BALASORE COLLEGE OF ENGINEERING AND TECHNOLOGY, SERGARH, BALASORE

Lecture Notes

On

STRUCTURAL ANALYSIS I

2nd Year

4th Semester

Prepared by -:

Ms. Punyamayee Pattanaik, Asst. Professor

Civil Engineering Department

Checked by

MODULE WISE DISTRIBUTION OF LOADS

Module	Chapter with title	Assigned Hour (as per BPUT)	Actual Session Needed	Range of Marks of Questions to be being asked (BPUT)
I	Introduction, SI, KI, methods of SA, 3 moment theorem, moment area method, Conjugate beam method, consistent deformation method	8	20	35-45
II	Strain energy, castigliano's theorem, unit load method, maxwell's theorem, maxwell betti's theorem	7	11	15-20
III	Truss analysis, method of joints, method of sections, willot mohr diagram	7	11	20-30
IV	Rolling loads, ILD, ILD for reactions, SF, BM, point loads, UDL	7	13	15-20
V	Three hinged arches, Suspension cable, ILDs, normal thrust, radial shear	7	11	15-25
TOTAL		36 hours	66 hours	100 marks

SYLLABUS

Module 1: (8 hours)

Concept of determinate and indeterminate structures, determination of degree of static and kinematic indeterminacy in plane frame and continuous structures. Methods of Analysis: Equilibrium equations, compatibility requirements, Introduction to force and displacement methods. Analysis of propped cantilever by consistent deformation method, Analysis of fixed and continuous beams by Moment-Area method, Conjugate beam method and theorem of three moments.

Module 2: (7 hours)

Energy theorems and its application, Strain energy method, Virtual work method, unit load method, Betti's and Maxwell's laws, Castigliano's theorem, concept of minimum potential energy.

Module 3: (7 hours)

Analysis of redundant plane trusses. Deflection of pin jointed plane trusses. Analytical method and Williot –Mohr diagram. Introduction to space truss.

Module 4: (7 hours)

Rolling loads and influence lines for determinate structures, simply supported beams, cantilever, ILD for reaction, shear force and bending moment at a section, ILD for wheel loads, point loads and udl, maximum bending moment envelope.

Module 5: (7 hours)

Analysis of three hinged arches, Suspension cable with three hinged stiffening girders subjected to dead and live loads, ILD for Bending Moment, Shear Force, normal thrust and radial shear for three hinged arches.

Books

- Theory and Problems in Structural Analysis by L Negi, Mc Graw Hill
- Structural Analysis by T.S. Thandamoorthy, Oxford University Press
- Basic Structural Analysis by C S Reddy, McGraw Hill
- Elementary Structural Analysis by Norris and Wilber, McGraw Hill
- Structural Analysis by Aslam Kassimali, Cengage Learing
- Structural Analysis by R.C. Hibbeler, Pearson Education

Digital learning courses

Course name: Structural Analysis 1

Couse link: https://archive.nptel.ac.in/courses/105/105/105105166/

Course Instructor: Prof Amit Shaw

MODULE 1

CHAPTER 1

Session 1

<u>INTRODUCTION</u>

Learning objective:

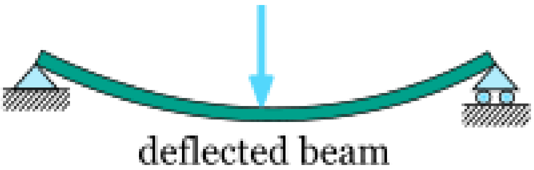
Students will be able to understand

- *What is structure.*
- *Various type of structures.*
- *What is equilibrium and compatibility conditions.*

1.1 Introduction

<u>Structure:</u>

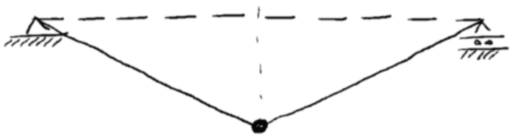
A system subjected to elastic deformation on applying loads, the deflected profile is non-linear because of internal resistance. If the load is removed and if it can comeback to its original position, it is a "structure".



[Fig. 1.1]

<u>Mechanism:</u>

Unstable structures are called mechanisms. On removal of load, deformation doesn't disappear i.e., system can't comeback to it's original state. Deflected profiles are linear as shown in Fig. 1.2



Mechanism

[Fig. 1.2]

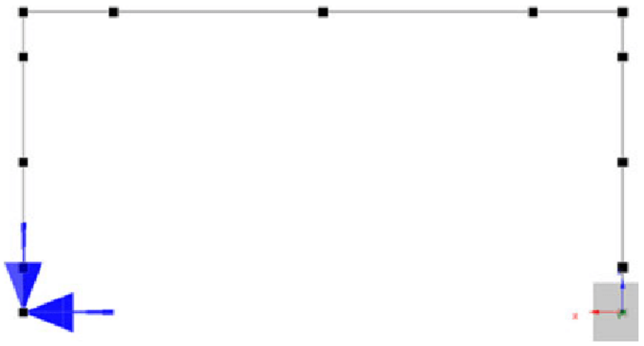
1.2 Classification of Structures:

1.2.1 Based on predominant dimensions of members

a. Skeletal structures

One dimension predominant. (Linear or non-linear)

E.g.:



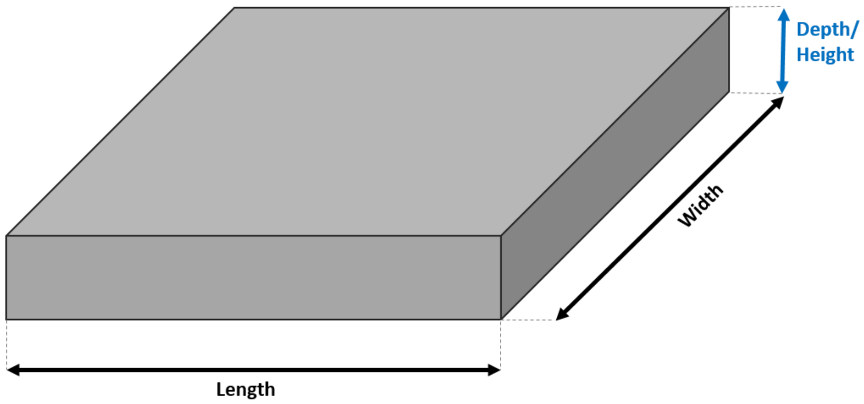
[Fig.1.3: Building Frame]

[Fig.1.4: Truss]

b. Surface Structure

Idealized to plane (or) curved surface.

E.g.:



[Fig.1.5: Slab]

c. Solid structures

Structures neither be idealized to a skeleton nor to a plane.

E.g.: massive foundation

1.2.2. Based on the dimensions of frames

a Plane frame

All the members assumed to be in one plane. If loaded, member is subjected to one axial force, one shear force, one bending moment.

E.g.: Pin jointed Plane frame, Rigid jointed plane frame.

b Space frames

All the members don't lie in one plane.

E.g.: Pin jointed space frame, Rigid jointed space frame.

1.2.3. Based on type of Joints

a. Pin jointed Structure

Structures are subjected to axial forces. Shear force and Bending moment are neglected.

b. Rigid jointed structure

The joints are assumed to be rigid, so that the angles between the members remain unchanged. These frames are subjected to bending moments, shear force, axial forces and twisting moments.

1.3 Assumptions:

- Materials are homogeneous and isotropic. Homogeneous material refers to the identical properties that exist in one direction throughout the material. And Isotropic material refers to that type of material which is identical in all directions.
- Stress-strain behavior/relations are linear, i.e., within proportionality limit. $(stress \propto strain)$
- Law of superposition holds good.
- Deflections & slopes are assumed to be small.

MODULE 1

CHAPTER 1

Session 2

Learning objective:

- *Understand the equilibrium condition.*
- *Understand the compatibility condition.*

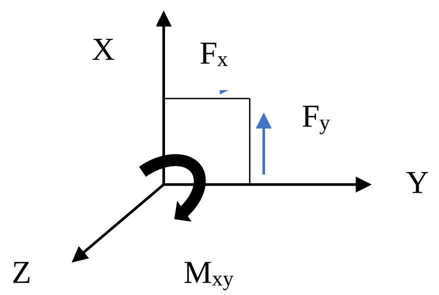
1.4 Equilibrium Conditions:

Deals with balancing of forces, so that the structure considered to be in equilibrium, if initially at rest remains rest and when subjected to system of forces will not undergo rigid body motion.

a. For plane frame –

$$\sum F_X = 0, \sum F_Y = 0, \sum M_{XY} = 0$$

- Minimum 3 equilibrium equations for plane frame (whether pin jointed or rigid jointed)
- The plane frame shall be safe against overturning, hence even for in jointed trusses $\sum M_{XY} = 0$



➢ Pin joint of plane frame-

$$\sum F_X = 0, \sum F_Y = 0$$

Two equilibrium equations.

