5. Depth of footing is calculated by the following three criteria and highest value so calculated is adopted in the design

(a) **<u>By one-way shear Criterion:</u>** The critical section for one-way shear is taken at a distance d (effective Depth) from the columns face.



Shear force at the critical section

$$V_u = p_u \times X \times \left(\frac{Y-b}{2} - d\right) \quad ...(i)$$

Shear force resisted by concrete $= \tau_c Xd$ ...(*ii*)

Equating (*i*) and (*ii*)

$$\tau_c\, Xd = p_u \times X \times \left(\frac{Y-b}{2} - d\right)$$

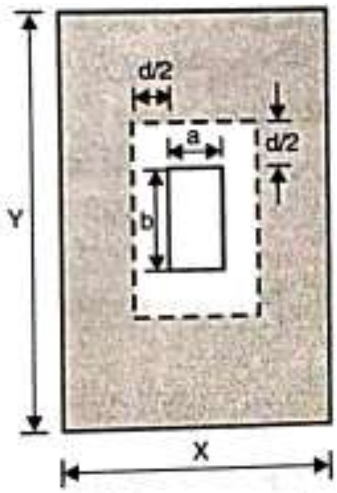
As exact percentage of reinforcement to be provided is not yet known, τ_c may be assumed as that corresponding to minimum reinforcement, *i.e.*, 0.2%. For M20, this value may be taken as 0.32 N/mm^2.

(*b*) **By two way shear criterion:-** The critical section for two way shear or punching shear as it is commonly called, is at a distance $d/2$ from the face of the column.

Referring this figure

$$= 2\left(a+\frac{d}{2}+\frac{d}{2}+b+\frac{d}{2}+\frac{d}{2}\right)$$

$$= 2(a+b+2d)$$



Area of concrete resisting punching shear

$$A = 2(a+b+2d)\times d$$

Punching shear on the critical section

$$= p_u\,(XY-(a+d)\times(b+d)) \qquad \text{...(iii)}$$

Punching shear resisted by the section

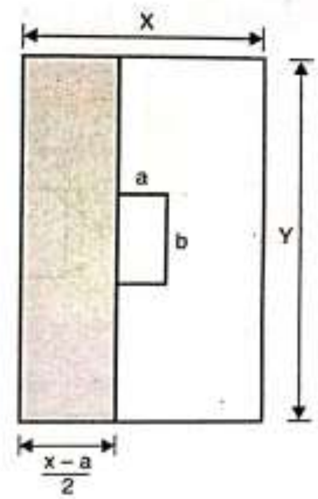
$$= \tau_c \times A$$

$$= \tau_c \times 2\,(a+b+2d)\times d \qquad \text{...(iv)}$$

where $\tau_c = 0.25\sqrt{f_{ck}}$

by equating the two expressions, *(iii)* and *(iv)* we can calculate the depth of footing.

(*c*) **By bending moment criterion:** The critical section for bending moment is shown in Fig. 14.11.



$$p_u\left(\frac{X-a}{2}\right)\left(\frac{X-a}{4}\right) = \frac{p_u}{8}(X-a)^2 \qquad \text{...(v)}$$

$$\text{B.M. in } Y \text{ direction} = \frac{p_u}{8}(Y-b)^2 \qquad \text{...(vi)}$$

Moment of resistance of section

$$= 0.36\,f_{ck}\cdot\frac{x_{u\lim}}{d}\left(1-\frac{0.42\,x_{u\lim}}{d}\right)Yd^2$$

Equating the (*v*) and (*vi*) with the moment of resistance we get the value of *d*.

The highest value of depth as obtained in steps (*a*), (*b*) and (*c*) above shall be adopted as effective depth of the footing.

6. Determine the area of reinforcement required by following equation.

$$M_u = 0.87\,f_y\,A_{st}\,d\left(1-\frac{A_{st}\,f_y}{Xd\,f_{ck}}\right)$$

The reinforcement area so calculated should not be less than the minimum reinforcement and distributed as per IS code provisions.

NOTE:	**For design of square footing, follow the above mentioned procedure and substitute $X = Y$ and $a = b$**

Learning Objectives

7.5.4 Numerical on footings

Problem

Design a square footing of uniform thickness for an axially loaded column of 450 mm X 450 mm size. The safe bearing capacity of soil is 190 KN/m2. Load on column is 850 Kn. Use M_{20} concrete and Fe_{415} Steel.

Solution. Given,

$$w_c = 850 \text{ kN}$$

$$\text{Bearing capacity} = 190 \text{ kN/m}^2$$

$$f_{ck} = 20 \text{ N/mm}^2,$$

$$f_y = 415 \text{ N/mm}^2$$

■ **Load calculation**

$$w_c = 850 \text{ kN}$$

$$\text{Self wt. of footing, } w_f = 10\% \text{ of } w_c = 85 \text{ kN}$$

$$w_c + w_f = 850 + 85 = 935 \text{ kN}$$

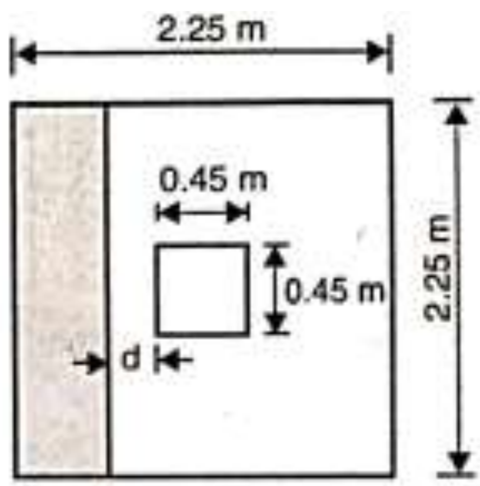
■ **Area of footing**

$$A = \frac{w_c + w_f}{q_u} = \frac{935}{190} = 4.92 \text{ m}^2$$

$$\text{Side of square footing} = \sqrt{4.92} = 2.22 \text{ m say } 2.25 \text{ m}$$

Factored soil pressure due to column load only

$$p_u = \frac{1.5 \times 850}{2.25 \times 2.25} = 251.85 \text{ kN/m}^2$$



Depth of footing by one way shear criterion

Critical section shall be at a distance d from the face of the column.

Shear force due to factored soil pressure at critical section

$$= 2.25 \times \left(\frac{2.25 - 0.45}{2} - d \right) \times 251.85$$

$$= 566.66\ (0.9 - d) \qquad \text{...}(i)$$

Assuming 0.2% steel, $\tau_c = 0.32$ N/mm^2 from Table 5.1

Shear force resisted by the section

$$= \tau_c\ Xd$$

$$= \frac{0.32 \times 10^6}{10^3} \times 2.25 \times d \qquad \left[\tau_c = \frac{0.32 \times 10^6}{10^3}\ \text{kN/m}^2 \right]$$

$$= 720\ d \qquad \text{...}(ii)$$

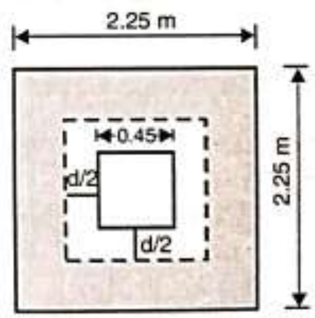
Equating (i) and (ii) we get

$$566.66\ (0.9 - d) = 720\ d$$

$$1286.66d = 509.99$$

$$d = \frac{509.99}{1286.66} = 0.396 \text{ m} \qquad \text{...[A]}$$

Depth of footing by two way shear



Considering critical section is at $\frac{d}{2}$ from the face of column.

Perimeter of critical section = $4(0.45 + d) = 1.80 + 4d$

Shear force at critical section

$$= 251.85 \times (2.25 \times 2.25 - (0.45 + d)^2)$$

$$= 1274.99 - 251.85\ (0.2025 + d^2 + 0.9d) \qquad \text{...}(iii)$$

Shear force resisted by the critical section

Maximum allowable shear stress = $0.25\sqrt{f_{ck}}$

$$= 0.25\sqrt{20} = 1.118 \text{ N/mm}^2 = 1118 \text{ kN/m}^2$$

$$\text{Shear force resisted} = 1118\,(1.80 + 4d) \times d = 2012.4d + 4472\,d^2 \qquad \text{...}(iv)$$

Equating (*iii*) and (*iv*)

$$1274.99 - 251.85\,(0.2025 + d^2 + 0.9d) = 2012.4d + 4472d^2$$

$$d^2 - 0.423d - 29 = 0$$

$$d = \frac{-0.423 \pm \sqrt{(-0.423)^2 + 4 \times (0.29)}}{2}$$

$$d = 0.367 \text{ m.} \qquad \text{...[B]}$$

■ **Depth of footing by bending moment criterion**

Critical section is at the face of column.

Bending moment at the critical section

$$M_u = 251.85 \times 2.25 \times \left(\frac{2.25 - 0.45}{2}\right) \times \left(\frac{2.25 - 0.45}{4}\right)$$

$$= 566.66 \times \frac{1.80^2}{8}$$

$$= 229.498 \text{ kNm}$$

$$= 229.498 \times 10^6 \text{ Nmm} \qquad \text{...}(v)$$

Moment of resistance at critical section:

$$\frac{x_{u\max}}{d} = 0.48 \quad \text{and} \quad R_u = 2.76 \text{ for M20 concrete and Fe 415 steel}$$

$$M_{u\,\lim} = R_u \times bd^2$$

$$= 2.76 \times 2250 \times d^2 = 6210\,d^2 \qquad \text{...}(vi)$$

Equating (*v*) and (*vi*), we get

$$229.498 \times 10^6 = 6210\,d^2$$

$$d = 192.24 \text{ mm}$$

$$= 0.192 \text{ m} \qquad \text{...[C]}$$

From Eqs. [A], [B] and [C], the highest value of d obtained is 0.396 m.

Let us adopt **d = 400 mm.**

Overall depth = 400 + 8 + 50

= 458 say 460 mm [Taking clear cover = 50 mm and 16 mm dia. bars]

■ **Area of steel reinforcement**

$$M_u = 0.87\, f_y\, A_{st}\, d\left(1 - \frac{A_{st}\, f_y}{bd\, f_{ck}}\right)$$

$$229.498 \times 10^6 = 0.87 \times 415 \times A_{st} \times 400\left(1 - \frac{A_{st} \times 415}{2250 \times 400 \times 20}\right)$$

or $A_{st}^2 - 43369.37\, A_{st} + 68918318.32 = 0$

$$A_{st} = \frac{43369.37 \pm \sqrt{43369.37^2 - (4 \times 68918318.32)}}{2}$$

$$= 1652.03 \text{ mm}^2$$

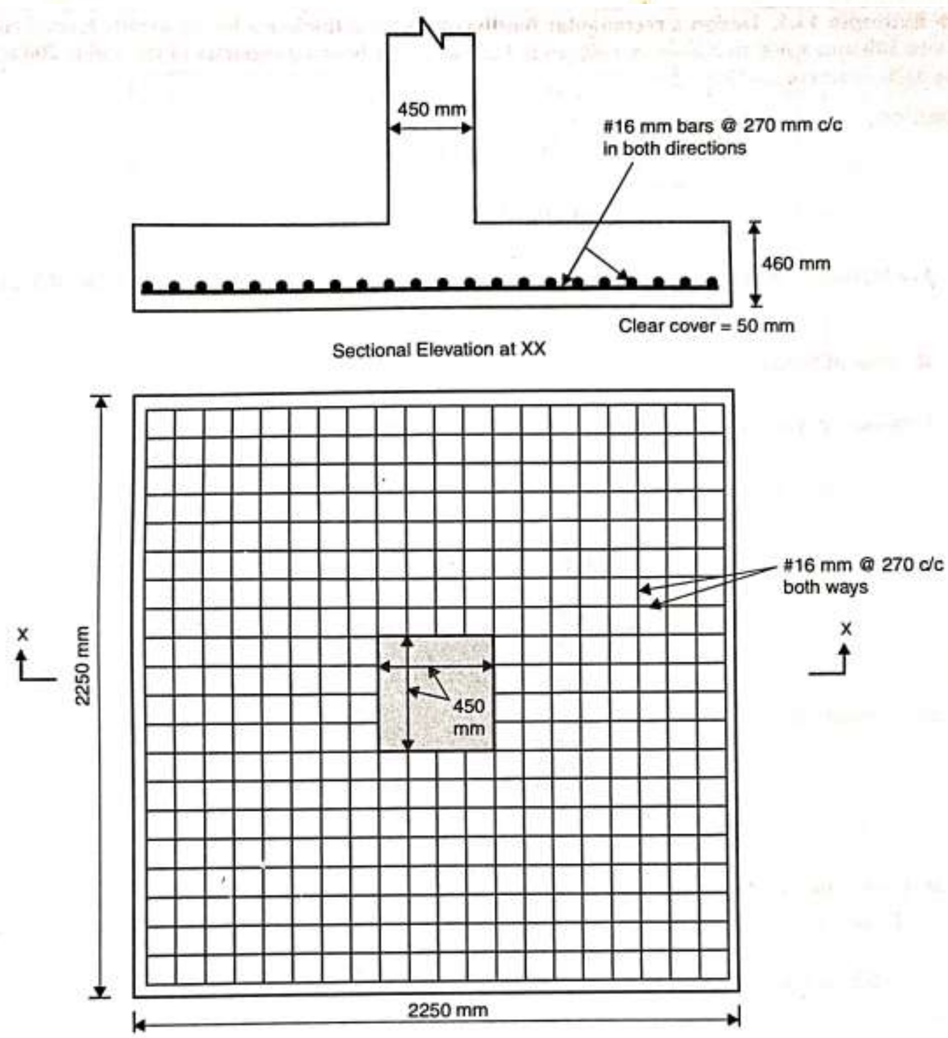
$$\text{Minimum reinforcement reqd.} = \frac{0.12 \times 2250 \times 460}{100}$$

$$= 1242 \text{ mm}^2 < 1652.03 \text{ mm}^2$$

Using 16 bars, $A_\phi = \frac{\pi}{4} \times 16^2 = 201 \text{ mm}^2$

$$\text{Spacing} = \frac{201 \times 2250}{1652.03} = 273 \text{ mm}$$

Provide 16 ϕ bars @ 270 mm c/c in each direction.



Plan

■ **Check for development length**

$$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}} \qquad [\tau_{bd} = 1.92 \text{ N/mm}^2 \text{ for M20 Fe 415}$$

$$= \frac{0.87 \times 415 \times 16}{4 \times 1.92} = 752.2 \text{ mm}$$

$$\text{Available length of bars} = \frac{2250 - 0.45}{2} \times 1000 = 900 \text{ mm. Hence OK}$$

Possible Short Type Questions

1. **What is a column? Give the classification of column?**

   A column is defined as a vertical compression member which is mainly subjected to axial loads and the effective length of which exceeds three times its least lateral dimension.

2. **What is the function of transverse reinforcement in a column?**

   The transverse reinforcement is provided along the lateral direction of the column in the form of ties or spirals enclosing the main steel. The function are as follows

   - To hold the longitudinal bars in position.
   - To prevent buckling of the longitudinal bars.
   - To impact ductility to the column.

3. **What is minimum eccentricity in column?**

   It is not possible to have a perfectly axially loaded column. Some eccentricity will always be there due to non – homogeneity of materials and imperfections of the construction or due to some other reasons. Therefore, IS 456:2000 recommends that all types of columns shall be designed for minimum eccentricity as mentioned above and given in clause 25.4 of the code.

   E_{min} should not be greater than 20 mm.

4. **Define slenderness ratio?**

   The slenderness ratio of a compression member is defined as the ratio of effective length to the least lateral dimension.

Possible Long Type Questions

1. Design a short R.C.C column to carry an axial load of 1600 KN. It is 4 m long, effectively held in position and restrained against rotation at both ends. Use M_{20} concrete and Fe_{415} steel.
2. Design a column of size 450 mm X 600 mm and having 3 m unsupported length. The column is subjected to a load of 2000 KN and is effectively held in position but not restrained against rotation. Use M_{20} concrete and Fe_{415} steel.
3. Design a circular column of diameter 400 mm subjected to a load of 1200 KN. The column is having spiral ties. The column is 3 m long and is effectively held in position at both ends but not restrained against rotation. Use M_{20} concrete and Fe_{415} steel.

MODULE-4 **Chapter - 8**
Session - 41

Learning Objectives
8.1 Introduction on Retaining walls
8.1.1 Types of retaining walls

8.1 Introduction on Retaining walls

Retaining walls are used to retain earth or other loose materials. These walls are commonly constructed in the following cases:

(1) In the construction of building basements.
(2) As wing wall or abutment in the bridge construction.
(3) In the construction of embankments.

The material which is retained by the retaining wall is called as Backfill. The sloping backfill is called as "Inclined Surcharge". The term surcharge means the backfill above the level of the top of the wall. The backfill exerts a push or lateral pressure on the retaining wall which tries to overturn, bend and slide the retaining wall.

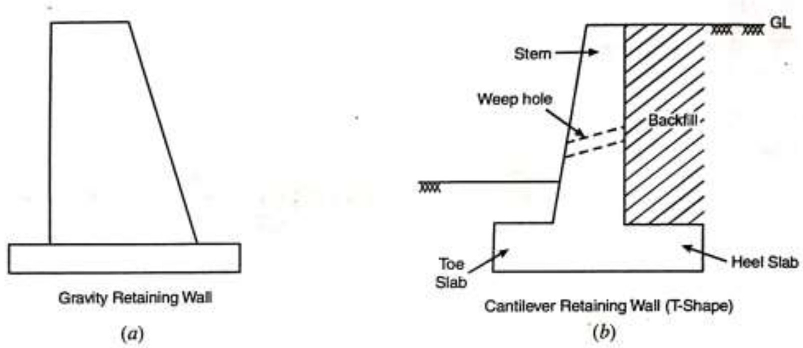
8.1.1 Types of retaining walls

Following are the common types of retaining walls:

1. Gravity retaining wall.
2. Cantilever retaining wall
3. Counterfort retaining wall
4. Buttress retaining wall.

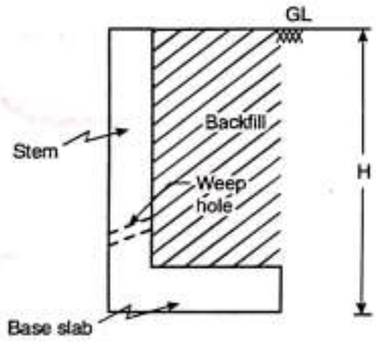
(1) Gravity Retaining Wall

A gravity retaining wall is that retaining wall in which the weight of the retaining wall provides stability against the pressure exerted by the backfill. Gravity retaining walls are made up of massive stone masonry or plain concrete. The principle of design of gravity retaining wall is that tension is not developed anywhere in the section. Therefore, the wall is designed on the basis of "Middle Third Rule".