➢ Rigid joint of a plane frame-

$$\sum F_X = 0, \sum F_Y = 0, \sum M_{XY} = 0$$

b.For space frames-

$$\sum F_X = 0, \sum F_Y = 0, \sum F_Z = 0$$

$$\sum M_{YZ} = 0, \sum M_{xz} = 0, \sum M_{XY} = 0$$

- Pin joint-

$$\sum F_X = 0, \sum F_Y = 0, \sum F_Z = 0$$

- Rigid joint-

$$\sum F_X = 0, \sum F_Y = 0, \sum F_Z = 0$$

$$\sum M_{YZ} = 0, \sum M_{xz} = 0, \sum M_{XY} = 0$$

1.5 Compatibility Condition:

Deals with balancing of slopes & deflections (or) member displacements.

1.5.1 For 2-D systems (Plane frame)

$$\theta = 0, \delta V = 0, \delta H = 0$$

1.5.2 For 3-D systems (space frame)

No. of compatibility equation at a support

= no. of reaction components

Probable Questions:

i.	What is difference between structure and mechanism?	(2 marks)
ii.	What are the types of structure?	(6 marks)
iii.	What are the assumptions made?	(2 marks)
iv.	What is the Equilibrium condition?	(2 marks)
v.	What is the compatibility condition?	(2 marks)

MODULE 1

CHAPTER 2

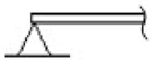
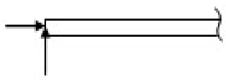
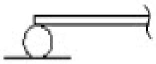
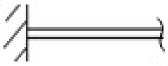
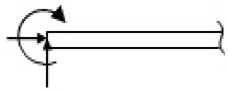
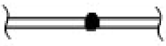
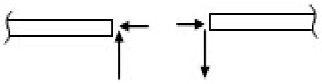
Session 3

Learning objectives

Students will be able to understand

- *Types of supports with reaction forces.*
- *Types of end conditions with compatibility condition.*
- *Various stability condition.*

2.1 Types of Supports & its reactions

Sl no.	Type	Sketch	Constraints	Reactions	Displacements	no. of reactions
1	Pinned		Horizontal and vertical translation.		0 0	2
2	Roller		Vertical translation		0	1
3	Fixed		Horizontal and vertical translation. Rotation		0 0 0	3
4	Internal Hinge		Relative displacements of member ends		0 0 0	4

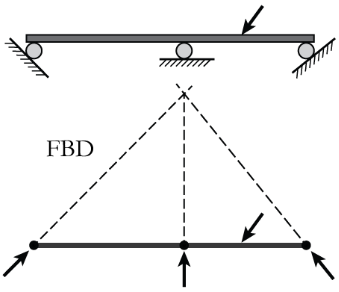
2.2 Types of Supports and its compatibility condition:

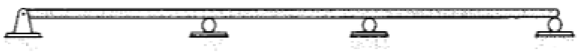
2.3 Stability Of Structures:

2.3.1 External Stability

- Minimum reaction components shall be developed for external stability. To ensure equilibrium, the members and the whole structure should be stable.

- Partial constraint:
  $reactions(r) < no. of\ equlibrium\ equations(3)$
- Improper constraint: When the reactions are concurrent or parallel



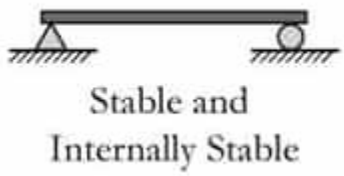


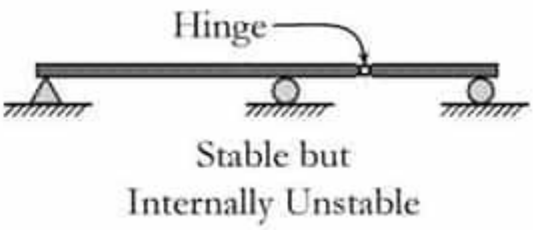
[Fig. 2.2: Parallel reactions]

[Fig. 2.1: Concurrent reactions]

2.3.2 Internal Stability

In internally stable structure is one that would maintain its shape if all the reaction supports were removed. A structure that is internally unstable may still be stable if it has sufficient external support reactions.





[Fig. 2.3] [Fig. 2.4]

2.4 External Determinacy:

The ability to calculate all of the external reaction component forces using only static equilibrium. A structure that satisfies this requirement is externally statically determinate. A structure for which the external reactions component forces cannot be calculated using only equilibrium is externally statically indeterminate.

2.5 Internal Determinacy:

The ability to calculate all of the external reaction component forces *and* internal forces using only static equilibrium. A structure that satisfies this requirement is internally statically determinate. A structure for which the internal forces cannot be calculated using only equilibrium is internally statically indeterminate. Typically, if one talks about 'determinacy', it is an internal determinacy that is meant.

2.6 Redundant:

Indeterminate structures effectively have more unknowns than can be solved using the three equilibrium equations (or six equilibrium equations in 3D). The extra unknowns are called redundant.

2.7 Degree of Indeterminacy:

The degree of indeterminacy is equal to the number of redundant. An indeterminate structure with 2 redundant may be said to be statically indeterminate to the second degree.

Probable Questions:

1. Write down types of end conditions and its reactions and compatibility condition. (6 marks)

MODULE 1 CHAPTER 3

Session 4

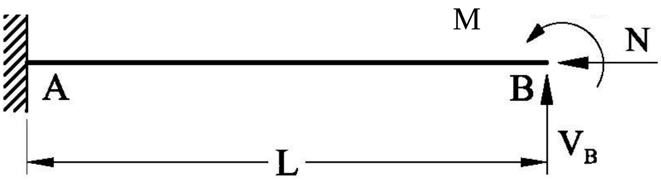
Learning objective

Students will be able to

- *Understand statically determinate and indeterminate.*
- *Formulate the statically indeterminacy.*

3.1 Statically Determinate:

A statically determinate structure is one that is stable and all unknown reactive forces can be determined from the equations of equilibrium alone.



[Fig. 3.1: Determinate beams]

3.2 Statically Indeterminate:

If a structure cannot be analyzed for external and internal reactions using static equilibrium conditions alone then such a structure is called indeterminate structure.

$$D_S = D_{Se} + D_{Si}$$

Where,

D_S = Degree of static-indeterminacy

D_{Se} = External static-indeterminacy

D_{Si} = Internal static-indeterminacy

3.2.1 External static indeterminacy:

It is related with the support system of the structure and it is equal to number of external reaction components in addition to number of static equilibrium equations.

$D_{Se} = r_e - 3$ For 2D

$D_{Se} = r_e - 6$ For 3D Where, r_e = total external reactions

3.2 Internal static indeterminacy:

It refers to the geometric stability of the structure. If after knowing the external reactions it is not possible to determine all internal forces/internal reactions using static equilibrium equations alone then the structure is said to be internally indeterminate.

For geometric stability sufficient number of members are required to preserve the shape of rigid body without excessive deformation.

$D_{Si} = 3C - r_r$ …… For 2D

$D_{Si} = 6C - r_r$ …… For 3D

where, C = number of closed loops.

and

r_r = released reaction

- $r_r = \sum (m_j - 1)$ …… For 2D

$r_r = 3\sum (m_j - 1)$ ……. For 3D

where m_j = number of members connecting with J number of joints.

and J = number of hybrid joint

Hence static indeterminacy,

$D_s = m + r_e - 2j$, For 2D truss

$D_{Se} = r_e - 3$ & $D_{Si} = m - (2j - 3)$

$D_S = m + re - 3j$, For 3D truss

$D_{se} = re - 6$ & $D_{si} = m - (3j - 6)$

$D_S = 3m + r_e - 3j - r_r$ …... 2D Rigid frame

$D_s = 6m + r_e - 6j - r_r$ …... 3D rigid frame

$D_S = (r_e - 6) + (6C - r_r)$ …... 3D rigid frame

MODULE 1

CHAPTER 3

Session 5

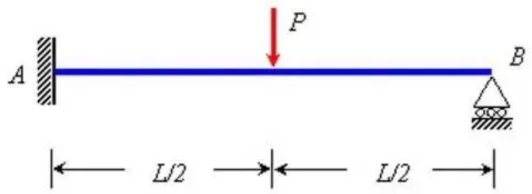
Learning objectives:

Students will be able to

Solve various numerical on static indeterminacy

3.3 Solved Examples: Find out static indeterminacy of the following figures.

1.



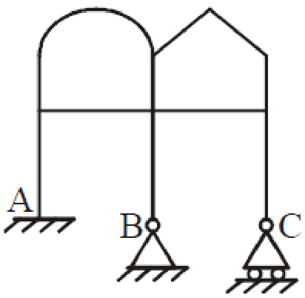
Ans: -

$$Total\ reaction\ components, r = 4$$

$$Total\ reaction\ required = 3$$

Degree of static indeterminacy = 4 – 3 = 1

2.



$$\text{Reaction at A} = 3,$$

$$\text{Reaction at B} = 2,$$

$$\text{Reaction at C} = 1$$

$$\text{Total reaction} = 6,$$

$$\text{No. of equilibrium equation} = 3,$$

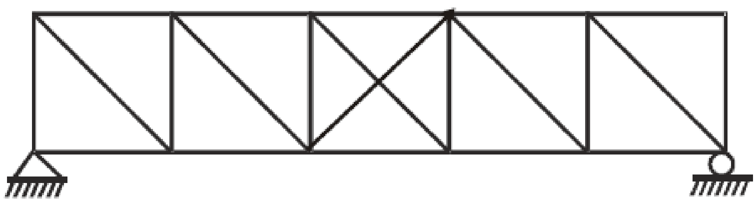
$$\text{Dse} = \text{r} - 3 = 6 - 3 = 3$$

D_{Si} = 3C for rigid jointed plane frames, Where
C = no. of closed boxes

$D_{si} = 3 \times 2 = 6$

$D_s = D_{se} + D_{si} = 3 + 6 = 9$

3.