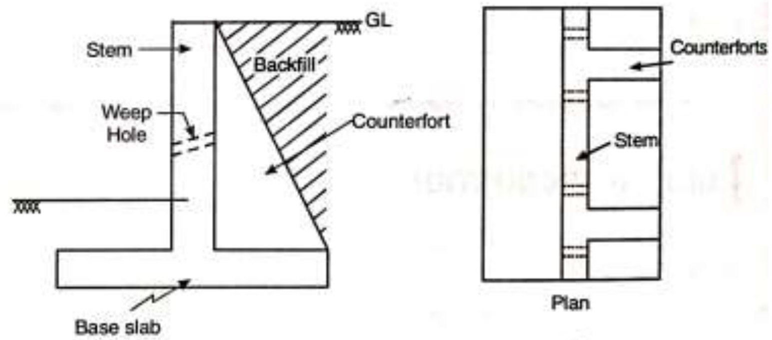


Gravity Retaining Wall
(a)

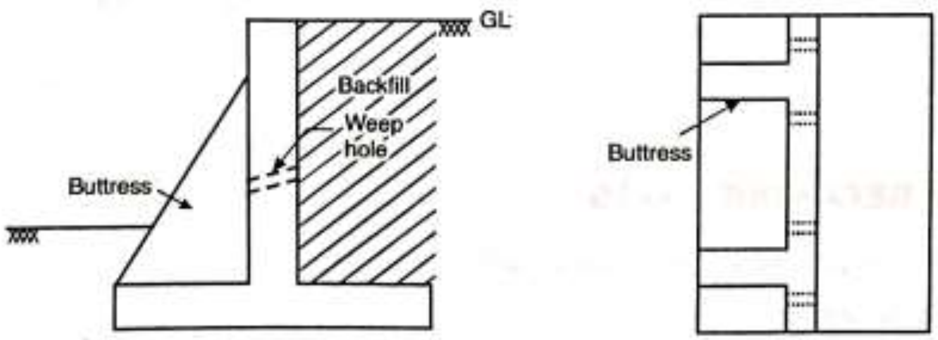
Cantilever Retaining Wall (T-Shape)
(b)



(*c*) Cantilever retaining wall (L-shape)



(*d*) Counterfort retaining wall



(*e*) Buttress retaining wall

(2) Cantilever Retaining Wall

It is the most common type of retaining wall which consists of a vertical wall called as stem, heel slab and the toe slab as shown in Fig. 16.1(*b*) and (*c*). As all the three components of this wall act as cantilevers, the wall is called as cantilever retaining wall. The stem, heel and toe, all resists the earth pressure by bending. These walls can be L or inverted T shaped. The cantilever retaining walls are used up to a height of 6 m. The weight of the earth on the heel slab and the weight of the retaining wall together provide stability to the wall.

(3) Counterfort Retaining Wall

When the backfill of greater height is to be retained and the required height of cantilever retaining wall exceeds 6 m, then it becomes uneconomical to provide cantilever retaining wall. In such cases, counterfort retaining wall is to be provided. In these walls, counterforts are provided at some suitable interval along the length of the wall, on the backfill side, as shown in Fig. 16.1(*d*). These counterforts are concealed in the backfill and tie the vertical stem and heel slab together. In a counterfort retaining wall, the stem and the heel do not act as a cantilever slab but as a continuous slab because of the counterfort supports. This results in reduction in the maximum bending moment and shear force. The weight of the retaining wall and the weight of the earth retaining on the heel slab together impart stability to the wall.

(4) Buttress Retaining Wall

A buttress retaining wall is similar to the counterfort retaining wall as shown in Fig. 16.1(*e*), but with the difference that in buttress retaining wall, the counterforts, called as 'Buttresses', are provided on the opposite side of the backfill. These buttresses tie the stem and the toe slab together. These buttresses are designed as compression members and hence economical but still not preferred. It is because counterforts are concealed but buttresses are visible and they occupy the space in front of the wall which could have been used for some other purpose.

In addition to the retaining walls described above, there are other retaining walls such as bridge abutments and box culverts. In bridge abutments, the bridge deck provides an additional horizontal restraint to the vertical stem at the top. Therefore, the stem is designed as a beam fixed at the base and simply supported or partially supported at the top. In the case of box culverts, the side walls support the backfill as a retaining wall.

MODULE-4 **Chapter - 8**
Session - 42

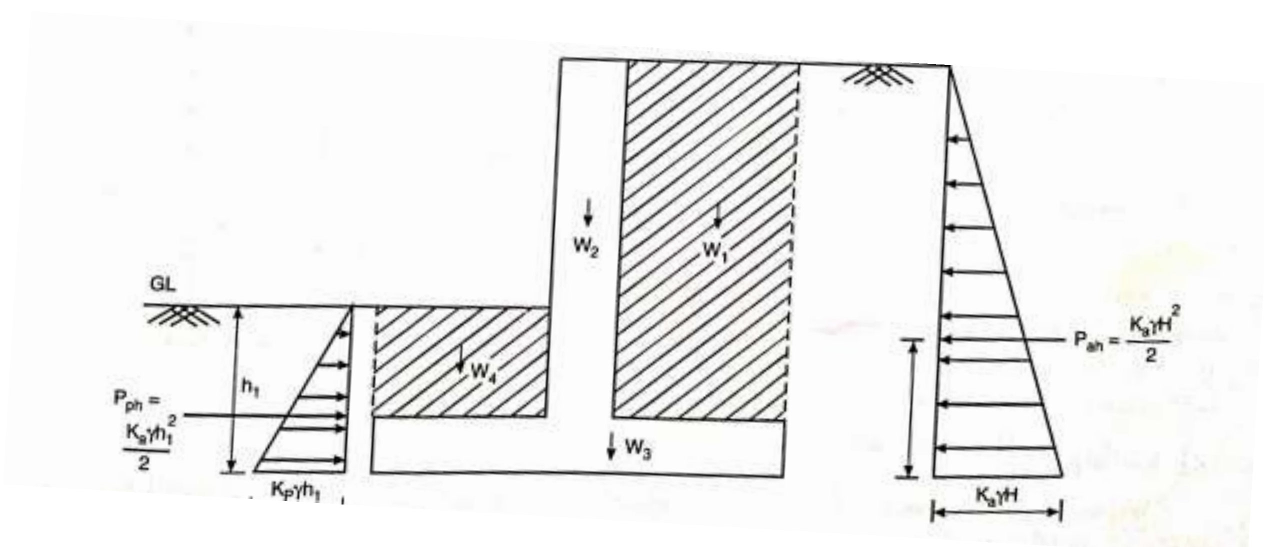
Learning Objectives

8.1.2 Forces on a cantilever retaining wall

8.1.3 Stability of retaining wall

Consider cantilever retaining wall shown in figure. The forces acting on the wall as follows

(1) The lateral force (P_{ah}) due to active earth pressure acting at a height H/3 from the base
(2) Weight of the earth supported on heel slab(W_1)
(3) Weight of the stem(W2).
(4) Weight of the base(W_3)



Forces acting on cantilever retaining wall

8.1.3 Stability of retaining wall

A cantilever retaining wall may fail in the following ways:

(1) Overturning

(2) Sliding

(3) Failure of the undersoil

(1) Overturning

A retaining wall is subjected to overturning moments under the action of lateral force developed due to lateral earth pressure, which tries to overturn the wall about the toe end. The overturning moment (M_0) is given as:

$$M_0 = P_{ah} \times \frac{H}{3}$$

$$= \frac{1}{2}(K_a \gamma H).H.\frac{H}{3}$$

$$\therefore \quad M_0 = \frac{K_a \gamma H^3}{6}$$

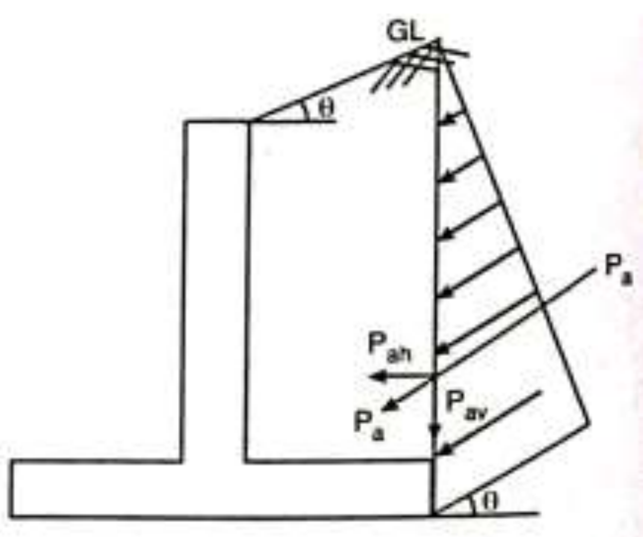
The resisting moment (M_R) is provided by the weight of backfill, surcharge and self weight of the retaining wall. If ΣW is the resultant vertical load made up of self weight of retaining wall and the weight of backfill on the base slab, then resisting moment is given as:

$$M_R = \Sigma W.\bar{x}$$

where $\bar{x}$ is the position of the resultant vertical load (ΣW) from toe end.

As per code IS456:2000 Clause 20.1, the stability of the retaining wall against overturning should be ensured so that the resisting moment is not less than 1.4 times the maximum overturning moment due to characteristic imposed loads (the lateral earth pressure in the case of retaining walls). If the dead load provides the restoning moment, then as per code, only 0.9 times the characteristic dead load should be taken into consideration.

GL



Therefore, $$f_{s1} = \frac{0.9 M_R}{M_0}$$

$$f_{s1} \geq 1.4$$

or $$\frac{0.9(\sum W.\bar{x})}{\frac{K_a \gamma H^3}{6}} \geq 1.4$$

For inclined backfill the vertical components of active earth pressure (P_{av}) also contributes to the restoring moment but it is neglected to make the calculations simpler and to be on the conservative side.

(2) Sliding

The lateral earth pressure tries to slide the retaining wall away from the backfill. This is opposed by the frictional force developed between the base slab and the soil. If μ is the coefficient of friction between the concrete and soil, then the frictional force resisting the sliding is given as:

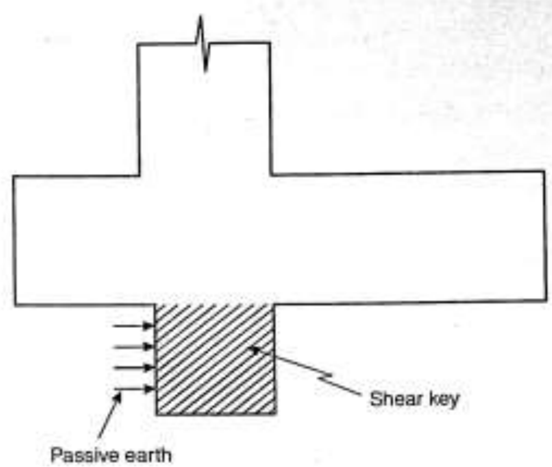
$$F_R = \mu \sum W$$

The lateral force causing the sliding is P_{ah}.

$$F_S = P_{ah} = \frac{K_a \gamma H^2}{2}$$

Then factor of safety against sliding (f_{s2}) is given as

$$f_{s2} = \frac{F_R}{F_S} = \frac{\mu \sum W}{P_{ah}}$$



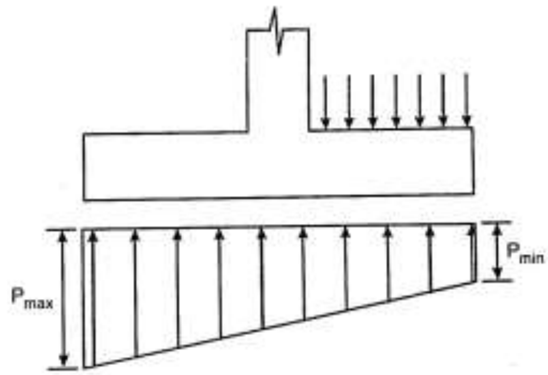
As per IS 456:2000, a minimum factor of safety of 1.4 is to be ensured against sliding and only 0.9 times the characteristic dead load is to be considered for restoring force.

$$\therefore \quad \frac{0.9(\mu \sum W)}{P_{ah}} \geq 1.4$$

If the factor of safety against sliding comes out be less than 1.4 then a shear key may be provided as shown. The shear key increases the resistance against sliding as the passive earth pressure developed on the shear key provides additional resistance against sliding.

(3) Failure of the Under Soil

The base width of the retaining wall is designed in such a way that the maximum pressure on the under soil caused due to load distribution must not exceed the safe bearing capacity of the soil. In addition to that it is to be ensured that, no tension is developed anywhere on the section *i.e.*, the resultant load must fall in the middle third zone [as per the middle third rule] so that negative pressure (tension) is not developed any where. As you have already studied the middle third rule, it is not explained here. The resultant pressure distribution under the base slab is Shown in fig.



Base pressure distribution

$$p_{max} = \frac{\sum W}{b}\left[1 + \frac{6e}{b}\right]$$

$$p_{min} = \frac{\sum W}{b}\left[1 - \frac{6e}{b}\right]$$

The maximum pressure at the base *i.e.*, p_{max} should be less than the safe bearing capacity of soil. The minimum pressure *i.e.*, p_{min} should not be negative.

Here, e is the eccentricity of the resultant load and can be obtained as below:

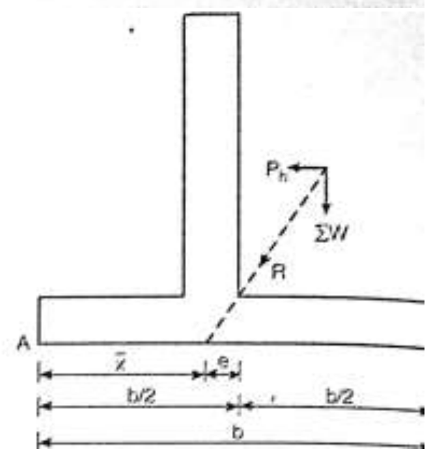
Total moment at toe end A

= Resisting moment about A – Overturning moment at A

$= M_R - M_0$

Total vertical load $= \sum W$

$$\bar{x} = \frac{M_R - M_0}{\sum W}$$



Eccentricity, $e = \frac{b}{2} - \bar{x}$

Learning Objectives

8.1.4 Proportioning of the cantilever retaining wall

Design of a retaining wall involves the determination of its dimensions and the amount of steel required. Before starting the actual analysis of a retaining wall, some preliminary dimensions are to be assumed. The preliminary dimensions of a retaining wall are assumed on the basis of some thumb rules which are explained below:

(1) Depth of Foundation

The minimum depth of the foundation is determined on the basis of Rankine's formula

$$h_{min} = \left(\frac{1-\sin\phi}{1+\sin\phi}\right)\frac{q_0}{\gamma}$$

where h_{min} is the depth of foundation below the earth surface

q_0 = safe bearing capacity of the soil

γ = unit weight of the soil

ϕ = angle of internal friction or angle of repose

(2) Height of the Retaining Wall (H)

The height of the material to be retained (h) is given. The depth of foundation is added to the height of the material to be retained, to get the total height of the retaining wall (H)

$$H = h + h_{min}$$

(3) Base Width (b)

The width of the base slab can be determined by considering the equilibrium of various forces at the base. Based on exact analysis and experience, it is found that the base width (b) varies from $0.4H$ to $0.6H$.

(4) Thickness of Base Slab

For preliminary analysis, the thickness of base slab is assumed to be $\frac{H}{10}$ to $\frac{H}{15}$, where H is the total height of the retaining wall. The minimum thickness of base slab should not be less than 300 mm. The thickness assumed should be checked from bending moment and shear force requirements.

(5) Thickness of the Stem

The thickness of vertical stem or wall is governed by the bending moment criteria. As the stem behaves like a cantilever, subjected to lateral pressure which is increasing with depth, it is economical to have a trapezoidal section of the stem with minimum thickness of 150 mm at top. The thickness at the base of stem should not be less than 300 mm. Initially, the thickness of stem may be assumed to be about 8 to 10% of the total height of the retaining wall or can be found from the bending moment criteria.

The preliminary dimensions of the various components of the cantilever retaining wall are used for checking the various stability criteria like overturning, sliding, safe bearing pressure and the depth requirement for maximum bending moment. If these criteria/requirements are satisfied then the dimensions are adopted and design calculations regarding the area of steel etc. are done otherwise the dimensions are revised.

MODULE-4 **Chapter - 8**
Session - 44

Learning Objectives
8.1.5 Design Procedure of cantilever retaining wall.(Problems)

Problem

Design a cantilever retaining wall to retain horizontal earthen embankment of height 4 m above the ground level. The earthen backfill is having a density of 18 KN/m³ and angle of internal friction as 30⁰. The safe bearing capacity of the soil is 180 KN/m². The coefficient of friction between soil and concrete is assumed to be 0.45. Use M_{20} concrete and Fe_{415}

Solution. Given:

$$f_{ck} = 20 \text{ N/mm}^2$$
$$f_y = 415 \text{ N/mm}^2$$
$$\phi = 30°$$
$$\mu = 0.45$$
$$\gamma = 18 \text{ kN/m}^3$$

Safe bearing capacity of soil $= q_0 = 180 \text{ kN/m}^2$

Height of earthen embankment $= 4.0$ m

■ **Coefficient of active earth pressure (K_a)**

$$K_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 30°}{1+\sin 30°}$$

$$K_a = \frac{1}{3}$$

■ **Minimum depth of foundation (h_{min})**

$$h_{min} = \frac{q_0}{\gamma}\left(\frac{1-\sin\phi}{1+\sin\phi}\right)^2$$

$$= \frac{180}{18}\left(\frac{1}{3}\right)^2$$

$$h_{min} = 1.11 \text{ m say } 1.2 \text{ m}$$

∴ Providing the depth of foundation as 1.2 m

Total height of retaining wall = Depth of foundation + Height of embankment

$= 1.2 + 4.0$

Total height of retaining wall (H) = 5.2 m

■ **Preliminary dimensions of the retaining wall**

(1) Base Width (b): It varies from 0.4 H to 0.6 H

Assuming $b = 2.8$ m

Length of toe slab $= 0.3b$ to $0.4b$

$= 850$ mm (say)

(2) Thickness of Base Slab

Thickness of base slab is assumed to be $\frac{H}{10} \simeq 500$ mm.

(3) Thickness of vertical wall or stem:

Thickness of stem may be assumed as $\frac{H}{12}$ at base but here depth required from BM consideration is calculated. = 433.33 mm

Pressure at the base of the stem $= K_a \gamma h$ $\quad [h = 5.2 - 0.5 = 4.7 \text{ m}]$

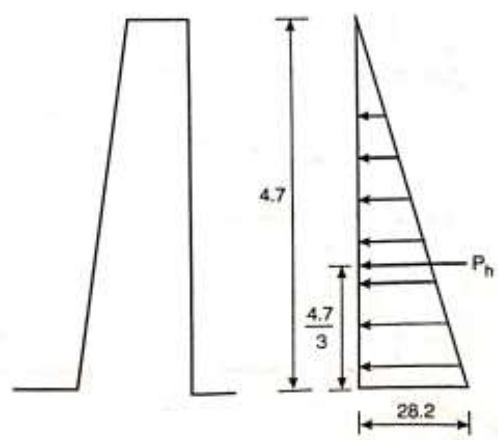
$$= \frac{1}{3} \times 18 \times 4.7$$

$$= 28.2 \text{ kN/m}^2$$

Moment at the base of the stem $= \frac{1}{2}(K_a \gamma h) \, . \, h \, . \, \frac{h}{3}$

$$= \frac{1}{2} \times (28.2) \times 4.7 \times \frac{4.7}{3}$$

$$= 103.83 \text{ kNm}$$



Ultimate moment at the base of the stem

$$= 1.5 \times 103.83$$

$$= 155.74 \text{ kNm}$$

Factor of safety

Minimum depth required for a balance section is

$$d_{\text{reqd}} = \sqrt{\frac{M_u}{R_u \, . \, b}}$$

$R_u = 2.76$, for M20 concrete and Fe 415 steel

$$d_{\text{reqd}} = \sqrt{\frac{155.74 \times 10^6}{2.76 \times 1000}}$$

$$= \sqrt{\frac{155.74 \times 10^6}{2.76 \times 1000}}$$

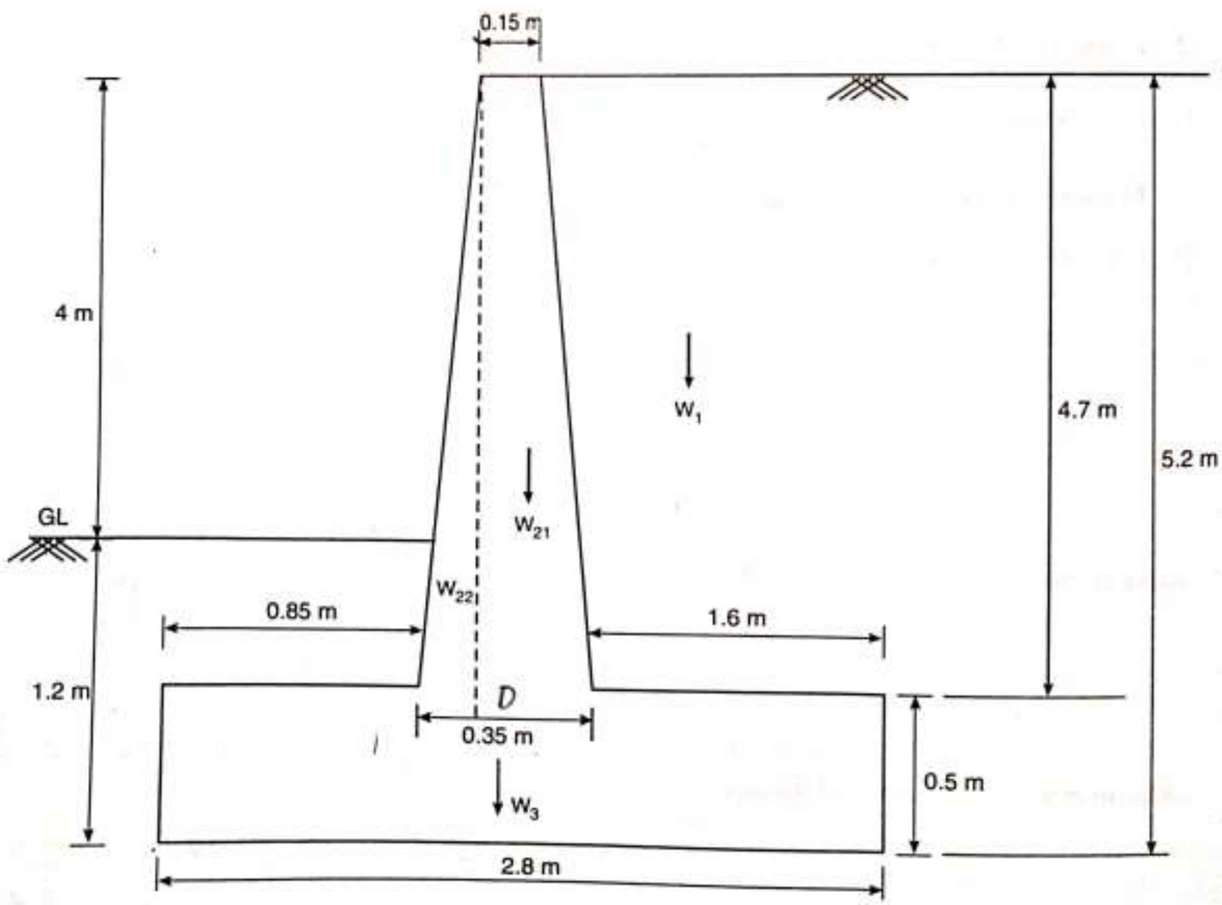
$$= 238 \text{ mm}$$

Assuming 60 mm cover,

$$\text{Total depth required} = 238 + 60$$

$$= 298 \text{ mm} \simeq 35$$

Hence taking $D = 350$ mm at base of stem and reducing it to 150 mm at top



Type of Force	Magnitude of Force (kN)	Position of force from toe end O (m)	Bending moment at toe end O (kNm)
(1) **Overturning force** $P_{ah} = \frac{1}{2}(K_a \gamma H).H$	$\frac{1}{2} \times \left(\frac{1}{3} \times 18 \times 5.2\right) \times 5.2$ $= 81.12$	$\frac{H}{3} = \frac{5.2}{3} = 1.733$	81.12×1.733 $= 140.61$ $\Sigma M_0 = 140.61$

(2) **Restoring forces**			
(a) Weight of backfill (W_1)	$1.6 \times 4.7 \times 18 = 135.36$	$2.8 - \frac{1.6}{2} = 2.0$	270.72
(b) Weight of stem			
(i) Weight of rectangular portion (W_{21})	$0.15 \times 4.7 \times 25$ $= 17.625$	$0.85 + 0.35 - \frac{0.15}{2}$ $= 1.125$	19.828
(ii) Weight of triangular portion (W_{22})	$\frac{1}{2} \times 0.2 \times 4.7 \times 25$ $= 11.75$	$0.85 + \frac{2}{3} \times 0.2 =$ 0.983	11.554
(c) Weight of base slab (W_3)	$0.5 \times 2.8 \times 25 = 35$	$\frac{2.8}{2} = 1.4$	49
	$\Sigma W = 199.735$		$\Sigma M_R = 351.1$

■ **Stability Checks**

(1) **Overturning**

$$\frac{0.9 M_R}{M_0} = \frac{0.9 \times 351.10}{140.61}$$

$$= 2.2 > 1.4 \text{ hence o.k.}$$

(2) **Sliding**

$$\frac{0.9 F_R}{F_S} \geq 1.4$$

$$F_R = \mu \Sigma W = 0.45 \times 199.735 = 89.88 \text{ kN}$$

$$F_S = P_{ah} = 81.12 \text{ kN}$$

$$\frac{0.9 F_R}{F_S} = \frac{0.9 \times 89.46}{81.12} = 0.99 < 1.4$$

Hence, shear key is to be provided to increase the resistance against sliding.

(3) **Base Pressure**

$$\text{Resultant moment at toe end } O = M_R - M_0$$

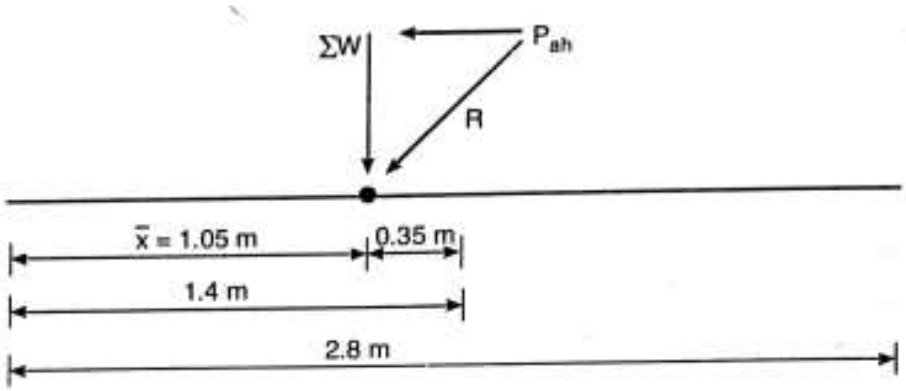
$$= 351 - 140.61$$

$$= 210.49 \text{ kNm}$$

The resultant vertical load $= \Sigma W = 199.73$ kN

It acts at a distance $\bar{x}$ from the toe end O

$$\bar{x} = \frac{210.49}{199.73} = 1.05 \text{ m}$$



$$e = \frac{b}{2} - \bar{x} = 1.4 - 1.05$$

$$e = 0.35 \text{ m}$$

which lies in the middle third zone *i.e.*, $\frac{b}{6}$ from centre (0.466 m). Hence OK

b = 2.8m

■ **Maximum pressure at toe end *O***

$$p_{max} = \frac{\sum W}{b}\left[1 + \frac{6e}{b}\right]$$

$$= \frac{199.73}{2.8}\left[1 + \frac{6 \times 0.35}{2.8}\right]$$

$$p_{max} = 124.83 \text{ kN/m}^2 < 180 \text{ kN/m}^2 \text{ (safe } BC \text{ of soil).}$$ Hence OK

■ **Minimum pressure at heel end = p_{min}**

$$p_{min} = \frac{\sum W}{b}\left[1 - \frac{6e}{b}\right]$$

$$= \frac{199.73}{2.8}\left[1 - \frac{6 \times 0.35}{2.8}\right]$$

$= 17.83$ kN/m², which is positive.

Hence OK, as no tension develops anywhere on the base slab.

Design of Stem

The depth required for stem is already checked while assuming the preliminary dimensions

$\therefore$

$$D = 350$$

$$d = 350 - 60$$

$$= 290 \text{ mm}$$

Maximum moment at base of stem = 155.73 kNm = ultimate moment at the base of the stem

Area of steel (A_{st}) in stem

$$M_u = 0.87 f_y A_{st} d \left[1 - \frac{f_y A_{st}}{f_{ck} bd}\right]$$

$$155.73 \times 10^6 = 0.87 \times 415 \times A_{st} \times 290 \left[1 - \frac{415 A_{st}}{20 \times 1000 \times 290}\right]$$

$$A_{st}^2 - 13979.23 A_{st} + 20794392.5 = 0$$

On solving the equation

$$A_{st\ reqd} = 1693\ \text{mm}^2$$

Using 16 mm diameter bars, $A_\phi = 201\ \text{mm}^2$ $\frac{\pi}{4} \times 16^2$

$$\text{Spacing required} = \frac{201 \times 1000}{1693} = 118\ \text{mm} \cong 100\ \text{mm}$$

Hence, provide 16 mm diameter, Fe 415 bars @ 100 mm c/c.

Distribution steel

Distribution steel is provided @ 0.12% of total x-sectional area (page-48) IS456:2000

$$A_{st} = \frac{0.12}{100} \times 1000 \times \left(\frac{150 + 350}{2}\right) \qquad \left[\left(\frac{150 + 350}{2}\right) \text{is the average thickness of the stem}\right]$$

$(A_{st} = 300\ \text{mm}^2)$

Using 8 mm diameter bars, $A_\phi = 50.3\ \text{mm}^2$

$$\text{Spacing required} = \frac{50.3 \times 1000}{300} = 167.5\ \text{mm} \doteq 150\ \text{mm}$$

Hence, provide 8 mm diameter Fe 415 bars @ 150 mm c/c, on the inner face of the stem as distribution steel.

Similarly provide 8 mm diameter Fe 415 bars @ 150 mm c/c in both directions at the outer face (front face) of the stem as temperature and shrinkage reinforcement since this face is exposed to weather.

Check for shear

The critical section for shear is at a distance 'd' from base of stem *i.e.,* $h = 4.7 - 0.29 = 4.41$

$$\text{Shear force at this section of the stem} = \frac{1}{2}\left(\frac{1}{3} \times 18 \times 4.41\right) \times 4.41$$

$d = 290\ \text{mm}$
$= 0.29\ \text{m}$

$$V = 58.3\ \text{kN}$$

factored shear $V_u = 1.5 \times 58.3$

$$V_u = 87.52\ \text{KN}$$

$$\text{Nominal shear stress} = \frac{V_u}{bd}$$

$$\tau_v = \frac{87.52 \times 1000}{1000 \times 290} = 0.30\ \text{N/mm}^2$$

For

$$p_t = \frac{201 \times 1000}{1000 \times 290} = 0.69\%$$

$$\tau_c = 0.54\ \text{N/mm}^2$$

$$\tau_c = 0.54\ \text{N/mm}^2 > \tau_v \quad \text{hence OK.}$$

MODULE-4 **Chapter - 8**
Session - 45

Learning Objectives

8.1.5 Design Procedure of cantilever retaining wall.(Problems continues)

Curtailment of tension reinforcement

As the stem of retaining wall behaves like a cantilever, the bending moment goes on reducing towards the top of the wall and becomes zero at the top. Therefore, tension reinforcement can be curtailed along the height of the stem.

Development length, L_d, for 16 mm diameter bars

$$L_d = \frac{0.87 f_y \phi}{4\tau_{bd}}$$

$$L_d = \frac{0.87 \times 415 \times 16}{4 \times 1.6 \times 1.2}$$

$$= 752 \text{ mm}$$

Therefore, no bar can be curtailed up to a distance of 752 mm from base of the stem. Curtailing bars a a distance 1000 mm from base of the stem *i.e.*,

$$4700 - 1000 = 3700 \text{ mm from top of the stem} \quad = 3.7\text{m}$$

$$\text{Total depth at this section} = 150 + \frac{200 \times 3700}{4700}$$

$$= 307 \text{ mm}$$

$$d = \text{Effective depth at this section} = 307 - 60 = 247 \text{ mm}$$

Moment due to earth pressure at 3.7 m from top

$$= \frac{K_a \gamma h^3}{6}$$

$$= \frac{1}{6}\left[\frac{1}{3} \times 18 \times 3.7^3\right]$$

$$= 50.7 \text{ kNm}$$

$$M_u = 1.5 \times 50.7$$

$$M_u = 76 \text{ kNm}$$

Area of steel required for an ultimate bending moment of 76 kNm

$$76 \times 10^6 = 0.87 \times 415 \times A_{st} \times 247\left[1 - \frac{415 A_{st}}{20 \times 1000 \times 247}\right]$$

On solving, we get $A_{st\ reqd} = 924 \text{ mm}^2$

Using 16 mm diameter bars,

$$\text{Spacing required} = \frac{201 \times 1000}{924} = 217 \text{ mm}$$

Hence half of the bars can be curtailed but as per IS code, 12φ or d distance, whichever is more, is to be provided beyond the point of curtailment. Hence curtailment the bars at 1.3 m from base or 3.4 m from top of stem. Thus providing 16 mm diameter bars @ 200 mm c/c after a distance of 1.3 m from base of stream.