$D_s = m + r_e - 2j$,

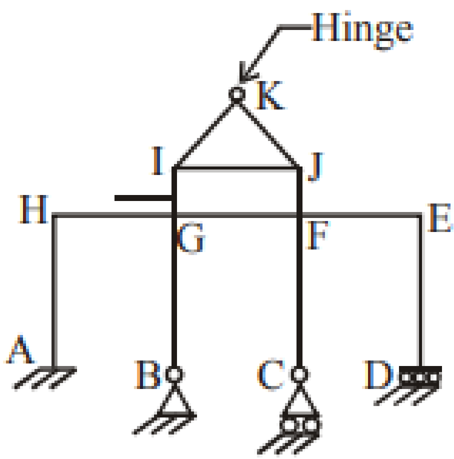
Where m = no. of members = 24

And r = total reactions = 3

J= no. of joints = 13

$D_s = 24+3-(2 \text{ X } 13) = 1$

4.



Reactions at A = 3, Reactions at B = 2, Reaction at C = 1, Reactions at D = 2

Total reactions (r) = 8

Dse = r – equilibrium equations
= r – 3 = 5

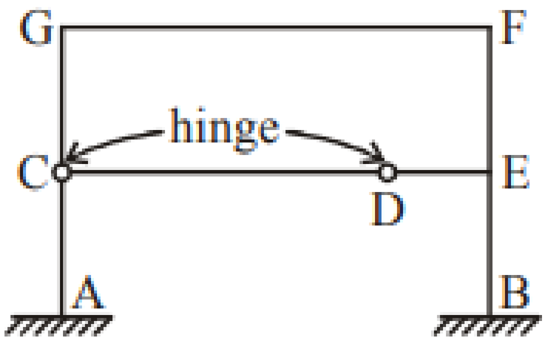
Dsi = 3C = 3 × 2 = 6

At ‘k’ a moment hinge exists.
Force release at a joint moment hinge = no. of members connected to hinge – 1 = 2 – 1 = 1

Ds = Dse + Dsi – no. of force release

= 5 + 6 – 1 = 10

5.



D_{se} = 6 – 3 = 3

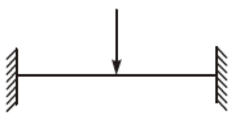
D_{si} = 3C = 3 × 1 = 3

Force Releases @ C = 3 – 1 = 2

Force Releases @ D = 2 – 1 = 1

$D_s = D_{se} + D_{si}$ – release = 3 + 3 – (2 + 1) = 3

6. A beam fixed at the ends and subjected to lateral loads only



Total number of reactions = 2 + 2 = 4

Equilibrium equation with lateral load only = 2

D_{Se} = External indeterminacy = R_e – equilibrium equation = 4 – 2 = 2

D_{Si} = Internal indeterminacy = 0

Total static indeterminacy
$D_S = D_{Se} + D_{Si}$ = 2 + 0 = 2

MODULE 1

CHAPTER 3

Session 6

Learning objective

Students will be able to

- *Analyse the effect of force release.*

3.4 Effect of Force Release

3.4.1 Moment hinge

Moment release at internal hinge = no. of members at joint – 1 = (m'-1)

…….for rigid jointed plane frame

= 3(m'-1) ……………… for space frame

3.4.2 Horizontal Shear Release

No. of force release = 1

3.4.3 Vertical shear Release

No. of force release = 1

3.4.4 Link

No. of force release = 2 (one release is along the force and other is moment)

MODULE 1

CHAPTER 3

Session 7

Learning objective

Students will be able to

- *Differentiate between static determinacy and indeterminacy*

3.5 Differentiation between Statically determinate and indeterminate

STATICALLY DETERMINATE STRUCTURES	**STATICALLY INDETERMINATE STRUCTURES**
1. Equilibrium equations are sufficient	**1**. Equilibrium equations + Compatibility equations are required
2. No thermal stress. In determinate structures temperature change will not cause deformation.	**2**. Temperature stress will develop. Due to temperature change, resistance against deformation occurs, hence thermal stress develop.
3. No effect of material & sectional properties on forces.	**3.** Forces are affected by material and sectional properties.
4. No effect (forces) of sinking of support.	**4.** There is effects of sinking of supports (by δ)
5. No stresses due to lack of fit. (If the length of member is either less or more than the actual length slightly, it is called lack of fit)	**5.** In case of statically indeterminate structure, if the length of a member is shorter, it shall be pulled to place it in the present position. Hence it will be subjected to axial tension. If member is longer, than shall be subjected to axial compression.
6. The design moment or forces of statically determinate are more.	**6.** Design moment (or) forces are less. Hence indeterminate structures are economical.

MODULE 1

CHAPTER 4

Session 8

Learning objective

Students will be able to

- *What is kinematic indeterminacy and its condition.*

4.1 Kinematic Indeterminacy

If the number of unknown displacement components are greater than the number of compatibility equations, for these structures additional equations based on equilibrium must be written in order to obtain sufficient number of equations for the determination of all the unknown displacement components. The number of these additional equations necessary is known as degree of kinematic indeterminacy or degree of freedom of the structure.

Note: A fixed beam is kinematically determinate and a simply supported beam is kinematically indeterminate.

4.2 Degree of freedom with various support condition:

a. Each joint of plane pin jointed frame has 2 degrees of freedom.

b. Each joint of space pin jointed frame has 3 degrees of freedom.

c. Each joint of plane rigid jointed frame has 3 degrees of freedom.

d. Each joint of space rigid jointed frame has 6 degrees of freedom.

Hence kinematic indeterminacy is given by,

- $D_k = 3j - r_e$ ………. For 2D Rigid frame when all members are axially extensible.
- $D_k = 3j - r_e - m$ ………. For 2D Rigid frame if 'm' members are axially rigid / inextensible.
- $D_k = 3(j + j') - r_e - m + r_r$ …… For 2D Rigid frame when J' = Number of Hybrid joints is available.
- $D_k = 6(j + j') - r_e - m + r_r$ ….. For 3D Rigid frame
- $D_k = 2(j + j') - r_e - m + r_r$ ….. For 2D Pin jointed truss.
- $D_k = 3(j + j') - r_e - m + r_r$ …… For 3D Pin jointed truss.

i_e is degree of Indeterminacy

e_c is the number of equations of condition,

Internal Support Type	Equations of Condition
Hinge	$e_c = n - 1$
Roller	$e_c = 2 * (n - 1)$

MODULE 1

CHAPTER 4

Session 9

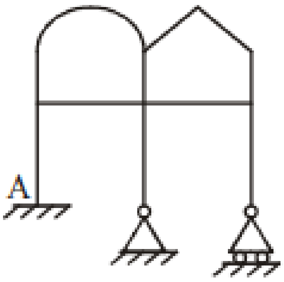
Learning objectives

Students will be able to

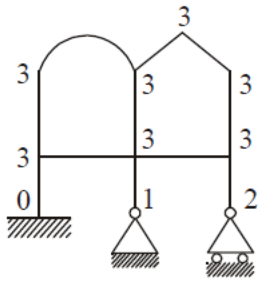
Solve various numerical on kinematic indeterminacy.

4.2 Find out kinematic indeterminacy:

1.



Degrees of freedom of various supports (or) joints are shown in figure



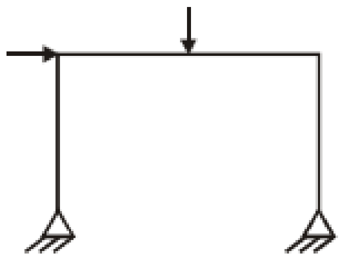
$D_k = 0 + 3 \times 7 + (1 + 2)$

= 24 (with axial deformation)

= 24 – 11 = 13

(Neglecting axial deformation)

2.



Degree of freedom (Dk)

= No. of unknown joint displacements

At pinned support DOF = 1 (rotation)

At rigid joint of plane frame = 3

Dk = 1 + 3 + 3 + 1 = 8

(Considering axial deformations)

Dk = 8 – no. of members = 8 – 3 = 5

(Neglecting axial deformations)

3. The kinematic indeterminacy of single bay portal frame fixed at the base is

At fixed support DOF = 0

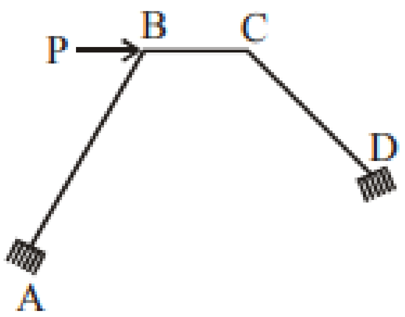
D_k = 0 + 3 + 3 + 0 = 6

(Considering axial deformation)

= 6 – 3 = 3

(Neglecting axial deformation)

4. Rigid frame with clamped ends at A and D shown in the figure



D_k = 0 + 3 + 3 + 0
= 6 (with axial deformation)
= 6 – 3 = 3

(Neglecting axial deformation)

MODULE 1 CHAPTER 5

Session 10

Learning objectives

Students will be able to

- *Understand Methods of structural analysis*
- *What is Force method*
- *What is displacement method*

5.1 Introduction to force and displacement methods of structural analysis:

We have two distinct methods of analysis for statically indeterminate structure:

a. Force method of analysis
b. Displacement method of analysis

In the force method of analysis,primary unknown are forces.In this method compatibility equations are written for displacement and rotations (which are calculated by force displacement equations). Solving these equations, redundant forces are calculated. Once the redundant forces are calculated, the remaining reactions are evaluated by equations of equilibrium.