Similarly, one more curtailment can be done at 1.5 m from top of stem.

$$\text{Moment at this section} = \left(\frac{18 \times 1.5^3}{3 \times 6}\right)$$

$$M_u = 1.5 \times 3.375$$
$$= 5.1 \text{ kNm}$$

$$\text{Depth at this section} = 150 + \frac{200}{4700} \times 3200$$
$$= 286 \text{ mm}$$
$$d = 286 - 60 = 226 \text{ mm}$$

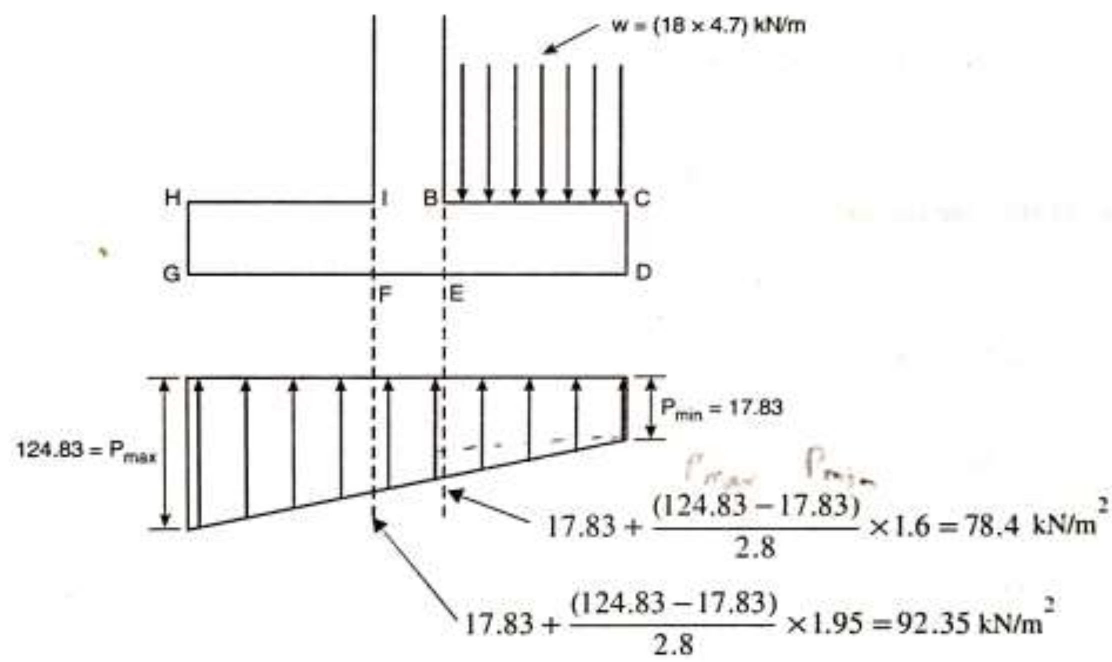
$$5.1 \times 10^6 = 0.87 \times 415 \times A_{st} \times 226\left[1 - \frac{415 A_{st}}{20 \times 1000 \times 226}\right]$$

$$A_{st\ reqd} = 65 \text{ mm}^2 < A_{st\ min}\ i.e.,\ 300 \text{ mm}^2$$

Hence curtailing another half of the bars at 1.5 m from top and providing 16 mm diameter bars @ 400 mm c/c.

2. Design of Heel Slab

The pressure distribution on heel slab is shown in Fig.



Weight of earth supported on heel = $18 \times 4.7 = 84.6$ kN/m

Self weight of heel slab $= 0.5 \times 1.0 \times 25 = 12.5$ kN/m

Total load $= 97.1$ kN/m

$$\text{Maximum bending moment at } B = \frac{97.1 \times 1.6^2}{2} - \frac{17.83 \times 1.6^2}{2} - \frac{1}{2}(78.4 - 17.83) \times 1.6 \times \frac{1.6}{3}$$

$$= 101.5 - 25.8$$

$$= 75.7 \text{ kNm}$$

$$M_u = 1.5 \times 75.7 = 113.6 \text{ kNm}$$

0·5 ~ 0·50

$$d_{reqd} = \sqrt{\frac{113.6 \times 10^6}{2.76 \times 1000}} = 202 \text{ mm} < 440 \text{ mm. Hence OK.}$$

Area of steel for heel slab

$$113.5 \times 10^6 = 0.87 \times 415 \times 440\left(1 - \frac{415 A_{st}}{20 \times 1000 \times 440}\right)$$

$$A_{st} = 741 \text{ mm}^2$$

$$\therefore \quad \text{Spacing of 12 mm bars} = \frac{113 \times 1000}{741} = 152 \text{ mm}$$

$\therefore$ Provide 12 mm diameter bars @ 150 mm c/c at the top face of the heel slab *i.e., BC.*

Distribution steel is provided @ 0.12% of sectional area in the other direction

$$\frac{0.12}{100} \times 1000 \times 500 = 600 \text{ mm}^2$$

Using 10 mm diameter bars, $A_\phi = 78.5 \text{ mm}^2$, spacing required = 100 mm

Hence, provide same 10 mm dia bars @ 100 mm c/c in the other direction.

3. Design of Toe Slab

The weight of frontfill above the toe slab is neglected and maximum moment is calculated at the face of the stem.

$$\text{Maximum moment} = \frac{92.35 \times 0.85^2}{2} + \frac{1}{2}(124.83 - 92.35) \times 0.85 \times \frac{2}{3} \times 0.85$$

$$= 33.36 + 7.82 = 41.2 \text{ kNm}$$

$$M_u = 1.5 \times 41.2 = 61.8 \text{ kNm}$$

Area of steel for toe slab

$$61.80 \times 10^6 = 0.87 \times 415 \times A_{st} \times 440\left[1 - \frac{415 A_{st}}{20 \times 1000 \times 440}\right]$$

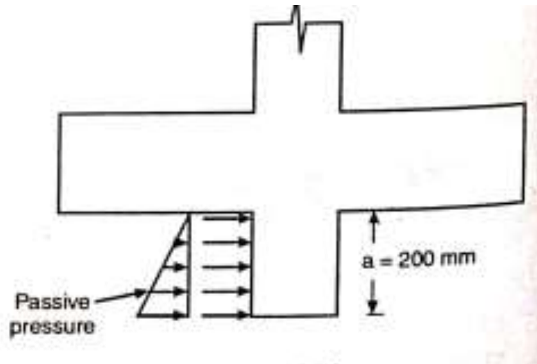
$$A_{st}^2 - 21209.88 A_{st} + 8251001.3 = 0$$

$$A_{st} = 396 \text{ mm}^2 < A_{st\,min}\ (600 \text{ mm}^2)$$

Hence providing minimum area of steel of 600 mm^2. Therefore provide 10 mm diameter bars @ 100 mm c/c in both directions.

4. Design of Shear key

As the wall is not safe in sliding, shear key is to be provided below the stem as shown in Fig



$$\text{Pressure at face of shear key} = 92.35 \text{ kN/m}$$

$$\text{Coefficient of passive earth pressure} = \frac{1 + \sin\phi}{1 - \sin\phi}$$

$$K_P = \frac{1.5}{0.5} = 3$$

$$\text{Let the depth of key} = a$$

$$\text{Resistance offered by shear key} = 3 \times 92.35 \times a$$

$$= 277.05a$$

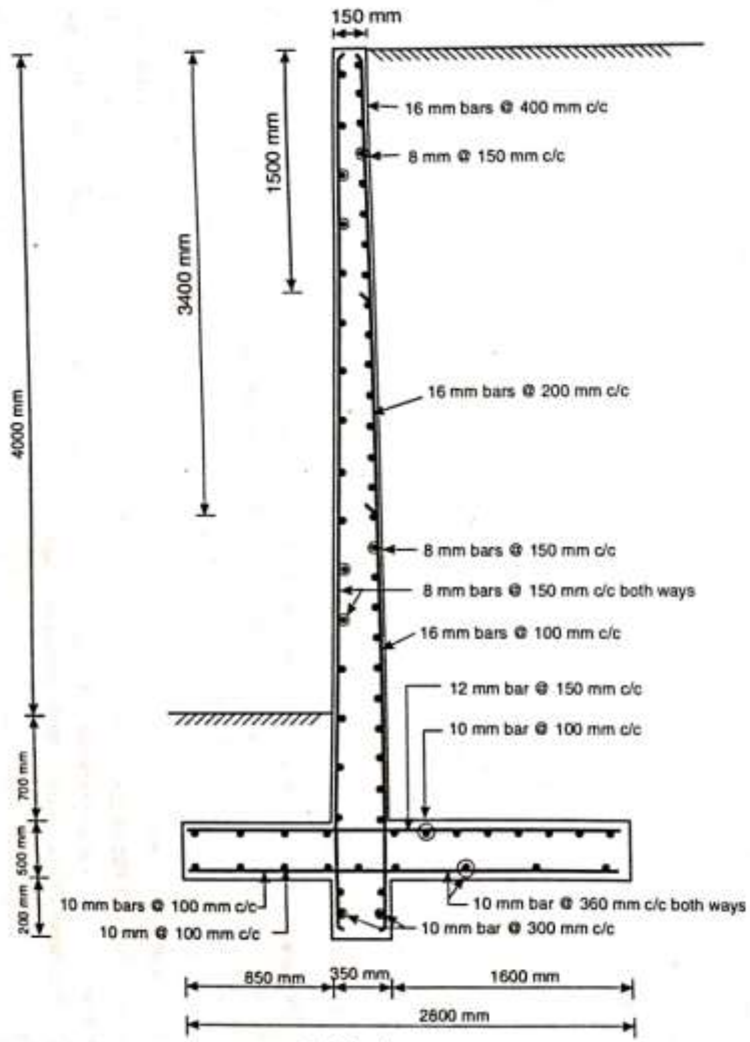
Factor of safety against sliding alongwith shear key

$$= \frac{0.9\mu \sum W + 277.05a}{P_{ah}}$$

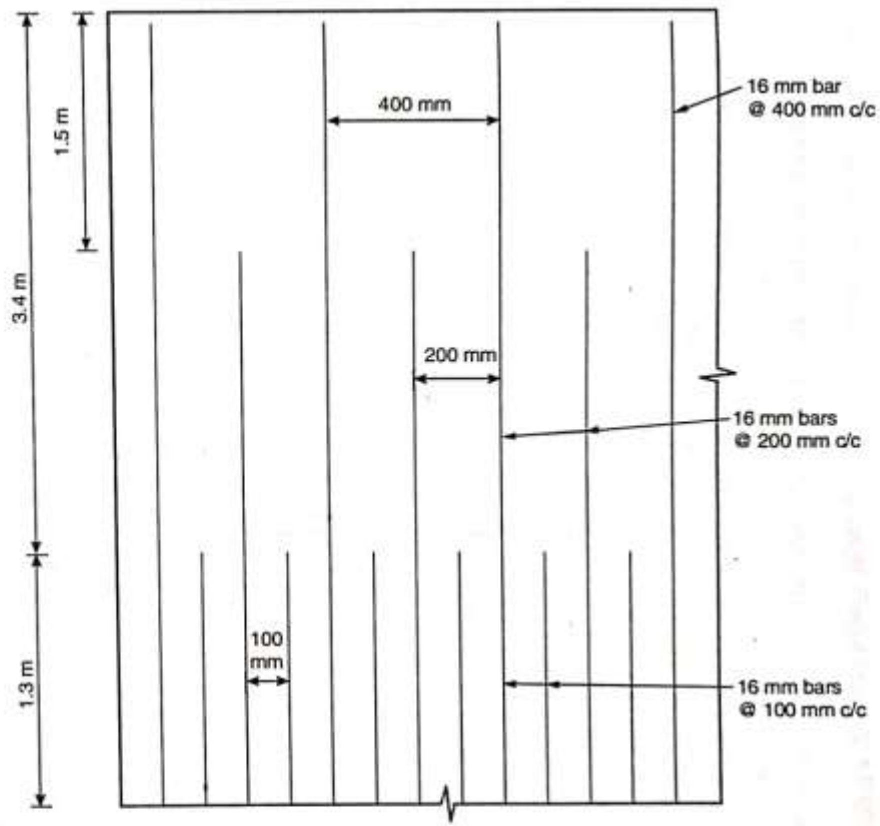
$$= \frac{0.9 \times 89.88 + 277.05a}{81.12} = 1.4$$

$$\Rightarrow \quad a = 0.118 \text{ m}$$

However, provide a 200 mm × 200 mm shear key.



(*a*) Section



(*b*) L-Section of Stem

MODULE-4 **Chapter - 8**
Session – 46, 47

Learning Objectives
8.1.6 Design Procedure of Counterfort retaining wall. (Problems continues)

Problem
Design a counterfort retaining wall to retain 4 m earth above ground level. The top of the earth is to be level. The density of earth is 15 KN/m^2. The angle of internal friction of soil is 30^0. The safe bearing capacity of soil is 200 KN/m^2 and the coefficient of friction between soil and wall is 0.6.

Solution. Given:

$$\gamma = 15\ \text{kN/m}^2,\quad \phi = 30^\circ,\quad \mu = 0.6,\quad q_0 = 200\ \text{kN/m}^2,\quad h = 4.0\ \text{m}$$

Using M20 concrete and Fe 415 steel

$$f_y = 415\ \text{N/mm}^2,\quad f_{ck} = 20\ \text{N/mm}^2$$

■ **Minimum Depth of Foundation**

$$h_{min} = \frac{q_0}{\gamma}\left(\frac{1-\sin\phi}{1+\sin\phi}\right)^2$$

$$= \frac{200}{15}\left(\frac{1-\sin 30^\circ}{1+\sin 30^\circ}\right)^2$$

$$h_{min} = 1.48\ \text{m}$$

Taking depth of foundation = 1.5 m

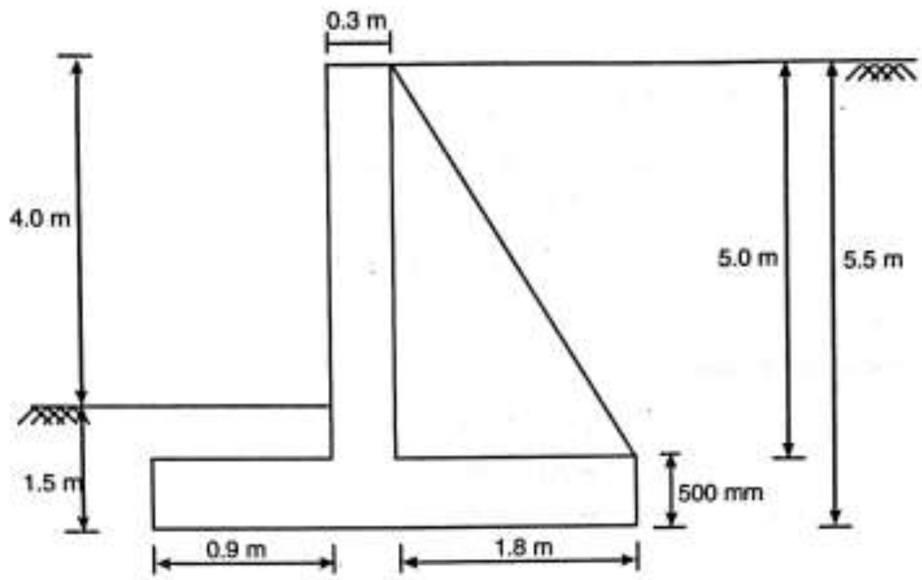
Overall depth of wall = 4.0 + 1.5 = 5.5 m

■ **Proportioning of retaining wall**

1. Width of base slab is kept approximately as 0.6 H

∴ $b = 3.0$ m

2. Assuming thickness of base slab = $\frac{H}{12}$ say 500 mm.
3. Toe projection = 0.3 b say 0.9 m
4. Spacing of counterforts = 3.0 m.
5. Width of counterforts = 0.05 H say 300 mm
6. Thickness of stem = $\frac{H}{20}$ say 300 mm, (equal to the thickness of counterforts).



■ **Forces Acting on the Retaining Wall**

Force (Type)	**Force (kN)**	**Distance from toe edge m**	**Moment about toe edge (kNm)**
1. Overturning force $P_h = \frac{1}{2}(K_a \gamma H)H$	$\frac{1}{2} \times \left(\frac{1}{3} \times 15 \times 5.5\right) 5.5 = 75.625$	$\frac{H}{3} = \frac{5.5}{3} = 1.833$	138.65
	$F_S = 75.625$ kN		$M_0 = 138.65$
2. Restoring forces			
(*i*) Weight of backfill (W_1)	$15 \times 5 \times 1.8 = 135$	$3.0 - \frac{1.8}{2} = 2.1$	283.5
(*ii*) Weight of stem (W_2)	$0.3 \times 5.0 \times 25 = 37.5$	$0.9 + \frac{0.3}{2} = 1.05$	34.31
(*iii*) Weight of base slab (W_3)	$0.5 \times 3 \times 25 = 37.5$	$\frac{3.0}{2} = 1.5$	56.25
	$\Sigma W = 210$ kN		$M_R = 374.06$ kNm

■ **Stability Checks:**

(1) **Overturning:**

$$\text{Factor of safety against overturning} = \frac{0.9 M_R}{M_0}$$

$$= \frac{0.9 \times 374.06}{138.65}$$

$$= 2.4 > 1.4 \quad \text{hence OK}$$

(2) Sliding:

$$\text{Factor of safety against sliding} = \frac{0.9\mu \sum W}{F_S}$$

$$= \frac{0.9 \times 0.6 \times 210}{75.625}$$

$$= 1.49 > 1.4 \quad \text{hence o.k.}$$

(3) Base pressure check

$$\text{Net moment about toe edge} = M_R - M_0$$

$$= 374.06 - 138.65$$

$$= 235.41 \text{ kNm}$$

The point of application of resultant where it cuts base;

$$\bar{x} = \frac{\text{Net moment}}{\sum W} = \frac{235.41}{210}$$

$$\bar{x} = 1.121 \text{ m}$$

$$e = \frac{b}{2} - x = \frac{3.0}{2} - 1.121$$

$$e = 0.379 \text{ m} < \frac{b}{6} \text{ i.e., } 0.5 \text{ m} \quad \text{Hence OK}$$

$$p_{max} = \frac{\sum W}{b}\left[1 + \frac{6e}{b}\right]$$

$$= \frac{210}{3.0}\left[1 + \frac{6 \times 0.379}{3.0}\right]$$

$$p_{max} = 123.06 \text{ kN/m}^2 < 200 \text{ kN/m}^2 \text{ (safe B.C. of soil) Hence OK}$$

$$p_{min} = \frac{\sum W}{b}\left[1 - \frac{6e}{b}\right]$$

$$= \frac{210}{3.0}\left[1 - \frac{6 \times 0.379}{3}\right]$$

$$p_{min} = 16.94 \text{ kN/m}^2 \text{ which is +ve. Hence ok.}$$

■ **Design of stem**

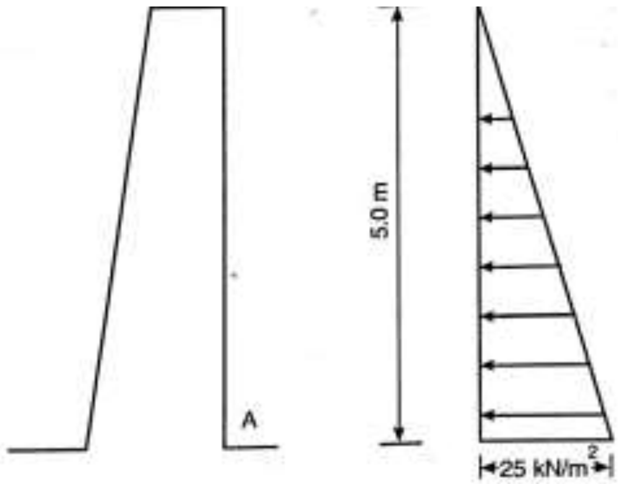
$$\text{Maximum horizontal pressure at the base of stem} = \left(\frac{1}{3} \times 15 \times 5\right)$$

$$P_h = 25 \text{ kN/m}^2$$

The stem acts as a horizontal slab supported on counterforts with $w = 25 \times 1 = 25$ kN/m

■ Maximum –ve moment at counterfort supports

$$= \frac{w.l^2}{12} = \frac{25 \times 3^2}{12} = 18.75 \text{ kNm}$$



$$M_u = 1.5 \text{ M}$$
$$= 1.5 \times 18.75$$
$$= 28.125 \text{ kNm}$$

Maximum positive moment at mid span

$$= \frac{w.l^2}{16} = \frac{25 \times 3^2}{16}$$
$$= 14.1 \text{ kNm}$$
$$M_u = 1.5 \times 14.1 = 21.1 \text{ kNm}$$

■ **Depth check**

$$d = \sqrt{\frac{M_u}{R_u.b}} = \sqrt{\frac{28.12 \times 10^6}{2.76 \times 1000}}$$
$$= 101 \text{ mm} < 250 \text{ mm} \quad \text{hence OK}$$

[assuming effective cover as 50 mm, $d = 300 - 50 = 250$ mm]

■ **Area of steel required**

$$28.125 \times 10^6 = 0.87 \times 415 \times A_{st} \times 250 \left[1 - \frac{415 A_{st}}{20 \times 1000 \times 250}\right]$$
$$A_{st} = 320 \text{ mm}^2$$

Using 10 mm diameter bars

$$A_\phi = 78.5 \text{ mm}^2$$
$$\text{Spacing required} = \frac{78.5 \times 1000}{320} = 245 \text{ mm}$$

$$A_{st\,min} = 0.12\% \text{ of x-sectional area}$$
$$= 0.12 \times 1000 \times 0.3 = 360 \text{ mm}^2 > 320 \text{ mm}^2$$

Hence $A_{st\,min}$ is to be provided

$$\text{Spacing required} = \frac{78.5 \times 1000}{360} = 218 \text{ mm}$$

Hence provide 10 mm diameter bars @ 200 mm c/c in both direction, all along the height of stem. It also takes care of +ve moment *i.e.*, 21.1 kNm at mid span. The spacing can be increased to 300 mm near the top of the stem as the pressure decrease towards the top of the stem.

■ **Shear check:** Maximum shear force at the face of counterfort

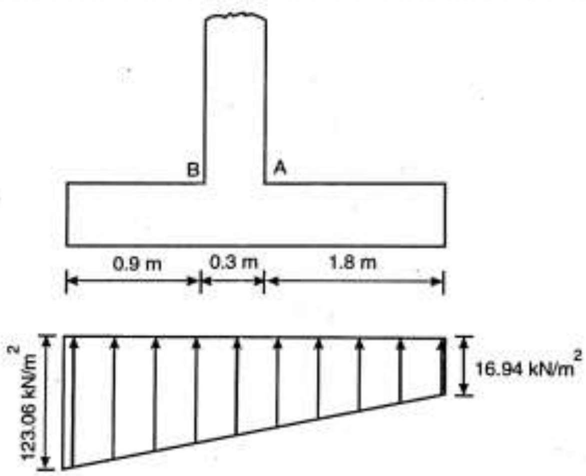
$$= \frac{25 \times (3 - 0.3)}{2} = 33.75 \text{ kN}$$

$$V_u = 1.5 \times 33.75$$
$$= 50.625 \text{ kN}$$

$$\tau_v = \frac{V_u}{bd} = \frac{50.625 \times 10^3}{1000 \times 250}$$

$$\tau_v = 0.2 \text{ N/mm}^2$$
$$p_t = 0.157\%$$
$$\tau_c = 0.28 \text{ N/mm}^2 \quad \text{[From table 19 IS 456}$$
$$\tau_v < \tau_c \quad \text{Hence ok.}$$

■ **Design of toe slab:** The pressure distribution under the base slab is as shown below



$$\text{Pressure below point } A = 16.94 + \left(\frac{123.06 - 16.94}{3.0}\right) \times 1.8$$
$$= 80.61 \text{ kN/m}^2$$

$$\text{Pressure below point } B = 16.94 + \frac{(123.06 - 16.94)}{3.0} \times 2.1$$
$$= 91.22 \text{ kN/m}^2$$

Neglecting the weight of earth retained on the toe slab, the cantilever moment at the section B is

$$= 91.22 \times \frac{0.9^2}{2} + \frac{1}{2}(123.06 - 91.22) \times 0.9 \times \frac{2}{3} \times 0.9$$

$$= 45.54 \text{ kNm}$$

$$M_u = 1.5 \times 45.54$$
$$= 68.31 \text{ kNm}$$

$$d_{reqd} = \sqrt{\frac{68.31 \times 10^6}{2.76 \times 1000}}$$

$$= 157 \text{ mm} < d \text{ provided.} \quad \text{Hence o.k.}$$

$$\text{Total depth} = 500 \text{ mm}$$
$$\text{Effective cover} = 60 \text{ mm}$$
$$d_{provided} = 500 - 60 = 440 \text{ mm}$$

Area of steel required for toe slab:

$$68.31 \times 10^6 = 0.87 \times 415 \times A_{st} \times 440\left[1 - \frac{415 A_{st}}{20 \times 1000 \times 440}\right]$$

$$A_{st \text{ reqd}} = 440 \text{ mm}^2$$

$$A_{st \text{ min}} = \frac{0.12}{100} \times 1000 \times 500 = 600 \text{ mm}^2 > 440 \text{ mm}^2$$

hence provide $A_{st} = 600 \text{ mm}^2$

Using 12 mm diameter bars, $A_\phi = 113 \text{ mm}^2$

$$\text{Spacing required} = \frac{113 \times 1000}{600} = 188 \text{ mm}$$

Hence provide 10 mm diameter bars @ 180 mm c/c in both directions in toe slab.

■ **Shear design:** The critical section for shear is at a distance 'd' from face of the stem *i.e.*, 0.44 m from stem or

$$0.9 - 0.44 = 0.46 \text{ m from toe edge}$$

$$\text{Pressure at this section} = 91.22 + \frac{1}{2}\left(\frac{123.06 - 91.22}{0.9}\right) \times 0.44$$

$$= 99.0 \text{ kN/m}^2$$

$$\text{S.F. at this section} = 99 \times 0.46 + \frac{1}{2}(123.06 - 99) \times 0.46$$

$$= 51.1 \text{ kN per m run}$$

$$V_u = 1.5 \times 51.1$$
$$V_u = 76.65 \text{ kN}$$

$$\tau_V = \frac{76.65 \times 10^3}{1000 \times 440} = 0.17 \text{ N/mm}^2$$

$$p_t = \frac{600 \times 100}{1000 \times 440} = 0.14\%$$

$$\tau_c = 0.28 \text{ N/mm}^2$$

$$\tau_v < \tau_c \quad \text{Hence OK}$$

■ **Design of heel slab**

The heel slab also acts as a continuous slab supported on counterforts like stem.

$$\text{Weight of backfill} = 1.0 \times 5.0 \times 15$$

$$= 75 \text{ kN/m}^2 \text{ per m run}$$

$$\text{Self weight of slab} = 1.0 \times 0.5 \times 25 = 12.5 \text{ kN/m}^2$$

$$\text{Total downward weight} = 75 + 12.5 = 87.5 \text{ kN/m}^2$$

Maximum downward pressure at the edge of the heel slab

$$= 87.5 - 16.94 = 70.56 \text{ kN/m}^2$$

$$M = \frac{70.56 \times 3^2}{12} = 52.92 \text{ kNm}$$

$$M_u = 1.5 \times 52.92$$

$$M_u = 79.38 \text{ kNm}$$

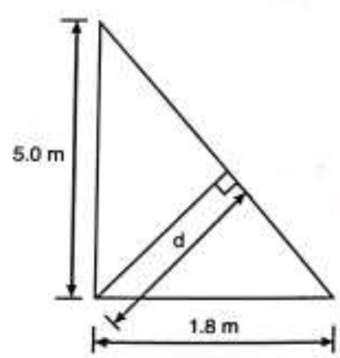
Area of Steel required:

$$79.38 \times 10^6 = 0.87 \times 415 \times A_{st} \times 440\left[1 - \frac{415 A_{st}}{20 \times 1000 \times 440}\right]$$

$$A_{st} = 512 \text{ mm}^2 < 600 \text{ mm}^2 \; (A_{st\,\min})$$

Hence provide 10 mm diameter @ 180 mm c/c in both directions.

■ **Design of counterforts:** Counterforts are designed as a triangular beam (beam of varying depth) supported on the stem and heel slab. It is also to be designed for the tension which tries to pull the counterfort away from stem and heel.



$$\tan\theta = \frac{5.0}{1.8} = 2.77$$

$$\theta = 43.71°$$

Depth of the triangular beam, (*d*)

$$d = 1.8 \sin\theta$$

$$= 1.8 \sin 43.71°$$

$$d = 1.243 \text{ m}$$

Maximum moment on the counterforts

$$= \left(\frac{1}{2}K_a\gamma h.h.\frac{h}{3}\right)\times L \quad \text{where } L \text{ is the spacing of counterforts}$$

$$= \left(\frac{1}{2}\times\frac{1}{3}\times 15\times 5\times 5\times\frac{5}{3}\right)\times 3.0$$

$$M = 312.5 \text{ kNm}$$

$$M_u = 1.5 \times 312.5 = 468.75 \text{ kNm}$$

Area of steel required:

$$468.75 \times 10^6 = 0.87\times 415\times A_{st}\times 1243\left[1-\frac{415A_{st}}{20\times 300\times 1243}\right]$$

$$A_{st} = 1114 \text{ mm}^2$$

$$A_{st\ min} = \frac{0.85bd}{f_y} = \frac{0.85\times 300\times 1243}{415}$$

$$= 763 \text{ mm}^2 < 1114 \text{ mm}^2 \quad \text{Hence OK}$$

Providing 4 bars bars of 20 mm diameter

$$A_{st\ provided} = 4\times\frac{\pi}{4}\times 20^2 = 1256 \text{ mm}^2 \quad \text{[curtailing 2 bars near the top]}$$

■ **Design for Horizontal Tension in Counterforts**

Horizontal ties are used for taking horizontal tension, caused due to the lateral earh pressure.

Considering the bottom 1 m height of the stem.

Maximum lateral pressure at the bottom

$$= K_a\gamma h$$

$$= \frac{1}{3}\times 15\times 5$$

$$= 25 \text{ kN/m}^2$$

Total lateral pressure to be taken by counterforts

$$= 25(3 - 0.3) \text{ per m run}$$

$$= 67.5 \text{ kN}$$

$$\text{Factored tensile force} = 1.5 \times 67.5$$

$$= 101.25 \text{ kN}$$

Area of steel required: $T = 0.87 f_y A_{st}$

$$A_{st} = \frac{101.25\times 1000}{0.87\times 415} = 281 \text{ mm}^2$$

Providing 10 mm bars $\qquad A_\phi = 78.5 \text{ mm}^2$

$$\text{Spacing required} = \frac{78.5 \times 1000}{281} = 279 \text{ mm}$$

Provide 10 mm diameter ties @ 260 mm c/c in the horizontal direction.

■ **Design for Vertical tension in counterforts**

The vertical tension in counterforts is caused due to the downward pressure which tries to separate out the counterfort and the heel.

Maximum downward pressure on the counterfort at the edge of heel = 70.56 kN/m^2

$$\text{Factored tensile force} = 1.5 \times 70.56$$
$$= 105.84 \text{ kN}$$

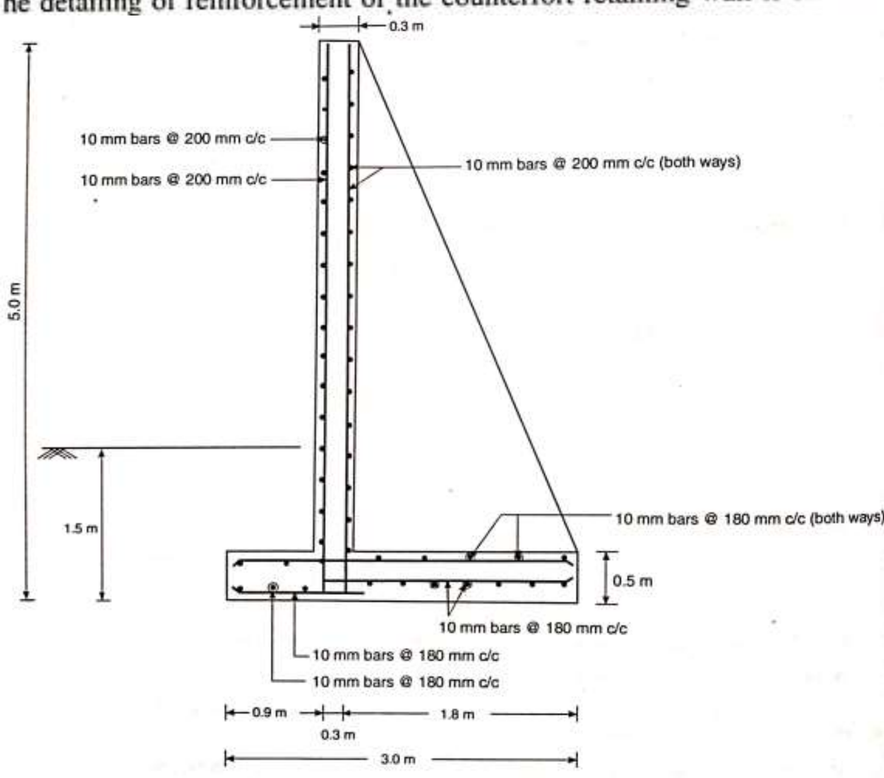
$$\text{Area of steel required } A_{st} = \frac{T}{0.87 f_y}$$
$$= \frac{105.84 \times 1000}{0.87 \times 415}$$
$$A_{st} = 293 \text{ mm}^2$$

Using 10 mm diameter bars,

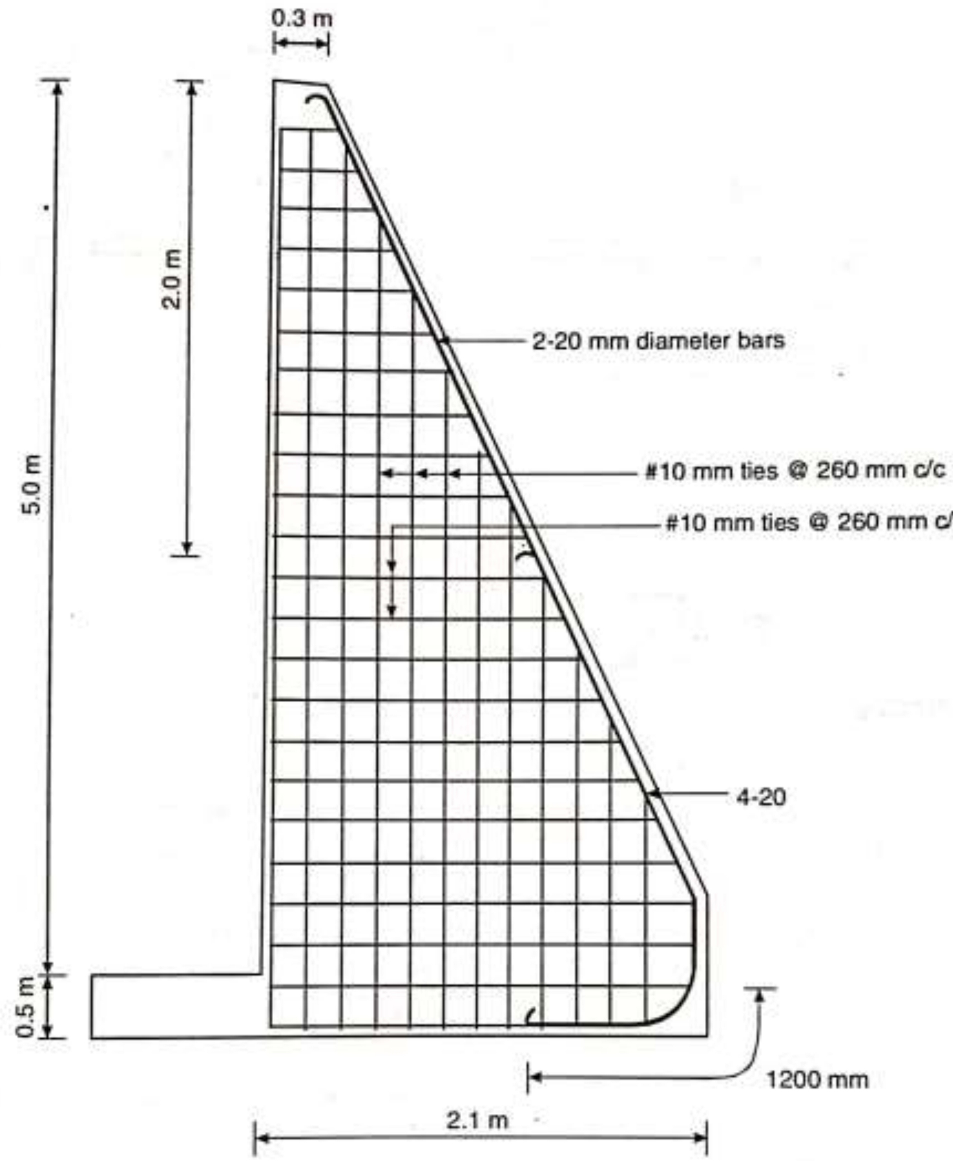
$$S_{v\,\text{reqd}} = 267 \text{ mm}$$

Hence provide 10 mm diameter ties @ 260 mm c/c.