In the displacement method of analysis,the primary unknowns are the displacements. In this method, first force -displacement relations are computed and subsequently equations are written satisfying the equilibrium conditions of the structure. After determining the unknown displacements, the other forces are calculated satisfying the compatibility conditions and force displacement relations

5.2 Difference between force & displacement methods

FORCE METHODS	DISPLACEMENT METHODS
1. Method of consistent deformation 2. Theorem of least work 3. Column analogy method 4. Flexibility matrix method	1. Slope deflection method 2. Moment distribution method 3. Kani's method 4. Stiffness matrix method
Types of indeterminacy- static indeterminacy	Types of indeterminacy- kinematic indeterminacy
Governing equations-compatibility equations	Governing equations-equilibrium equations
Force displacement relations- flexibility matrix	Force displacement relations- stiffness matrix

MODULE 1

CHAPTER 6

Session 11

Learning objectives

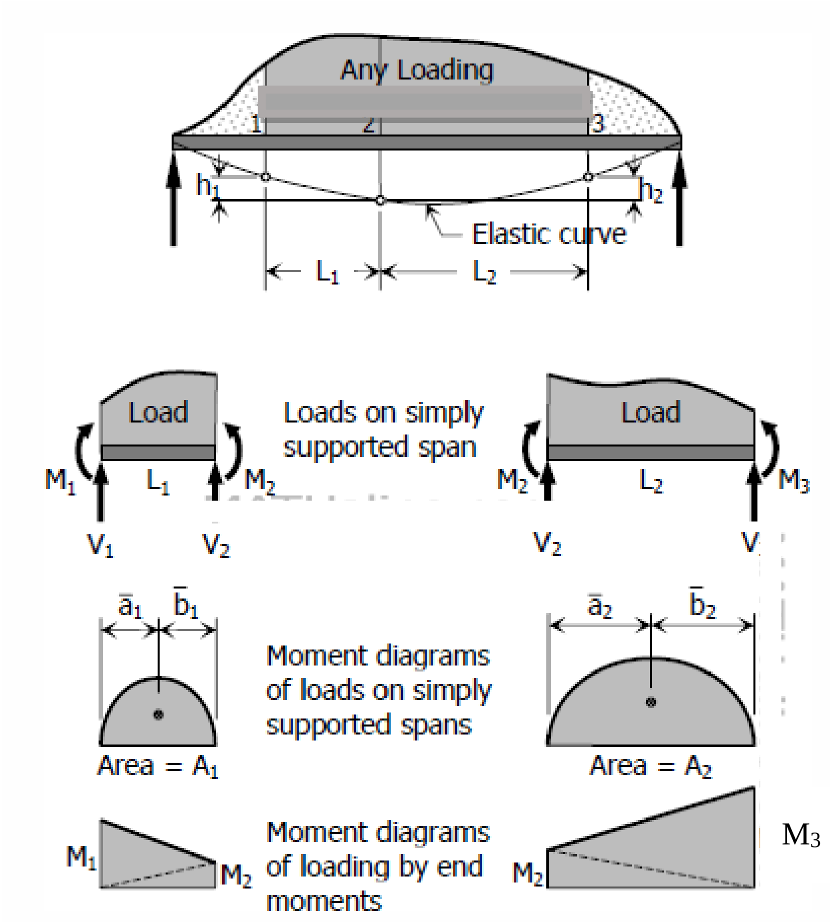
Students will be able to

- *Compute the unknown moments in the indeterminate beams subjected to - external load(s)*
- *Find reactions for indeterminate beams.*

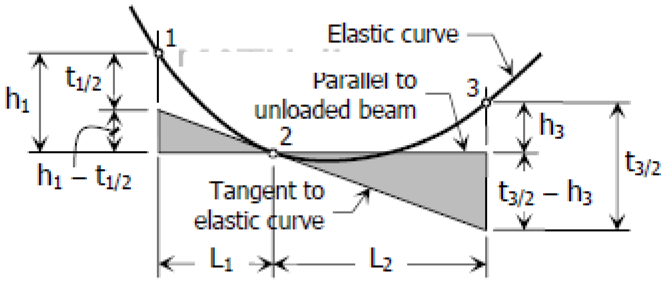
6.1 Derivation of Three moment Equation

The three-moment equation gives us the relation between the moments between any three points in a beam and their relative vertical distances or deviations. This method is widely used in finding the reactions in a continuous beam.

Consider three points on the beam loaded as shown.



[Fig. 5.1]: *internet source*



[Fig. 5.2]

From proportions between similar triangles:

$$\frac{h_1 - t_{\frac{1}{2}}}{L_1} = \frac{t_{\frac{3}{2}} - h_3}{L_2}$$

$$\frac{h_1}{L_1} - \frac{t_{\frac{1}{2}}}{L_1} = \frac{t_{\frac{3}{2}}}{L_2} - \frac{h_3}{L_2}$$

$$\frac{t_{\frac{1}{2}}}{L_1} + \frac{t_{\frac{3}{2}}}{L_2} = \frac{h_1}{L_1} + \frac{h_3}{L_2} \qquad \text{........ eqn(1)}$$

Values of $t_{\frac{1}{2}}$ & $t_{\frac{3}{2}}$

$$t_{\frac{1}{2}} = \frac{1}{E_1 I_1}\,(Area_{1-2})\,.\overline{X_1}$$

$$t_{1/2} = \frac{1}{E_1 I_1}\left[A_1\bar{a}_1 + (\tfrac{1}{2}M_1 L_1)(\tfrac{1}{3}L_1) + (\tfrac{1}{2}M_2 L_1)(\tfrac{2}{3}L_1)\right]$$

$$t_{1/2} = \frac{1}{6E_1 I_1}(6A_1\bar{a}_1 + M_1 {L_1}^2 + 2M_2 {L_1}^2)$$

$$t_{3/2} = \frac{1}{E_2 I_2}(\text{Area}_{2-3}) \cdot \bar{X}_3$$

$$t_{3/2} = \frac{1}{E_2 I_2}\left[A_2\bar{b}_2 + (\tfrac{1}{2}M_2 L_2)(\tfrac{2}{3}L_2) + (\tfrac{1}{2}M_3 L_2)(\tfrac{1}{3}L_2)\right]$$

$$t_{3/2} = \frac{1}{6E_2 I_2}(6A_2\bar{b}_2 + 2M_2 {L_2}^2 + M_3 {L_2}^2)$$

Substitute $t_{1/2}$ and $t_{3/2}$ to equation (1)

$$\frac{1}{6E_1I_1}\left(\frac{6A_1\bar{a}_1}{L_1}+M_1L_1+2M_2L_1\right)+\frac{1}{6E_2I_2}\left(\frac{6A_2\bar{b}_2}{L_2}+2M_2L_2+M_3L_2\right)$$
$$=\frac{h_1}{L_1}+\frac{h_3}{L_2}$$

Multiply both sides by 6

$$\frac{1}{E_1I_1}\left(\frac{6A_1\bar{a}_1}{L_1}+M_1L_1+2M_2L_1\right)+\frac{1}{E_2I_2}\left(\frac{6A_2\bar{b}_2}{L_2}+2M_2L_2+M_3L_2\right)$$
$$=6\left(\frac{h_1}{L_1}+\frac{h_3}{L_2}\right)$$

Distribute 1/EI

$$\frac{6A_1\bar{a}_1}{E_1I_1L_1}+\frac{M_1L_1}{E_1I_1}+\frac{2M_2L_1}{E_1I_1}+\frac{6A_2\bar{b}_2}{E_2I_2L_2}+\frac{2M_2L_2}{E_2I_2}+\frac{M_3L_2}{E_2I_2}=6\left(\frac{h_1}{L_1}+\frac{h_3}{L_2}\right)$$

Combine similar terms and rearrange

$$\frac{M_1L_1}{E_1I_1}+2M_2\left(\frac{L_1}{E_1I_1}+\frac{L_2}{E_2I_2}\right)+\frac{M_3L_2}{E_2I_2}+\frac{6A_1\bar{a}_1}{E_1I_1L_1}+\frac{6A_2\bar{b}_2}{E_2I_2L_2}=6\left(\frac{h_1}{L_1}+\frac{h_3}{L_2}\right)$$

If E is constant this equation becomes,

$$\frac{M_1L_1}{I_1}+2M_2\left(\frac{L_1}{I_1}+\frac{L_2}{I_2}\right)+\frac{M_3L_2}{I_2}+\frac{6A_1\bar{a}_1}{I_1L_1}+\frac{6A_2\bar{b}_2}{I_2L_2}=6E\left(\frac{h_1}{L_1}+\frac{h_3}{L_2}\right)$$