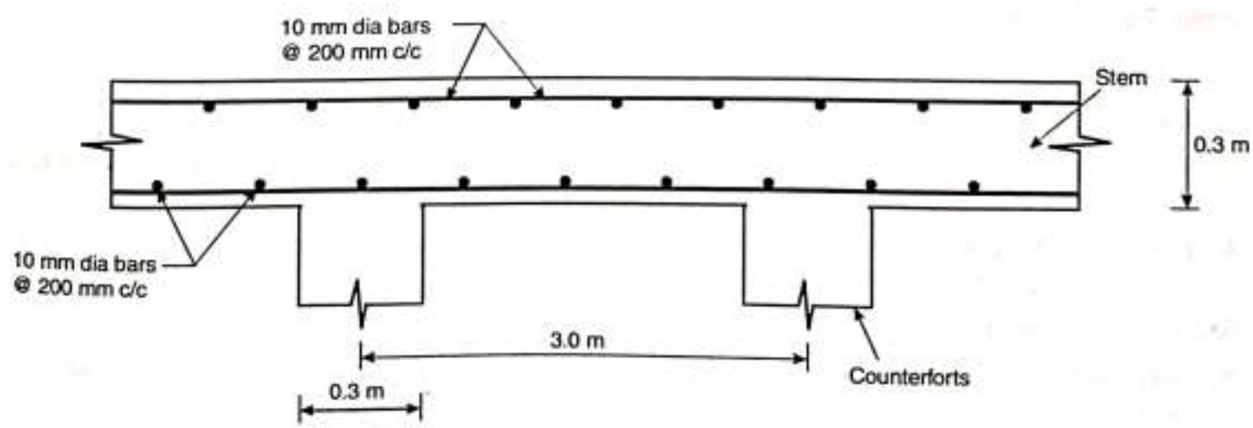
The detailing of reinforcement of the counterfort retaining wall is shown in Fig.



(*a*) Section at midspan of counterforts



(*b*) Section at counterfort



(*c*) Details of Stress reinforcement at base

POSSIBLE SHORT TYPE QUESTIONS

1. **What is retaining wall? How many types of retaining walls are there?**
   The wall which is used to retain earth or loose material is known as so. The material which is retained by retain wall are called as backfill.
2. **What is shear key?**
   Shear key is provided in retaining wall to increase the resistance against sliding.
3. **What are the parts of a cantilever retaining wall?**
   - Stem
   - Heel slab
   - Toe slab

POSSIBLE LONG TYPE QUESTIONS

1. Design a cantilever retaining wall to retain horizontal earthen embankment of height 4m above the ground level. The earthen backfill is having a density of 18 KN/m^3 and angle of internal friction as 30^0. The safe bearing capacity of soil is 180 KN/m^3 . The coefficient of friction between soil and concrete is assumed to be 0.45. Use M_{20} and Fe_{415} steel.
2. Design a counterfort retaining wall to retain 4 m earth above ground level. The top of the earth is to be level. The density of earth is 15 KN/m^2 . The angle of internal friction of soil is 30^0. The safe bearing capacity of soil is 200 KN/m^2 and coefficient of friction between soil and wall is 0.6.
3. Design a cantilever retaining wall for retaining an earth fill of 4.5 m height of 4.5 m above the ground level. The safe bearing capacity of soil is 130 KN/m^2 and unit weight of soil is 18 KN/m^3. The angle of internal friction is 30^0 and coefficient of friction between soil and concrete is 0.45. Use M_{20} concrete and Fe415 steel.

MODULE-5 **Chapter - 9**
Session - 52

Learning Objectives
9.1 Introduction on water tank
9.2 Design Philosophy
9.2.1 Methods of design

Tanks are widely used for storing liquids like water, chemicals and petroleum etc. The tanks are generally circular or rectangular in shape. They are broadly categorized into following three types:

1. Tanks resting on ground
2. Underground tanks
3. Elevated or overhead tanks.

The tanks resting on ground are supported on the ground directly. The sedimentation tanks, aeration tanks, filteration tanks and clear water storage reservoirs are generally of this type while the septic tank, imhoff tank and simple water tanks collecting water from the mains are generally constructed as underground tanks. Elevated or overhead water tanks, supported on staging, are commonly used in water distribution system. For constructing any type of liquid retaining structure, it is a must to ensure that the concrete is dense and impervious. It is essential not only from the leakage point of view, but also affects the durability, cracking and resistance against chemical attack and corrosion.

The Indian Standard Code of practicle for design of liquid retaining concrete structures *i.e.*, IS:3370 was first published in 1965. Presently, it is available in four parts as follows:

1. **IS 3370:2009 (Part 1):** Code of Practice for Concrete Structures for Storage of Liquids: General requirements.
2. **IS 3370:2009 (Part-2):** Code of Practice for Concrete Structures for Storage of Liquids: Reinforced Concrete Structures.
3. **IS 3370:1967 (Part-3):** Code of Practice for Concrete Structures for Storage of Liquids: Prestressed Concrete Structures.
4. **IS 3370:1967 (Part-4):** Design Tables for Design of Reinforced or Prestressed Concrete Structures for Storage of Liquids.

9.2 Design Philoshopy

Design of liquid retaining structures is based upon the fact that the concrete should not crack and hence the tensile strength of concrete should be within permissible limits. In order to control cracking, various requirements regarding material, joints and reinforcement detailing are listed in IS 3370 (Part 1): 2009, some of which are explained below:

1. Concrete mixes lower than M30 are not to be used for design of liquid retaining structures. The use of richer mixes results in less cracking.
2. The structure retaining the liquids should be designed as "subjected to Severe Exposure Conditions".
3. The cement content, not including fly ash and ground granulated blast furnace slag, should not exceed 400 kg/m^3 unless special consideration is taken for increased risk of cracking due to drying shrinkage etc.

4. Cracking can be controlled by using the plasticizers and by using minimum amount of cement content which will result in reduced water content per unit of concrete mix. The minimum cement content from durability criteria is 320 kg/m^3.
5. Cracking can also be controlled by reducing the steep changes in temperature and moisture content at early age of concrete. Curing should be done at least for a period 14 days.
6. Correct placing of reinforcement bars, use of deformed bars, bars closely spaced and use of small sized bars will also result in reduced cracking.
7. Crack width for reinforced concrete members in direct tension and flexural tension is considered satisfactory, if steel stress under service conditions does not exceed 130 N/mm^2 for high strength deformed bars.
8. The maximum calculated surface width of cracks for direct tension and flexure should not be more than 0.2 mm with specified cover.

9.2.1 Methods of design

The design of water tanks can be done by any of two methods given below:

(*i*) Limit state method of design.

(*ii*) Working stress method of design.

(i) Limit State Method of Design

In this method, all relevant limit states should be considered and satisfied with an adequate degree of safety and serviceability. The limit state of collapse and limit state of serviceability (Deflection and Cracking) should be followed as per IS 456:2000.

(ii) Working Stress Method of Design

The working stress method for design of water tanks is based on adequate resistance to cracking and strength.

The various assumptions of this method are as follows: (Refer Chapter 2)

(*a*) Plane sections remain plane before and after bending.

(*b*) Steel and concrete behave elastically and the modular ratio, m is given by:

$$m = \frac{280}{3\sigma_{cbc}}$$

Permissible Concrete Stresses in Calculations Related to Resistance to Cracking

S. No.	Grade of Concrete	Direct tension (N/mm^2)	Flexural tension (N/mm^2) σ_{bt}
1.	M25	1.3	1.8
2.	M30	1.5	2.0
2.	M35	1.6	2.2
3.	M40	1.8	2.4
4.	M45	2.0	2.6
5.	M50	2.1	2.8

Permissible Stresses in Concrete

S. No.	Grade of Concrete	Compressive Stress (N/mm^2)		Average Bond Stress for Plain bars in tension (N/mm^2) τ_{bd}
		Bending (σ_{cbc})	Direct (σ_{cc})	
1.	M25	8.5	6.0	0.9
2.	M30	10.0	8.0	1.0
2.	M35	11.5	9.0	1.1
3.	M40	13.0	10.0	1.2
4.	M45	14.5	11.0	1.3
5.	M50	16.0	12.0	1.4

Note:

1. **Bond stress in compression should be increased by 25%.**
2. **For deformed bars, the bond stress should be increased by 60%.**

Permissible Stresses in Steel

1. Resistance to Cracking: The permissible stresses in steel is limited by the fact that the permissible tensile stresses for resistance to cracking in concrete are not exceeded. In order to have the perfect bond between steel and concrete, the permissible stress in steel can be written as:

$$\sigma_{st} = m\sigma_{ct}$$

where σ_{st} = permissible tensile stress in steel

σ_{ct} = permissible tensile stress in concrete

m = modular ratio of steel and concrete.

2. Permissible Stresses for strength Calculations: For the purpose of strength calculations in liquid retaining structures, the permissible stresses should be as listed in table

S. No.	Type of Stress in Steel Reinforcement	Permissible Stresses (N/mm^2)	
		Plain mild steel bars	High yield strength deformed bars
1.	Tensile stress in members under direct tension, bending and shear	115	130
2.	Compressive stress in columns subjected to direct load	125	140

MODULE-5 **Chapter - 9**
Session - 53

Learning Objectives

9.3 IS Code recommendations regarding detailing in water tanks

9.4 Joints in Water Tank

9.4.1 contraction Joints

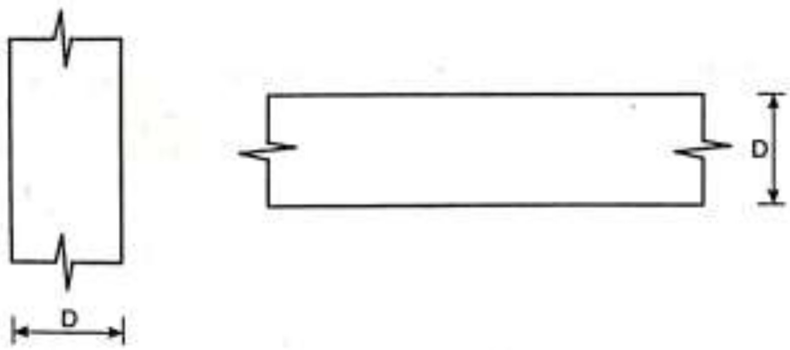
9.4.2 Expansion Joints

9.4.3 Sliding Joints

9.4.4 Construction Joints

9.4.5 Temporary open joints

1. The minimum reinforcement walls, floors and roofs in each of two directions at right angles within each surface zone should not be less than 0.35 % of the cross section of surface zone for HYSD bars and 0.64% for mild steel bars.

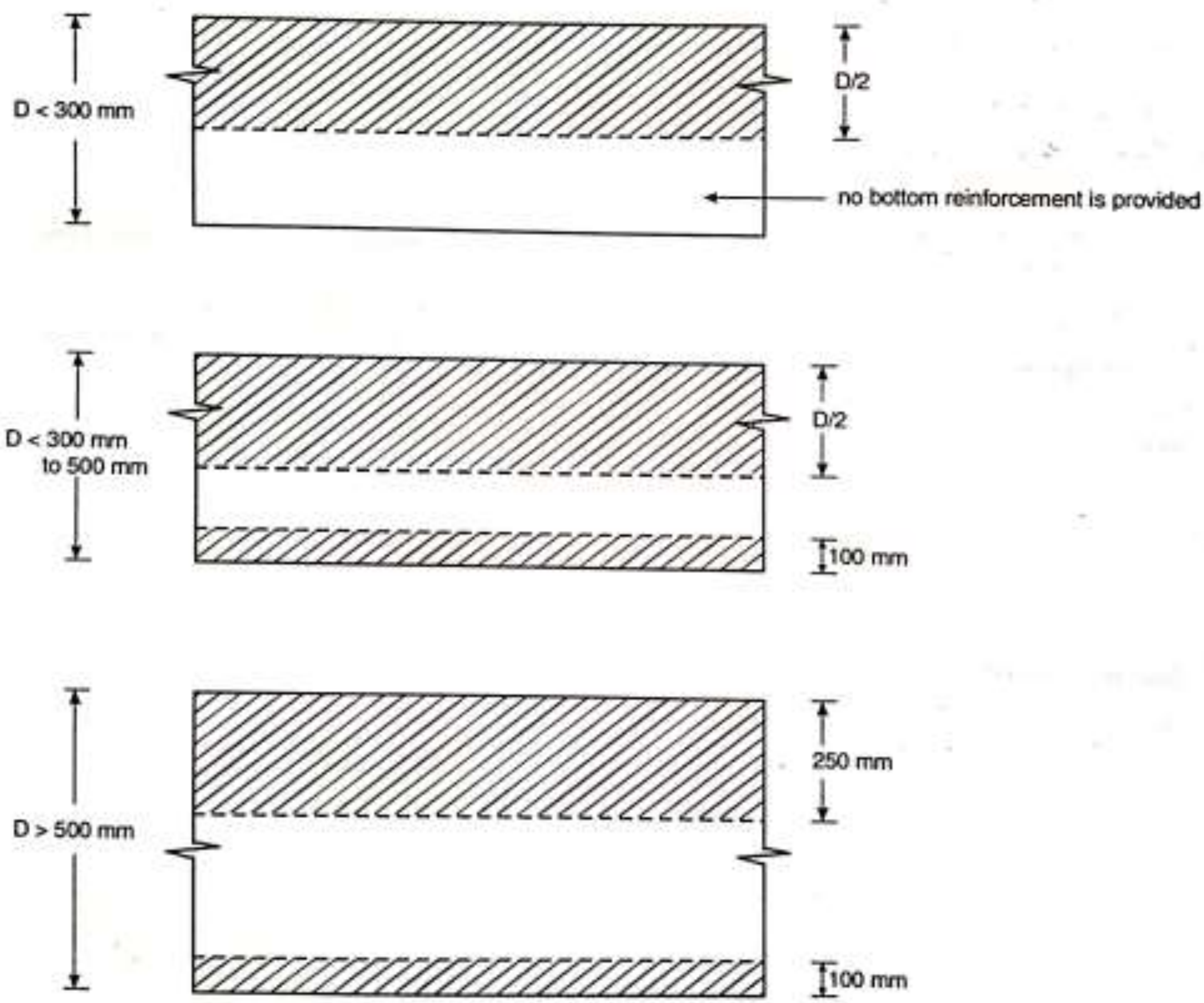


Surface zones: walls and suspended slabs.

2. The minimum reinforcement can be further reduced to 0.24% for deformed bars and 0.40% for plain bars, for tanks, not having any dimension more than 15 m.
3. In tank walls and slabs, having thickness less than 200 mm, the reinforcement can be placed in
4. For ground/base slab, having thickness less than 300 mm, the reinforcement should be placed on one face, as near as possible to the upper surface consistent with the cover.
5. The spacing of reinforcing bars should not exceed 300 mm or thickness of the section, whichever is less.
6. Size of bars, distance between bars, laps and bends should be as per IS 456:2000.

Note:

1. For $D \geq 500$ mm *i.e.*, thickness of the member greater than or equal to 500 mm, each reinforcement face controls half of the total depth ($D/2$) of concrete.
2. For $D < 500$ *i.e.*, thickness of the member less than 500 mm, each reinforcement face controls 250 mm depth of concrete, ignoring any central core beyond the surface depth.

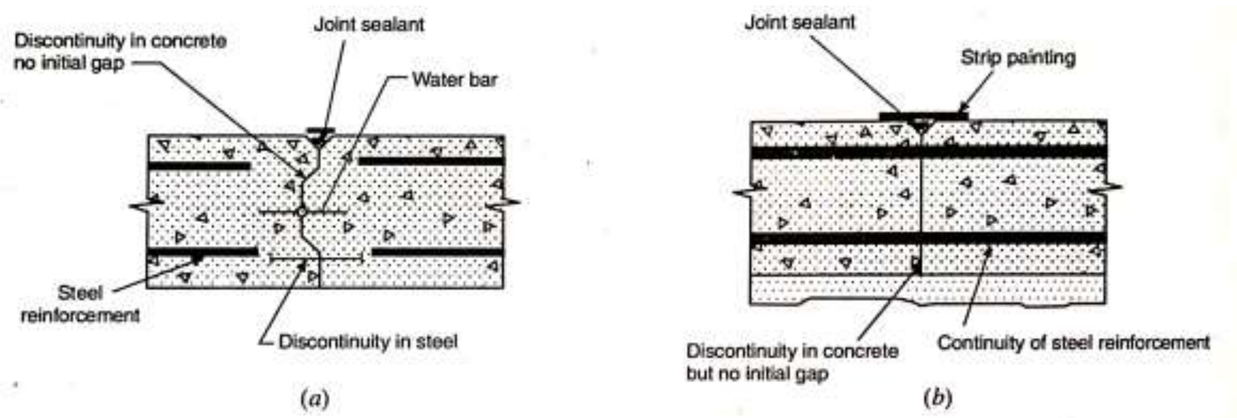


Surface zones in ground/base slabs.