If E and I are constant then,

$$M_1L_1+2M_2(L_1+L_2)+M_3L_2+\frac{6A_1\bar{a}_1}{L_1}+\frac{6A_2\bar{b}_2}{L_2}=6EI\left(\frac{h_1}{L_1}+\frac{h_3}{L_2}\right)$$

For the application of three-moment equation to continuous beam, points 1, 2, and 3 are usually unsettling supports, thus h_1 and h_3 are zero. With E and I constants, the equation will reduce to

$$M_1L_1+2M_2(L_1+L_2)+M_3L_2+\frac{6A_1\bar{a}_1}{L_1}+\frac{6A_2\bar{b}_2}{L_2}=0$$

MODULE 1

CHAPTER 6

Session 12

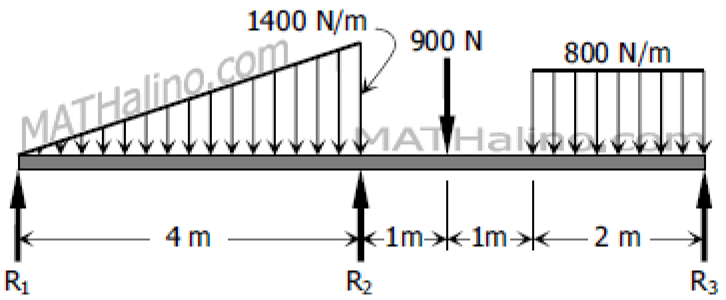
Learning objectives

Students will be able to

Solve numerical in continuous beam on three moment theorems

6.2 Continuous beam by three moment equation

1. Determine the moment over the support R_2 of the beam shown in Fig.



$$M_1 L_1 + 2M_2(L_1 + L_2) + M_3 L_2 + \frac{6A_1\bar{a}_1}{L_1} + \frac{6A_2\bar{b}_2}{L_2} = 0$$

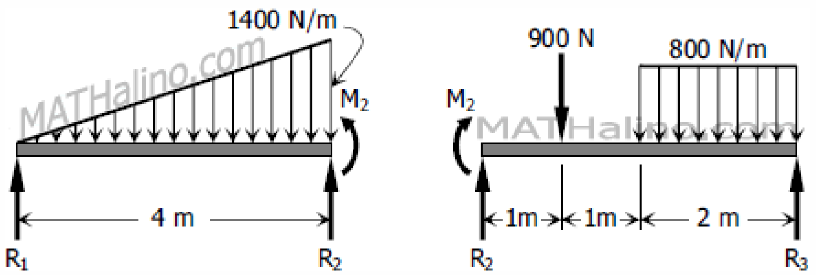
Where

$M_1 = M_3 = 0$

$L_1 = L_2 = 4\ \text{m}$

$$\frac{6A_1\bar{a}_1}{L_1} = \frac{8w_oL^3}{60} = \frac{8(1400)(4^3)}{60}$$

$$\frac{6A_1\bar{a}_1}{L_1} = 11\,946.67\ \text{N}\cdot\text{m}^2$$



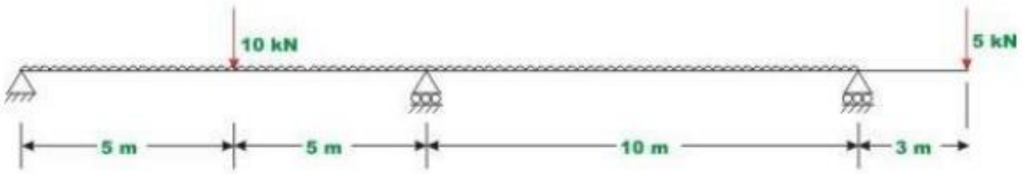
$$\frac{6A_2\bar{b}_2}{L_2} = \frac{Pb}{L}(L^2 - b^2) + \frac{w_o d^2}{4L}(2L^2 - d^2)$$

$$\frac{6A_2\bar{b}_2}{L_2} = \frac{900(3)}{4}(4^2 - 3^2) + \frac{800(2^2)}{4(4)}[\,2(4^2) - 2^2\,]$$

$$\frac{6A_2\bar{b}_2}{L_2} = 10\,325\ \text{N}\cdot\text{m}^2$$

Assignment

A continuous beam ABCD is carrying a uniformly distributed load of 1 kN/m over span ABC in addition to concentrated loads as shown in Fig. Calculate support reactions. Also, draw bending moment and shear force diagram. Assume EI to be constant for all members.



MODULE 1 **CHAPTER 7**

Session 13

Learning objectives

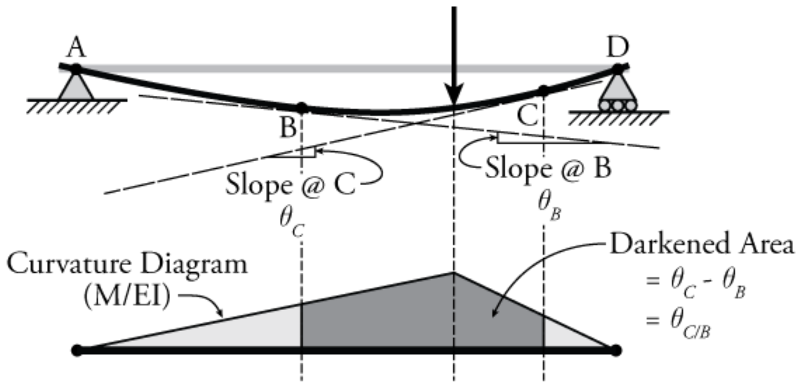
Students will be able to

- *Analyse moment area method.*

7.2 Moment Area theorem:

The moment area theorems provide a way to find slopes and deflections.

- The first moment area theorem is that the change in the slope of a beam between two points is equal to the area under the curvature diagram between those two points.

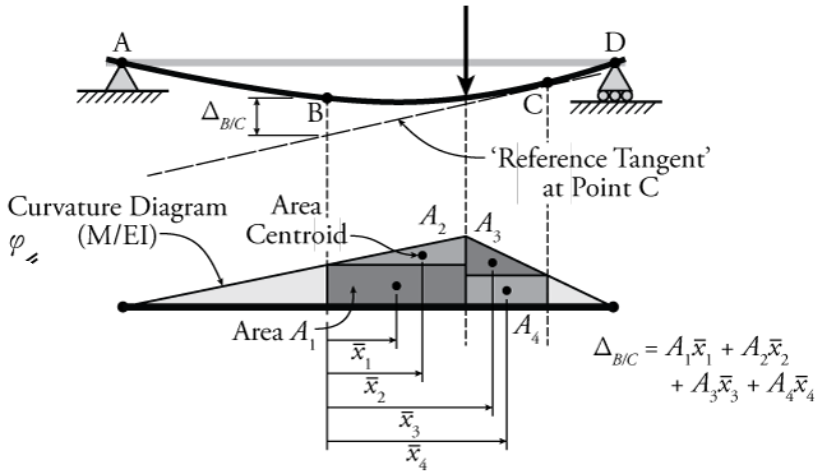


[Fig.7.1: For slope analysis]

$$\boxed{\theta_{CB} = \int_B^C \frac{M(x)}{E(x)I(x)}\,dx}$$

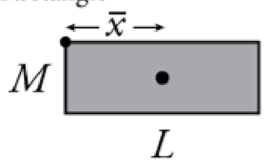
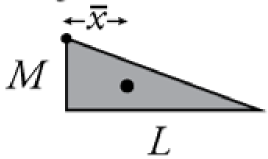
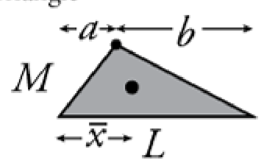
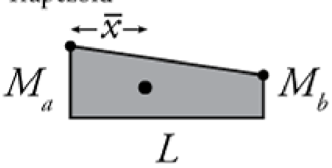
- The second moment area theorem is that the vertical distance between a reference tangent line that is tangent to the slope at one point on the beam and the deflected shape of the beam at another point, is equal to the moment of the area under the curvature diagram between the two points with the moments of the areas calculated relative to the point on the deflected shape.

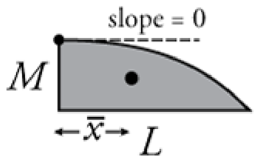
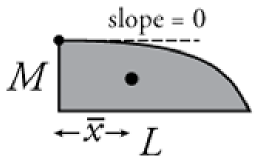
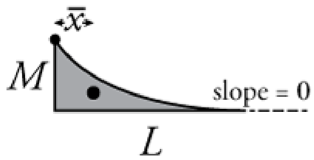
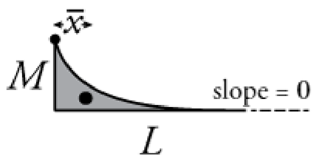
$$\boxed{\Delta_{BC} = \int_B^C \frac{M(x)}{E(x)I(x)}\,\bar{x}\,dx}$$



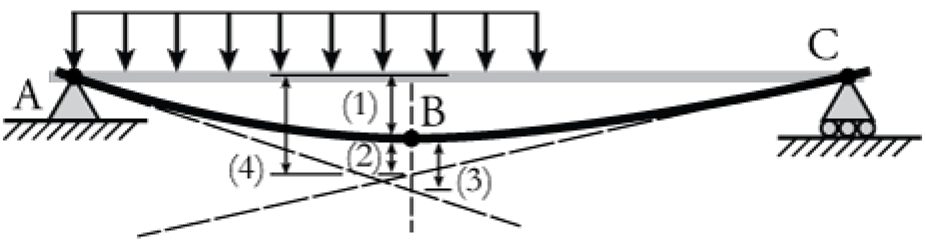
[Fig.7.2: Deflection Analysis]

7.2 Common equations for calculating the area and centroid of different shapes may be seen in

	Area A	Centroid $\bar{x}$
Rectangle 	LM	$\frac{L}{2}$
Triangle 	$\frac{LM}{2}$	$\frac{L}{3}$
Triangle 	$\frac{LM}{2}$	$\frac{1}{3}(L + a)$
Trapezoid 	$\frac{L}{2}(M_a + M_b)$	$\frac{L}{3}\left(\frac{M_a + 2M_b}{M_a + M_b}\right)$

Parabola 	$\frac{2LM}{3}$	$\frac{3L}{8}$
Cubic 	$\frac{3LM}{4}$	$\frac{2L}{5}$
Parabola 	$\frac{LM}{3}$	$\frac{L}{4}$
Cubic 	$\frac{LM}{4}$	$\frac{L}{5}$

Probable question



Which dimension in the figure shown represents $\Delta_{B/A}$?

What is the value of $\Delta_{B/A}$?

MODULE 1

CHAPTER 8

Session 14

Learning objectives

Students will be able to

- *What is conjugate beam method.*
- *Conversion of real supports to conjugate beam supports*
- *Draw conjugate beam*

8.1 Conjugate beam:

The conjugate beam method is the method used to determine the slope and deflection of the beam in which the imaginary conjugate beam is constructed from the real beam and the shear forces and bending moments of the conjugate beam are equal to the slope and deflection of the real beam.

Real beam: the beam with the actual loads and supports is known as a real beam

Conjugate Beam: It is an imaginary beam that has the same length as a real beam, but in this case, the loading is equal to the ratio of bending moment (M) of the real beam to the flexural rigidity (EI).

8.2 Conversion of real beam support to conjugate beam support

Sr. No.	Real Beam	Conjugate beam
1	Fixed support	Free end
2	Free end	Fixed support
3	Internal hinge/ Internal roller	Interior roller support/ Interior pin support
4	Interior roller support/ Interior pin support	Internal hinge/ Internal roller
5	Roller support/ Pin support	Roller support/ Pin support

8.3 Relation of Real beam to Conjugate beam

- Loading for conjugate beam: The ratio of bending moment of the real beam to the flexural rigidity is considered as loading in the case of a conjugate beam.

  e.g.: If the bending moment at a certain point of the real beam is 'M' then the load on the conjugate beam on that point is taken as (M / EI).

- Shear force at a certain point in the conjugate beam (V conjugate) is equal to the slope (**θ** real) at that point in a real beam.

- Bending moment at a certain point in conjugate beam (M conjugate) is equal to the deflection (X real) at that point in a real beam.

8.4 Steps used to draw the conjugate beam

Step 1: Draw the bending moment diagram for the real beam.

Step 2: Divide the magnitudes of bending moments by flexural rigidity and draw the

M/EI diagram.

Step 3: Draw the conjugate beam having the same length as a real beam.

Step 4: Plot the loading same as the M/EI diagram in step-2.

Step 5: Apply the supports to the conjugate diagram as describes before.

8.5 Steps used to solve the conjugate beam from the real beam

Step 1: Find the reactions of the conjugate beam using equilibrium conditions.
Step 2: Construct the shear force diagram for the conjugate beam.
Step 3: Construct the bending moment diagram for the conjugate beam.
Step 4: The values of the shear force in the conjugate beam diagram give the slope values in the real beam.
Step 5: The values of the bending moment in the conjugate beam diagram give the deflection values in the real beam.

MODULE 1

CHAPTER 8

Session 15

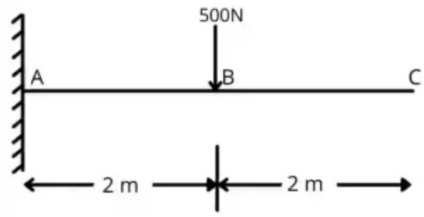
Learning objectives

Students will be able to

- *Solve conjugate beam method problems*
- *Draw BMD and conjugate beams*

8.6 Solve by conjugate beam method:

For the following beam calculate the slope and deflections at all points by the conjugate beam method. The beam has flexural rigidity, EI = 1.5 x 10^6 N.m^2



- Wood E= 12.9 Gpa = 12.9 x 10^9 N/m^2
- I= 1.16 x 10^-4 m^4
- EI = 1.5 x 10^6

By applying equation condition,

ΣFy = 0 [Take upward positive]

R_A−500=0 R_A-500=0

R_A=500N R_A=500N [Upward]

$\sum M_A$=0

$\sum M_A$=0 [Take clockwise positive]

[500 x 2]- M_A = 0

M_A = 1000 N.m

Draw bending moment diagram while calculating bending moments from A to C & consider clockwise moment as positive.

BM_{AJL}= 0 Nm

$BM_{AJR} = - M_A = -1000Nm$

$BM_B = -M_A+(R_A \times 2)$ $BM_B = -M_A+(R_A \times 2)$

$BM_B = -1000+(500 \times 2)$ $BM_B = -1000+(500 \times 2)$

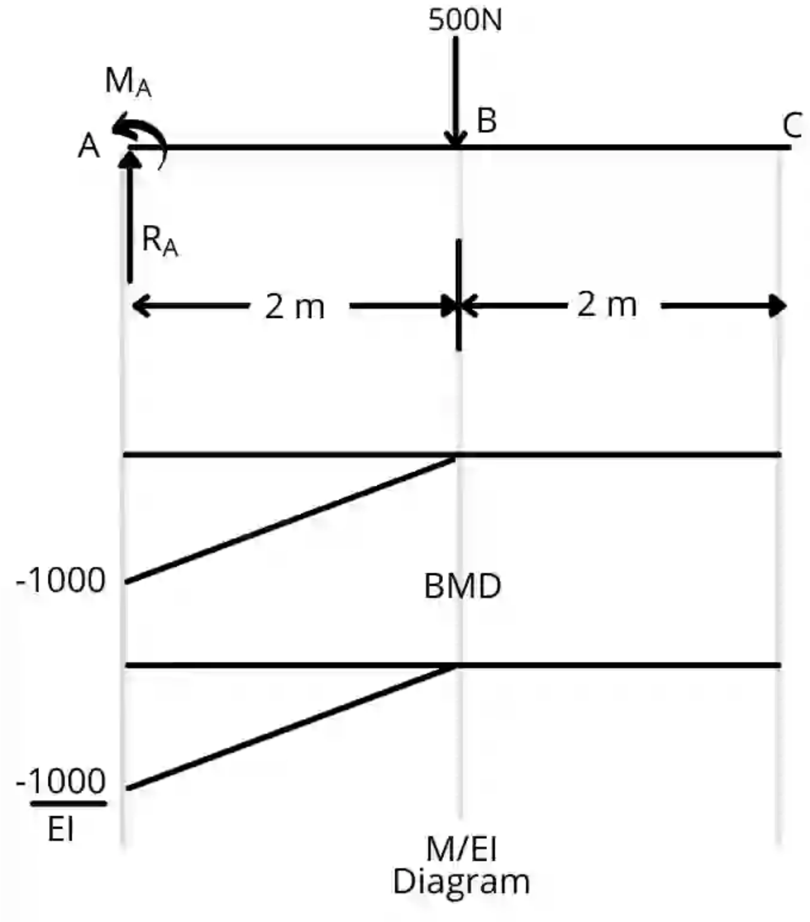
$BM_B = 0$ N.m.

$BM_C = (R_A \times 4) - M_A-(500 \times 2)$ $BM_C = (R_A \times 4)-M_A-(500 \times 2)$

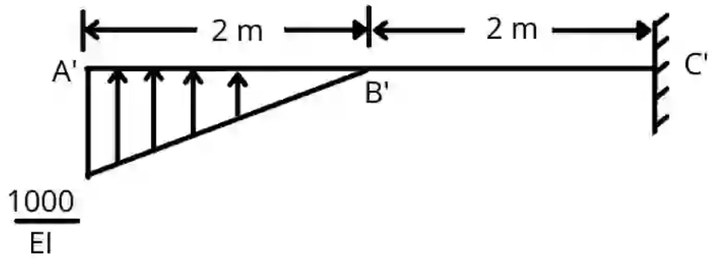
$BM_C = (500 \times 4) -1000-(500 \times 2)$

$BM_C = (500 \times 4)-1000-(500 \times 2)$

$BM_C = 0$ N.m.



Conjugate beam –



Draw the beam A’B’C’ of the same length as real beam.

Apply the conjugate supports as discussed above.

∴ The fixed support at A is replaced by Open end and the Open end at C is replaced by Fixed support.

Apply the load as the M/EI diagram.