9.4 Joints in Water Tank

9.4.1 Contraction Joints

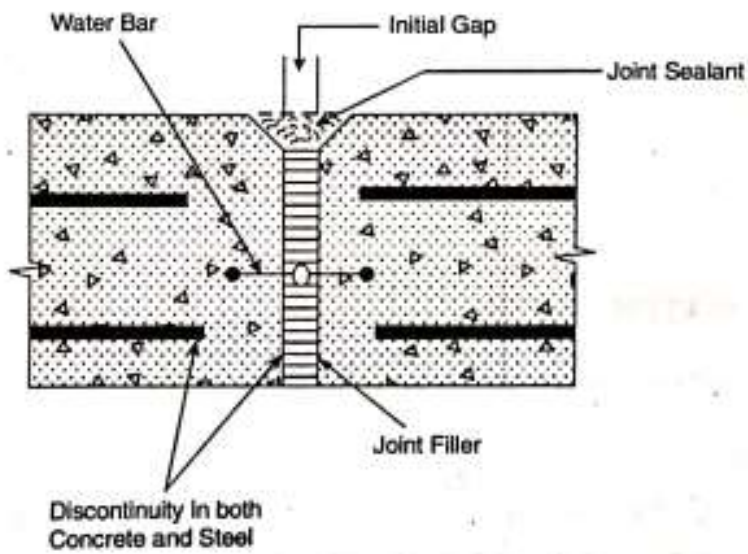
It is a movement joint, with deliberate discontinuity, without initial gap between the concrete on either side of the joint. The joint is designed to accommodate contraction of the concrete as shown in fig.



Contraction joints.

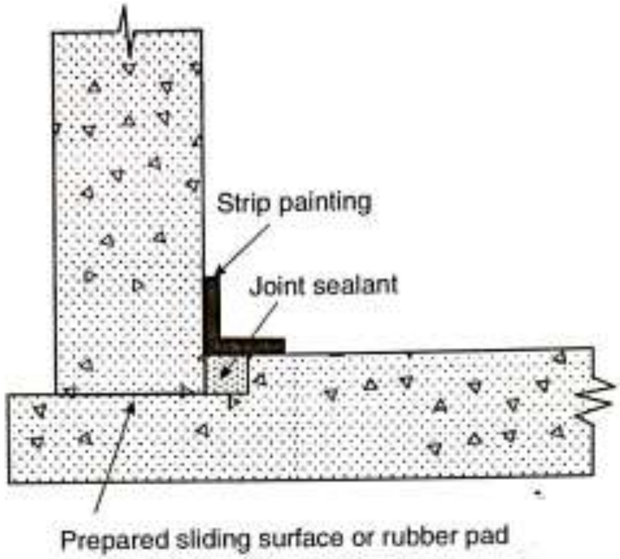
9.4.2 Expansion Joints

In this type of movement joint, complete discontinuity in both steel and concrete is provided to accommodate either expansion or contraction joint of the concrete. This joint has no restraint to movement. This type of joint requires an initial gap between the adjoining parts of as structure to accommodate expansion/contraction of the concrete of the concrete as shown in fig.



9.4.3 Sliding Joints

A movement joint which allows the adjoining parts of a structure to slide relative to each other with minimum restraint is known as sliding joint, In this joint, complete discontinuity is provided in both steel and concrete and at the discontinuity special provision is made to facilitate the relative sliding movement.

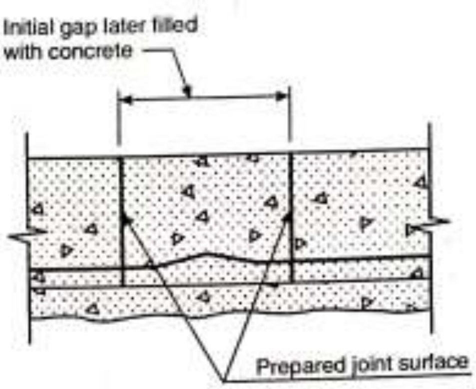


9.4.4 Construction Joint

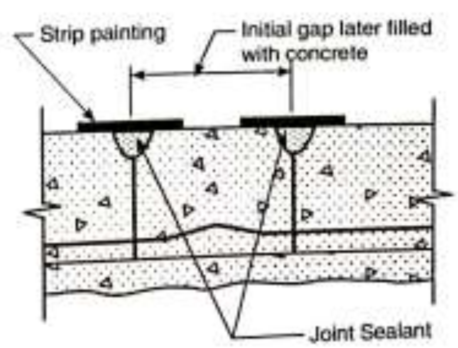
These joints are provided for convenience in construction. At these joints, special measures are incorporated to have subsequent continuity, without provision for further relative movement. These joints may be grouted and concrete at the joints should be bonded properly. The number of such joints should be kept as small as possible.

9.4.5 Temporary open joints

A gap is sometimes left temporarily between the concrete of adjoining parts of a water tank which is filled with mortar or with suitable jointing material, after a suitable interval of rime, before the structure is put to use.



(*a*)



(*b*)

Learning Objectives
9.5 Analysis of water tanks
9.6 Circular Tank resting on the ground

9.5 Analysis of water tanks

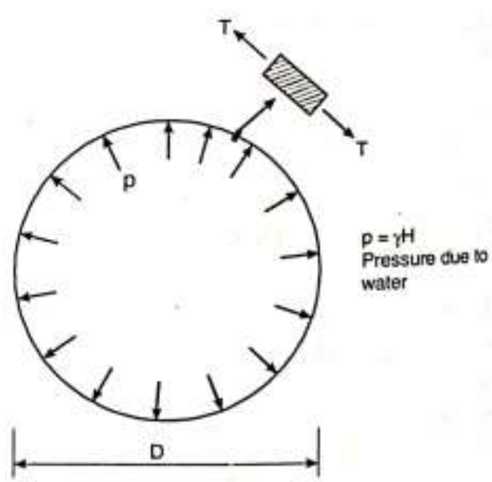
The exact analysis of water tanks is very complicated and time consuming. It requires thorough knowledge of theory of plates and cylinders based on finite element method, with appropriate boundary conditions and constraints. To simplify the analysis, IS code 3370 (Part-4): 1967 gives, ready to use, design tables for moments, hoop tension and shear coefficients values for cylindrical and rectangular tanks for different boundary conditions and various types of loadings. In addition to IS code method, approximate methods of analysis can also be used for design of simple tanks. These approximate methods do not reflect the exact behaviour of tanks but give results on the conservative side and save lot of time.

9.6 Circular Tank resting on the ground

The design of such tanks is based on the type of joint provided between the floor/base slab and the walls of the tank.

Tank With Flexible Joint Between the Floor and the Walls (Approximate Method)

In this type of tank, the flexible joint provided between the floor and the walls of the tank is flexible, thus allowing the horizontal, outward movement of the walls. Hence, no moments are developed at the joints. In such tanks, walls are subjected to hoop tension only which is developed because of the hydrostatic water pressure and is given by the following equation:



$$T = \gamma H \frac{D}{2}$$

where γ is the unit weight of water
H is the height of water
D is the diameter of the tank
T is the hoop tension

The thicknesss (t) of the tank wall may be calculated from the requirement thant tensile stress in concrete should be within the permissible limit. If σ_{ct} is the permissible tensile stress in concrete then

The reinforcement for this hoop tension is to be provided all along the height of the wall, in the form of hoops or rings suitably spaced apart. The area of steel required for carrying the hoop tension T, is calculated as follows:

$$T = \sigma_{st} . A_{st} = \gamma H \frac{D}{2}$$

$$A_{st} = \frac{\gamma H D}{2\sigma_{st}}$$

The thicknesss (t) of the tank wall may be calculated from the requirement thant tensile stress in concrete should be within the permissible limit. If σ_{ct} is the permissible tensile stress in concrete then

$$\sigma_{ct} > \frac{T}{A_{eq}} \qquad \text{where } A_{eq} \text{ is the area of the transformed section}$$

$$\sigma_{ct} > \frac{T}{A + (m-1) A_{st}}$$

$$\sigma_{ct} > \frac{T}{1000t + (m-1) A_{st}}$$

Since, the base slab of the tank rests on the ground, load gets transferred to the soil directly and hence a minimum thickness of 150 mm should be provided with minimum steel 0.35% in each direction.

MODULE-5 **Chapter - 9**
Session - 55

Learning Objectives

9.5 Design of on ground water tanks (Problems)

Problem

Design a circular tank with a flexible base for a tank of 1,00,000 litre capacity. The depth of water in the tank is 5m. Use M_{25} concrete and Fe_{415} Steel. Take unit weight of water as 9.8 KN/m²$^{}$.

Solution. Given: Volume of water in tank $= 1,00,000 l$

$$= \frac{100000}{1000} \text{ m}^3$$

Height of water in tank (H) $= 5.0$ m

σ_{st} = Permissible tensile stress in steel $= 130 \text{ N/mm}^2$ for HYSD bars

Permissible direct tensile stress in concrete= 1.3 N/mm^2 for M25 concrete

If D is the diameter of the tank then

$$\text{Volume of tank} = \frac{100000}{1000}$$

$$\frac{\pi}{4} . D^2 \times 5.0 = 100$$

$$D = 5.05 \text{ m}$$

Hence providing a diameter of 5.1 m.

■ **Maximum hoop tension (T)**

$$T = \gamma H \frac{D}{2}$$

$$= 9.8 \times 5.0 \times \frac{5.1}{2}$$

$$T = 124.95 \text{ kN per m height of the wall}$$

■ **Area of Steel**

$$A_{st} = \frac{T}{\sigma_{st}}$$

$$= \frac{124.95 \times 1000}{130}$$

$$A_{st} = 962 \text{ mm}^2$$

Using 12 mm diameter bars

$$\text{Spacing required} = \frac{113 \times 1000}{962}$$

$$= 117 \text{ mm}$$

Hence provide 12 mm diameter hoops (rings) @ 110 m c/c

$$A_{st \text{ provided}} = 1027 \text{ mm}^2$$

$T = \gamma \times 5.0 \times \frac{2.5}{2} = 62.5$ kN

$A_{st} = \frac{62.5 \times 1000}{130} = 481 \text{mm}^2$

At a distance 2.5 m from top $T = 62.5$ kN per m, and $A_{st \text{ reqd}} = 481$ mm^2, hence spacing can be doubled.

■ **Thickness of tank wall:** The thickness of the wall should be such that the tensile stress in concrete should not exceed the permissible value (σ_{ct})

$$\sigma_{ct} > \frac{T}{1000.t + (m-1)A_{st}}$$

$$1.3 > \frac{124.95 \times 1000}{1000.t + (11-1) \times 1027}$$

$$t > 85 \text{ mm}$$

Hence providing a thickness of 100 mm for tank wall

$$A_{st \min} = 0.35\% \text{ of X-section area of surface zone}$$

$$= \frac{0.35}{100} \times \left(1000 \times \frac{100}{2}\right)$$

$$= 175 \text{ mm}^2 < 1027 \text{ mm}^2$$

The spacing of hoops $\not>$ 300 mm or the thickness of section.

∴ Providing 12 mm diameter hoops @ 110 mm c/c along the height of the wall. The spacing is increased to 220 mm c/c at a distance 2.5 m from top.

■ Distribution Reinforcement

Distribution and temperature steel is provided @ 0.35%

$$= 175 \text{ mm}^2$$

Providing 8 mm diameter bars @ 250 mm c/c vertical steel

$\frac{\pi}{4} \times 8^2 : 50.26 \text{ mm}^2$

$$A_{st} = \frac{2 \times 50 \times 1000}{250}$$

$$= 200 \text{ mm}^2 > 175 \text{ mm}^2$$

Hence OK.

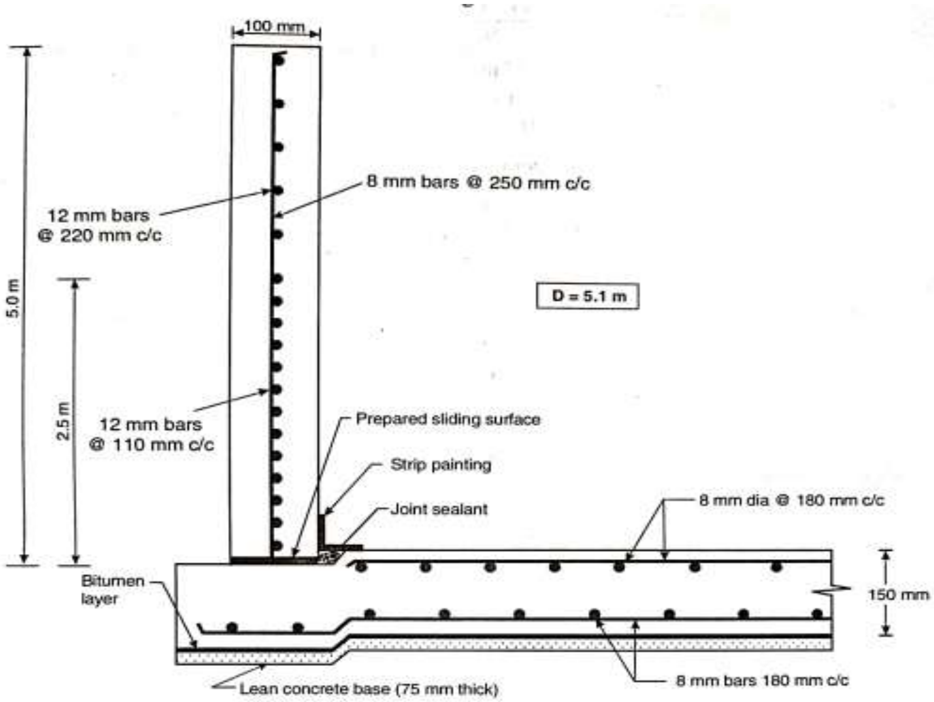
■ Design of Base/Floor Slab

Since the tank floor is resting on the ground, the load gets directly transferred to the soil. Hence providing a minimum thickness of 150 mm and 0.35% minimum steel in each direction

$$= \frac{0.35}{100} \times 1000 \times \frac{150}{2}$$

$$= 263 \text{ mm}^2$$

Hence provide 8 mm diameter bars @ 180 mm c/c in both directions at top and bottom face of the floor slab.



MODULE-5 **Chapter - 9**
Session - 56

Learning Objectives
9.5.1 Design of on ground water tanks (Problems)

Problems
Design a circular tank with flexible base for a capacity of 450 kl. The depth of water is 4.5 m. Allow suitable free board.

Solution. Using M30 concrete and Fe 415 steel

$$\text{Capacity of tank} = 450 \text{ kl} = 450{,}000 \text{ l} = \frac{450000}{1000} \text{ m}^3$$

Taking 200 mm as freeboard, effective depth of water in tank

$$= 4.5 - 0.2 = 4.3 \text{ m}$$

$$\left(\frac{\pi}{4}D^2\right) \times H = 450$$

$$D = \sqrt{\frac{450 \times 4}{\pi \times 4.3}} = 11.54 \text{ m}$$

Taking $D = 11.6$ m

- **Design Constants**

$$\left.\begin{aligned} \sigma_{cbc} &= 10 \text{ N/mm}^2 \\ \sigma_{bt} &= 2.0 \text{ N/mm}^2 \\ \sigma_{ct} &= 1.5 \text{ N/mm}^2 \end{aligned}\right\}$$

$$\sigma_{st} = 130 \text{ N/mm}^2$$

$$m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 10} = 9.33$$

$$k = \frac{m \,.\, \sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}} = 0.416$$

$$j = 0.85$$

$$R = \frac{1}{2} \times \sigma_{cbc} kj$$

$$= \frac{1}{2} \times 10 \times 0.416 \times 0.86$$

$$= 1.78$$

$= 1.78$

■ Design for Hoop Tension

$$\text{Maximum hoop tension} = \gamma H \frac{D}{2} \text{ at bottom of tank}$$

$$= 10 \times 4.5 \times \frac{11.6}{2}$$

$$= 261 \text{ kN per m height of wall}$$

$$A_{st\ reqd} = \frac{T}{\sigma_{st}} = \frac{261 \times 1000}{130}$$

$$A_{st} = 2008 \text{ mm}^2$$

Using 20 mm diameter hoops

$$\text{Spacing required} = \frac{314 \times 1000}{2008} = 156 \text{ mm}$$

Hence provide 20 mm diameter hoops @ 150 mm c/c at the bottom at the centre of the wall. The spacing can be increased near the top, say at 2 m from top, 20 mm hoops @ 300 mm c/c are provided.

$$A_{st\ provided} = 2093 \text{ mm}^2$$

$$\sigma_{ct} > \frac{T}{1000.t + (m-1)A_{st}}$$

$$\sigma_{ct} > \frac{261 \times 1000}{1000 \times t + (9.33 - 1)2093}$$

$$\therefore \quad \frac{261 \times 1000}{1000 \times t + 8.33 \times 2093} < 1.5$$

$$t > 157 \text{ mm}$$

Hence providing $t = 200$ mm with an effective cover = 30 mm

■ Vertical Steel or Distribution Steel

Vertical steel is provided @ 0.35% of the surface zone

$$t < 300 \text{ mm}$$

$$\therefore \quad A_{st\ minimum} = \frac{0.35}{100} \times \left(1000 \times \frac{200}{2}\right)$$

$$= 350 \text{ mm}^2$$

Providing 10 mm diameter bars @ 200 mm c/c in the vertical direction.