Find support reactions of conjugate beam

Now apply equilibrium conditions to fixed the reactions

1] ΣFy = 0 [Consider upward force as positive]

(100/EI) – Cy = 0

Cy=1000/EI

2] $\sum M_C$=0 [Consider clockwise moments positive]

[1000/EI×3.33] – M_C=0 [1000/EI×3.33] – M_C=0

M_C=3330/EI

Calculate the shear forces at all points of conjugate beam calculate from end A to end C & Consider upward force as positive.

SF_A=0
SF_B=1000/EI
SF_B=1000/EI
SF_{CJL}=1000/EI

SF_{CJL}=1000/EI
SF_{CJR}=(1000/EI)−(1000/EI)=0

Calculate bending moment at all points of conjugate beam
BM_A=0 BM_A=0

BM_B=[1000/EI×1.33] BM_B=[1000/EI×1.33]

BM_B=1330/EI BM_B=1330/EI

BM_{CJL}=[1000/EI×3.33] =3330/EI

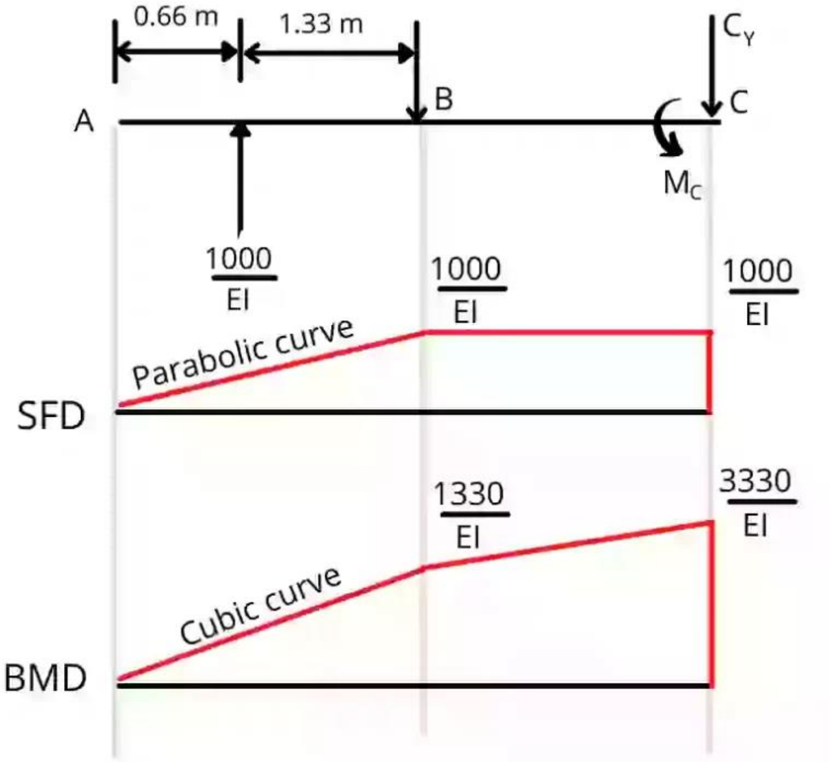
BM_{CJL}=[1000/EI×3.33] =3330/EI

BM_{CJR}=[1000/EI×3.33] −M_C

BM_{CJR}=[1000/EI×3.33]-M_C

BM_{CJR}=0

Draw the SFD and BMD for the conjugate beam



Slope and deflections of different points.

At point A :- Slope θ_A=0 Deflection X_A=0

At point B :- Slope, θ_B=1000/EI Deflection X_B=1330/EI

At point C :- Slope, θ_C=1000/EI Deflection X_C=3330/EI

MODULE 1

CHAPTER 9

Session 16

Learning objectives

Students will be able to

Apply and analyse by consistent deformation method.

9. Consistent deformation method

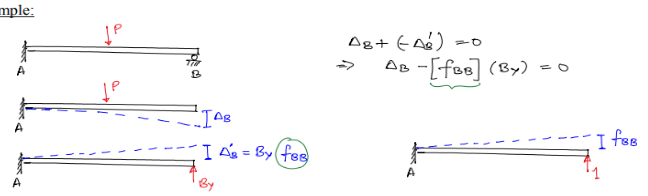
In this method of analysis, excess restraints on the structure are removed to get basic determinate structure. Such structure is also known as released structure. The released structure is analysed for given loading to get displacements in the direction of released restraint. Then redundant forces which are unknown are applied in the direction of restraints removed and the displacements in each direction of restraint removed, are obtained separately for each redundant force. These displacements are in terms of redundant forces. Then considering all these displacements of released structure total displacement due to loading and due to redundant forces in each restraint removed is found. Considering the displacement compatibility of original structure equations are assembled. These conditions result into as many equations as there are number of redundant forces. Solution of these simultaneous equations gives the values of redundant forces. Knowing these values moments and forces at any point in the structure can be found. The method is illustrated below by solving few typical cases.

Example: A propped cantilever of span L is fixed at A and is on roller at B. Analyse it when it is subjected to a concentrated load P at midspan. Assume uniform cross-section throughout.

Answer:

Total number of reactions = 3 + 1 = 4 Number of equilibrium equations available = 3. \ Degree of static indeterminacy = 4 – 3 = 1. By releasing support B restraint to vertical deflection is removed and we get a cantilever as basic determinate structure. This released structure is analysed for the given load and the redundant force R_B to get vertical displacements at B.

Example:



MODULE 2 **CHAPTER 10**

<u>STRAIN ENERGY</u> SESSION 17

Learning objective:

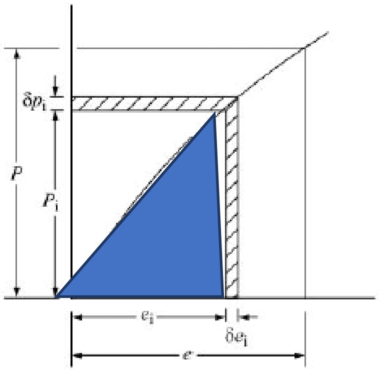
Students will be able to understand

- *What is strain energy and its significances.*
- *General formulations for strain energy.*

10.1 INTRODUCTION

Energy methods are extensively used for the determination of force or any internal stress resultant (for example, bending moment etc.) and displacements (linear and angular, both) of structures. It is particularly useful in the analysis of indeterminate structures. The energy theorems are applicable in elementary analysis as well as in advanced analysis and also in finite element methods. They are very convenient and general in their applications.

$$\text{Strain energy stored by a member (U)}$$
$$= \text{ Amount of the work done by the external forces to produce the deformation}$$



[Fig. 10.1 Stress strain relation]

Strain Energy is the energy stored up to proportionality limit (the blue shaded area in the Fig.10.1)

Area under stress strain curve (per unit volume) is the Strain Energy (U).

Area above the stress strain curve is called Complementary Energy (U^*).

For a linear elastic system, U= U^*

Significance:

- This method is suitable for calculating slopes and deflections.
- Analysis of truss and displacements of truss joints are worked out using energy principles.

10.2 STRAIN ENERGY

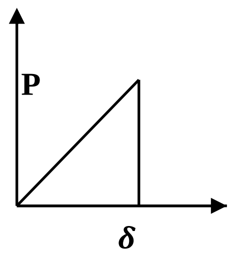
Loads

Gradually Applied loads Suddenly applied loads Impact loads

10.2.1 For Gradually applied load

$U = \frac{1}{2} \times P \times \delta$

$U = \frac{1}{2} \times \sigma \times \epsilon \times volume$



[Fig. 10.2]

10.2.2 Strain energy due to axial force

$$U = \frac{1}{2} \times P \times \frac{PL}{AE} = \frac{P^2 L}{2AE}$$

OR

strain energy stored by an elemental member *ds* be *dU*

$$dU = \frac{1}{2} \times P \times \frac{Pds}{AE} = \frac{P^2 ds}{2AE}$$

$$U = \int \frac{P^2 ds}{2AE}$$

10.2.3 Strain Energy due to shear force

Strain energy stored by an elemental member ds is dU subject to the shear force Q

$$dU = \frac{Q^2 ds}{2AG}$$

$$U = \int \frac{Q^2 ds}{2AG}$$

10.2.4 Strain Energy due to bending moment

Strain energy stored by an elemental member ds be dU, subject to the bending moment M.

$$dU = \frac{M^2 ds}{2EI}$$

$$U = \int \frac{M^2 ds}{2EI}$$

General Equation for strain Energy

$$U = \int \frac{P^2 ds}{2AE} + \int \frac{Q^2 ds}{2AG} + \int \frac{M^2 ds}{2EI}$$

Proof Resilience

The maximum strain energy stored at elastic limit is called Proof Resilience.

Modulus of Resilience

Strain Energy per unit volume.

MODULE 2 **CHAPTER 10**

STRAIN ENERGY **SESSION 18**

Learning objectives

Students will be able to understand

- *What is real and what is virtual work?*
- *What are its principles?*

10.3 VIRTUAL WORK

Work is done when the point of application of a force is moved and is given by the product of force x displacement. The word virtual indicates imaginary, so the virtual work is the hypothetical work consisting of real forces with virtual displacements or virtual forces with real displacements. The principle of virtual work was postulated by Aristotle in the 4th century BC. In fa&, all the energy methods can be developed from the principle of virtual work. The principle of virtual work is based on the physical principle of conservation of energy and is applicable to both linear and non-linear elastic systems of determinate and indeterminate structures.