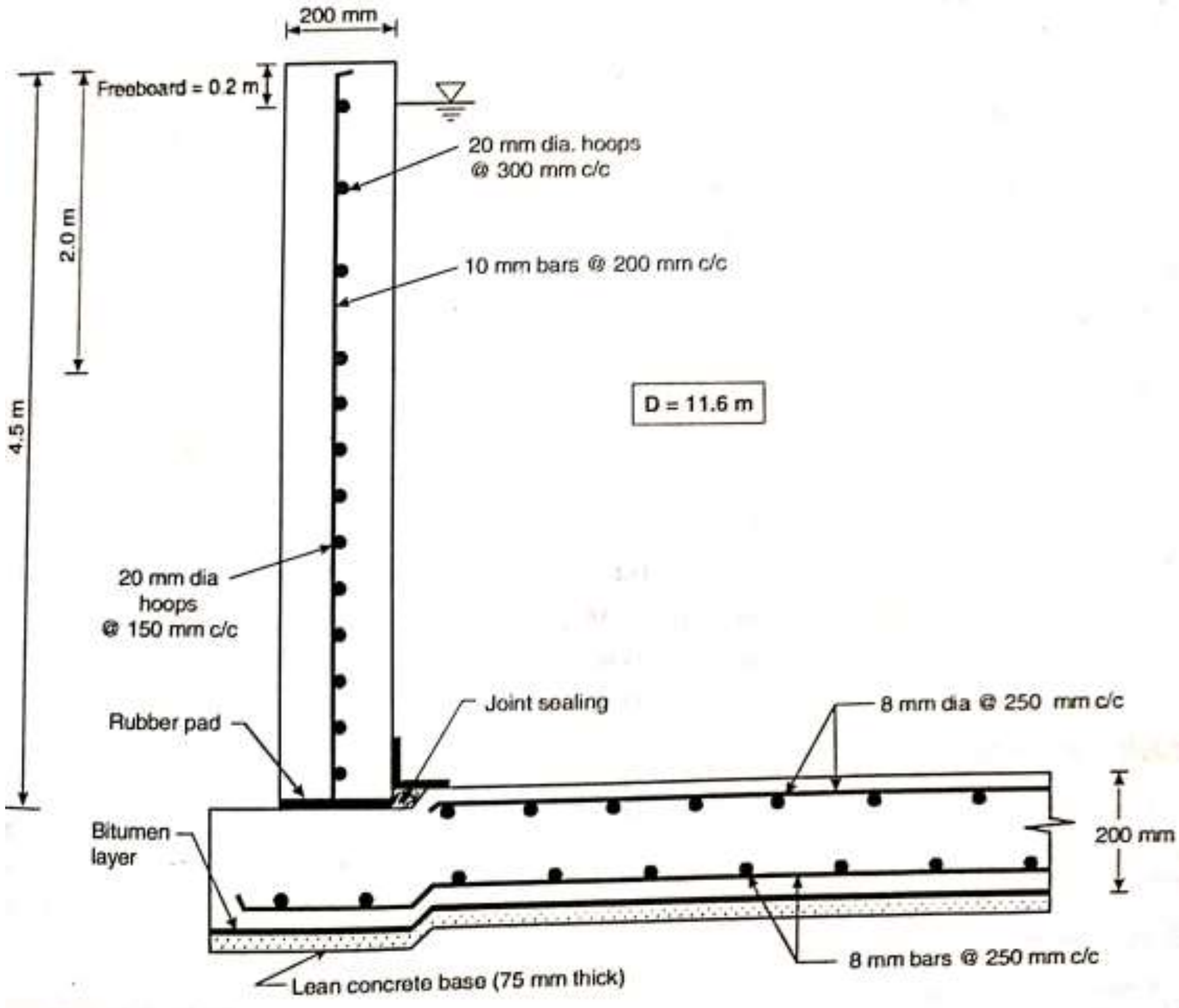
■ Design of Base Slab

As the tank is resting on ground, providing a thickness of 200 mm and minimum steel @ 0.35%.

$$A_{st} = 350 \text{ mm}^2$$

Providing half on each face *i.e.*, 175 mm²

Hence provide 8 mm diameter bars @ 250 mm c/c on each face. The details of reinforcement are shown **In fig.**



Circular tank with flexible joint

Learning Objectives

9.6 Design of elevated water tank(problem)

Design an elevated circular water tank 0f 500 Kl capacity with a top dome. The tank is supported on a masonry tower. The depth of water tank is 5 m. The unit weight of water= 10 KN/m^3. The love load on dome as 1.0 KN/m^2

Solution. Using M30 concrete and Fe 415 HYSD steel

Design constants:

$m = 9.33$

$k = 0.416$

$j = 0.86$

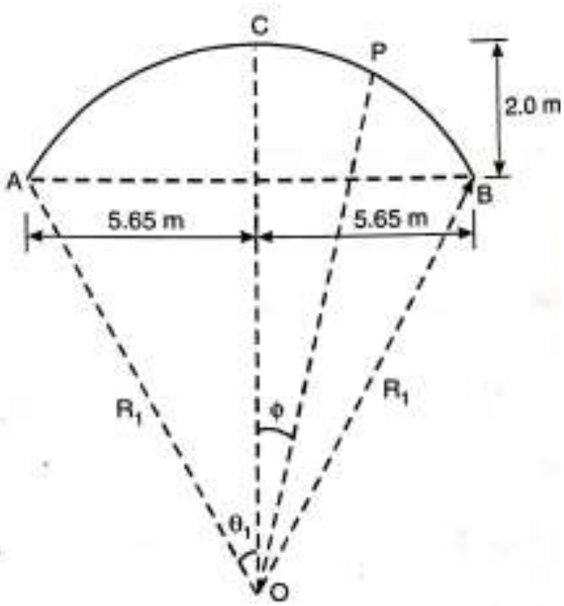
$R = 1.78$

For M20 concrete:

$\sigma_{ct} = 1.5 \text{ N/mm}^2$

$\sigma_{bt} = 2.0 \text{ N/mm}^2$

For Fe 415, HYSD steel $\sigma_{st} = 130 \text{ N/mm}^2$



■ **Diameter of Tank (*D*)**

Capacity of tank $= 500$ kl

$= 500 \times 1000$ litres

$$V = 500 \text{ m}^3$$

$$V = \left(\frac{\pi}{4} D^2\right) H$$

$$\frac{\pi}{4} D^2 \times 5.0 = 500$$

$$D = 11.28 \text{ m}$$ Hence taking $D = 11.3$ m

(1) Design of roof dome

Assuming the following data for dome and designing it using membrane analysis in which the dome is considered to be independent of walls

Thickness of dome = 100 mm

Rise of dome = 2.0 m

Live load = 1.0 kN/m² (given)

The radius R_1 of the dome is calculated as below:

$$(5.65)^2 = (2R - 2.0) \times 2.0$$

$$R_1 = 9.41 \text{ m}$$

$$\text{Self weight of dome} = 0.1 \times 25$$

$$= 2.5 \text{ kN/m}^2$$

$$\text{Live load on dome} = 1.0 \text{ kN/m}^2$$

$$\text{Total load} = 4.0 \text{ kN/m}^2$$

Semi central angle (θ) is given as below:

$$\sin\theta = \frac{5.65}{9.41} = 0.6$$

$$\theta = 36°53'55'' < 51°50'$$

As the semi central angle is less than 51°50′, whole dome is under hoop compression.

Design forces in dome:

$$\text{Meridonial thrust} = \frac{wR_1}{1+\cos\theta_1}$$

It gives maximum value at $\theta = 36°53'55''$

$$= \frac{4.0 \times 9.41}{1+\cos(36°53'55'')}$$

$$\text{Meridonial thrust} = 20.915 \text{ kN per m}$$

$$\text{Meridonial stress} = \frac{20.915 \times 1000}{\text{Thickness of dome}}$$

$$= \frac{20.915 \times 1000}{0.1}$$

$$= 209150 \text{ N/m}^2$$

$$= 0.209 \text{ N/mm}^2 < 1.5 \text{ N/mm}^2$$ Hence OK.

Hoop compression is maximum at $\theta = 0°$ *i.e.*, at the vertex C

$$= 0.618 \times 10 \times 5.0 \times \frac{11.3}{2}$$

$$T_{max} = 175 \text{ kN at 3.0 m } (0.6\ H) \text{ from top of wall}$$

(3) Maximum shear force at base of wall = $+0.152\gamma H^2$ (Table 11 of IS 3370)

$$= 0.152 \times 10 \times 25$$

$$= 38 \text{ kN}$$

■ **Design for Bending Moment**

$$M = 14.2 \text{ kNm at bottom of wall}$$

Effective thickness of wall required $= \sqrt{\dfrac{14.2 \times 10^6}{1000 \times 1.78}}$

$$= 91 \text{ mm}$$

Effective thickness of wall provided = 200 – 30 = 170 mm > 91 mm, Hence OK.

$$A_{st\ reqd} = \frac{M}{\sigma_{st} jd}$$

$$= \frac{14.2 \times 10^6}{130 \times 0.86 \times 170}$$

$$= 748 \text{ mm}^2$$

Using 12 mm diameter bars, spacing required:

$$= \frac{113 \times 1000}{748}$$

$$= 151 \text{ mm}$$

Therefore provide 12 mm diameter bars @ 140 mm c/c on the inner face. These bars are not required after a distance of 500 mm from bottom. However providing 12 mm dia @ 140 mm c/c up to a distance of 1 m from bottom and curtailing half of the bars at this point and continuing other half till top.

■ **Distribution Reinforcement**

Providing distribution steel @ 0.35% of x-sectional area of surface zone

$$= \frac{0.35}{100} \times \left(1000 \times \frac{200}{2}\right)$$

$$= 350 \text{ mm}^2$$

Providing 12 mm diameter @ 280 mm c/c on the outer face as distribution steel. No additional steel is required on the inner face as the steel provided for –ve moment will act as distribution steel also. The distribution steel provided on the outer face will also serve as steel required for +ve BM.

■ **Design for Hoop tension**

$$T_{max} = 175 \text{ kN}$$

$$A_{st\ required} = \frac{T}{\sigma_{st}} = \frac{175 \times 10^3}{130}$$

$$= 1347 \text{ mm}^2$$

Providing this steel on both faces *i.e.,* $\frac{1347}{2}$ on each face using 12 mm diameter hoops, spacing required:

$$= \frac{113 \times 1000}{\left(\frac{1347}{2}\right)}$$

$$= 167 \text{ mm}$$

Hence provide 12 mm diameter hoops @ 160 mm c/c on each face, this spacing may be increased towards the top as the hoop tension decreases

$$A_{st\ provided} = 1413 \text{ mm}^2$$

$$\text{Tensile stress in concrete} = \frac{T}{A + (m-1)A_{st}}$$

$$= \frac{175 \times 10^3}{1000 \times 200 + (9.33 - 1) \times 1413}$$

$$= 0.83 \text{ N/mm}^2 < 1.5 \text{ N/mm}^2 \qquad \text{Hence OK.}$$

(4) Design of Base Slab

The base slab is cast monolithic with the tank wall and hence it is assumed to be fixed at the edges. The base slab can be designed as a circular slab fixed along the edges and subjected to UDL on its entire Surface.

Let the thickness of slab = 500 mm

Pressure due to weight of water carried by the base slab

$$= 5 \times 10 = 50 \text{ kN/m}^2$$

Pressure due to self weight of base slab = $0.5 \times 25 = 12.5$ kN/m^2

Total pressure exerted on the base slab = 62.5 kN/m^2

(*i*) Circumferential moment in the circular slab:

(*a*) at centre $\quad M = \frac{+wr^2}{16} \qquad \left[r = \frac{D}{2} = \frac{11.3}{2}\right]$

$$= \frac{1}{16} \times 62.5 \times \left(\frac{11.3}{2}\right)^2$$

$$M = 125 \text{ kNm}$$

(*b*) At edges $\quad M = 0$

(*ii*) Radial moment in the circular slab:

(*a*) at centre $\quad M = +\frac{1}{16} wr^2$

$$= \frac{1}{16} \times 62.5 \times \left(\frac{11.3}{2}\right)^2$$

$$M = 125 \text{ kNm}$$

(*b*) at edges $\quad M = \frac{-2}{16} wr^2$

$$M = -250 \text{ kNm}$$

(*iii*) Radial shear force in the circular slab

$$= 0.5wr$$

$$= 0.5 \times 62.5 \times \frac{11.3}{2}$$

$$V = 176.5 \text{ kN}$$

$$M_{max} = 250 \text{ kNm}$$

$$\text{Thickness of slab required} = \sqrt{\frac{250 \times 10^6}{1000 \times 1.78}}$$

$$= 375 \text{ mm}$$

Hence providing a base slab of 450 mm depth with an effective cover of 50 mm

$$d = 400 \text{ mm}$$

$$500 - 50 = 450 \text{ mm}$$

Steel for –ve BM

$$A_{st} = \frac{250 \times 10^6}{130 \times 0.86 \times 450}$$

$$A_{st} = 4970 \text{ mm}^2$$

Using 30 mm diameter bars,

$$\text{Area of one bar} = \frac{\pi}{4} \times 30^2 = 706 \text{ mm}^2$$

$$\text{Spacing required} = \frac{706 \times 1000}{4970}$$

$$= 142 \text{ mm}$$

Hence providing 30 mm diameter bars radially @ 130 mm c/c at the edges upto a distance of 3 m. 3 rings of 30 mm diameter are provided to supported and tie them as shown in fig.

Steel for +ve BM

$$M = 125 \text{ kNm}$$

The effective depth available in one direction is 450 mm and 450 – 15 – 12.5 = 422.5 mm (using 25 mm diameter bars) in other direction.

$$A_{st\ required} = \frac{125 \times 10^6}{130 \times 0.86 \times 422.5}$$

$$= 191 \text{ mm}^2$$

Spacing required for 25 mm diameter bars

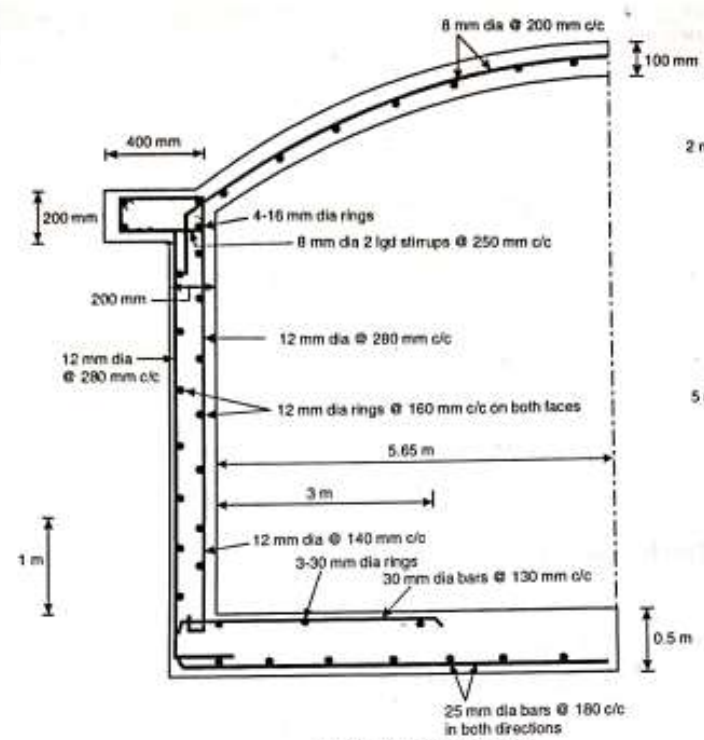
$$= 490 \times 1000$$

$$= 191 \text{ mm}$$

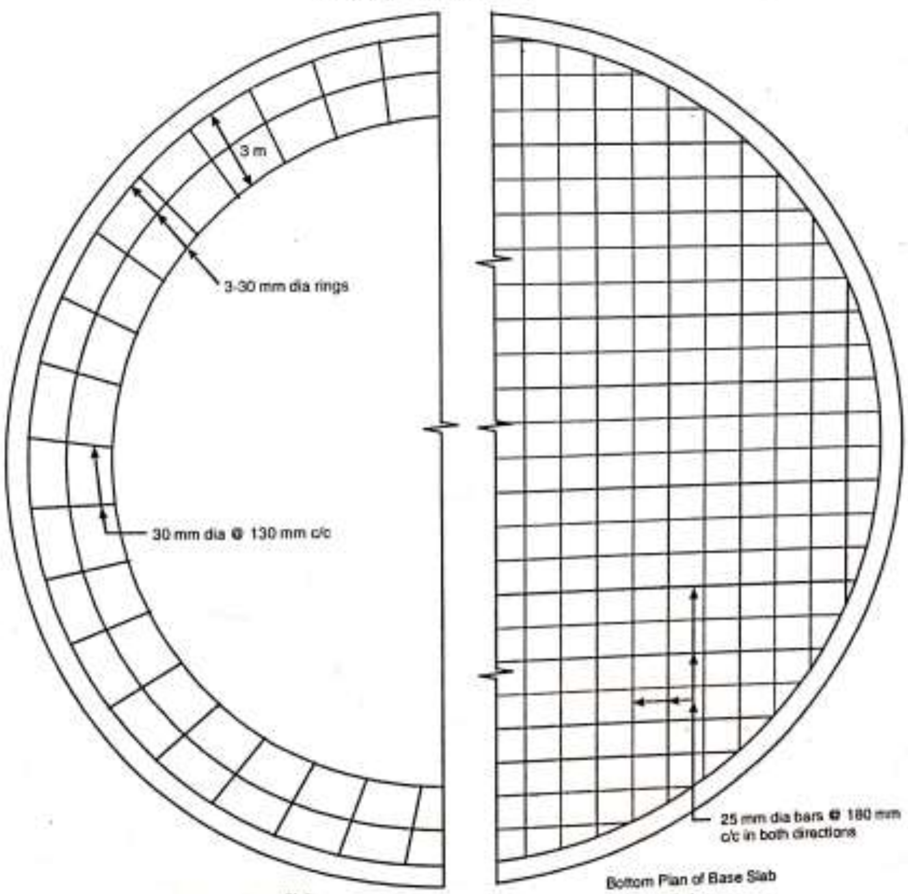
Hence providing 25 mm diameter bars @ 180 mm c/c in both the directions as a mesh at the ottom face of the slab as shown in Fig. 22.37(*c*).

■ **Check for shear**

$$V = 176.5 \text{ kN}$$



(*a*) Sectional Elevation



Top Plan of Base Slab
(*b*)

Bottom Plan of Base Slab
(*c*)

Possible Short Type Questions

1. **Define water tank, broadly categorized into how many types?**
   Tanks are widely used for storing liquids like water, chemicals, petroleum etc.
   They are broadly categorized into three types
   - Tanks resting on ground
   - Underground tanks
   - Elevated or overhead tanks.
2. **Which code is generally used for the design of liquid retaining structures?**
   Indian standard code of practice for design of liquid retaining concrete structures IS 3370:2009.
3. **How many types of joints in water tanks?**
   - Movement joints
   - Contraction joints
   - Expansion joints
   - Sliding joints

Possible Long Type Questions

1. Design a circular tank with a flexible base for a tank of 1,00,000 litre capacity. The depth of water in the tank is 5 m. Use M_{25} concrete and Fe_{415} steel. Take unit weight of water as 9.8 KN/m^3.
2. Design a circular water tank with flexible base for a capacity 0f 450 kl. The depth of water is 4.5 m. Allow suitable free board.