10.3.1 Principle of Virtual Displacements (Rigid Bodies)

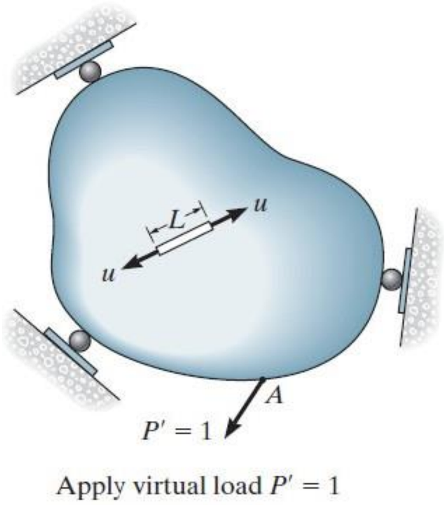
The total work done by a rigid body held in equilibrium by a system of forces and reactions during a small virtual displacement is zero.

This principle is useful in determining forces and influence lines. Unit displacement method is developed based on this concept.

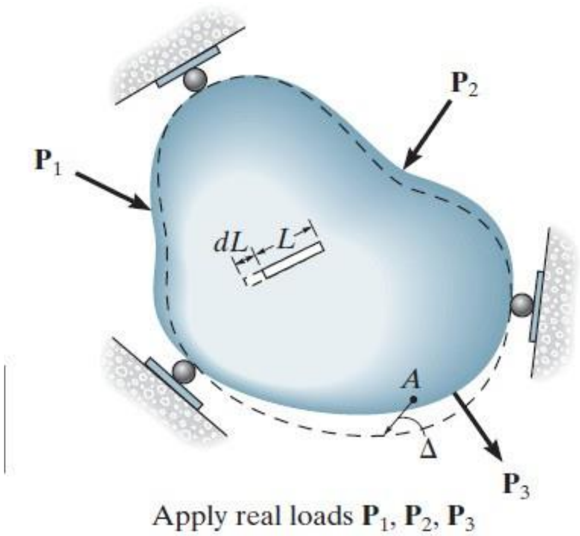
10.3.2 Principle of Virtual Forces

The total work done by a rigid body subjected to a deformation compatible with the support conditions, held in equilibrium, by virtual forces and reactions on the body is equal to zero.

This principle is useful in computing displacements in a structure. Unit load (for trusses) unit moment (for beams) and unit torsion (for shafts) have been developed based on this concept for determination of deformation of various structures.



[Fig. 10.3]



[Fig. 10.4]

In general, then, the principle of work and energy states $\sum P\Delta = \sum u\delta$ i.e., Work of external loads = work of internal loads

MODULE 2 **CHAPTER 11**

SESSION 19

Learning objectives

Students will be able to understand

- *Castigliano's theorem I and theorem II.*

11.1 CASTIGLIANO'S 1ST THEOREM

The partial derivative of the strain energy of a linearly elastic structure (represented in terms of displacements) with respect to any displacement Δ_j, at coordinate j is equal to the force P_j, at coordinate j.

Mathematically,

$$\frac{\partial U}{\partial \Delta_j} = P_j$$

This theorem is also applicable to the system of moments and the resulting angular deformations.

This principle is widely used in analysis of structures.

11.2 CASTIGLIANO'S 2ND THEOREM

The partial derivative of the strain energy of a linearly elastic structure (represented in terms of forces) with respect to any force P_j at coordinate j is equal to the displacement Δ_j, at coordinate j.

$$\frac{\partial U}{\partial P_j} = \Delta_j$$

This theorem is extensively used for determination of displacement in a structure of both the determinate and indeterminate types.

In fact, it is a powerful tool for the analysis of the structure.

11.3 MINIMUM ENERGY THEOREM

In any and every case of statically indeterminate structure, where an indefinite number of different values of the redundant forces and displacements satisfy the condition of statical equilibrium, their actual values are those that render the total strain energy stored to a minimum.

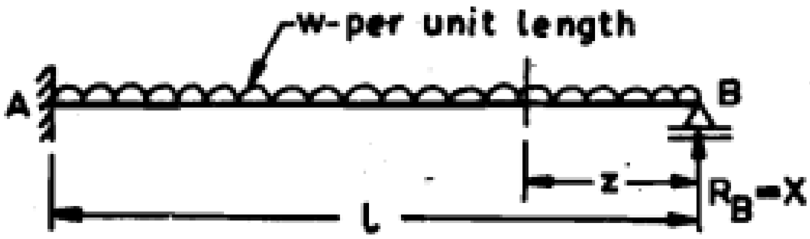
Therefore, $\frac{\partial U}{\partial X} = 0$ and $\frac{\partial^2 U}{\partial X^2}$ is positive

where X = redundant force.

The strain energy stored by a structure subjected to bending and/or axial loading is given by,

$$U = \int \frac{M^2 ds}{2EI} + \int \frac{P^2 ds}{2AE}$$

Example: Find the reaction at the prop of a propped cantilever beam loaded as shown in fig



Solution

Let X be the reaction at the prop (considered as the redundant reaction) i.e R_B

B.M. at any section distant Z from B, M $= Xz - \frac{wz^2}{2}$

.: Strain energy stored by the beam = U = $\int \frac{M^2 dz}{2EI} = \int_0^l \left(Xz - \frac{wz^2}{2}\right)^2 \frac{dz}{2EI}$

By the Minimum energy Principle $\frac{\partial U}{\partial X} = 0$

We get $\int_0^l 2\left(Xz - \frac{wz^2}{2}\right) Z \frac{dz}{2EI} = 0$

$$X = \frac{3}{8} wl$$

i.e. R_B= X, we can find out all the reactions at the support A.

MODULE 2 CHAPTER 12

SESSION 20

Learning objectives

Students will be able to understand

- *Unit load method.*

12.1 UNIT LOAD METHOD

It is the extension of Castigliano's Theorem to calculate displacements in various structures.

In case of truss,

$$\delta = \frac{\sum PkL}{AE}$$

Where *P*= Force in a member due to external load system

k = Force in a member by applying unit load in the direction at the point where deflection is desired after removing the given external loads.

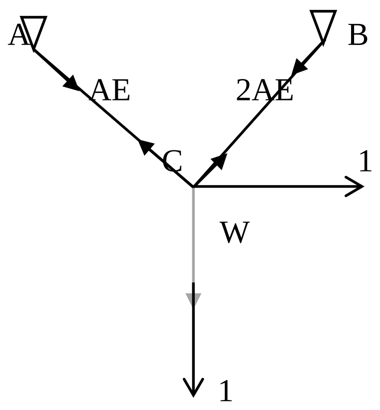
L = length of the concerned member.

AE = Axial rigidity of the member.

Sign Convention: Tension= -ve

Compression= +ve

Example: **Both AC & BC are of length l and separated by 45-degree angle.**



	P	k (vertical)	l	AE
AC	$\frac{-W}{\sqrt{2}}$	$\frac{-1}{\sqrt{2}}$	$\sqrt{2}l$	AE
BC	$\frac{-W}{\sqrt{2}}$	$\frac{-1}{\sqrt{2}}$	$\sqrt{2}l$	2AE

Applying unit load in

vertical direction,

$$\delta_V = \frac{\sum PkL}{AE}$$

Applying horizontal unit load

$F_{AC}\cos 45^o = F_{CB}\cos 45^o + 1$

$F_{AC} + F_{CB} = 0$

$F_{CB} = \frac{-1}{\sqrt{2}}$ (Compression)

Applying unit load in horizontal direction,

$$\delta_H = \frac{\sum Pk'L}{AE}$$

	k'
AC	$\frac{-1}{\sqrt{2}}$
BC	$\frac{-1}{\sqrt{2}}$

Members	P	K	k'	L	AE
AC	$\frac{-W}{\sqrt{2}}$	$\frac{-1}{\sqrt{2}}$	$\frac{-1}{\sqrt{2}}$	$\sqrt{2}l$	AE
BC	$\frac{-W}{\sqrt{2}}$	$\frac{-1}{\sqrt{2}}$	$\frac{-1}{\sqrt{2}}$	$\sqrt{2}l$	2AE

$$\delta_V = \frac{\sum PkL}{AE} = \frac{\frac{-w}{\sqrt{2}} \times \frac{-1}{\sqrt{2}} \times \sqrt{2}l}{AE} + \frac{\frac{-w}{\sqrt{2}} \times \frac{-1}{\sqrt{2}} \times \sqrt{2}l}{2AE}$$

$$= \frac{3wl}{2\sqrt{2}AE}$$

$$\delta_H = \frac{\sum Pk'L}{AE} = \frac{\frac{-w}{\sqrt{2}} \times \frac{-1}{\sqrt{2}} \times \sqrt{2}l}{AE} + \frac{\frac{-w}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times \sqrt{2}l}{2AE}$$

$$= \frac{wl}{2\sqrt{2}AE}$$

Note: $\boldsymbol{\delta} = \frac{\sum PkL}{AE} = \sum k\delta'$; δ' represents the actual deflection in a member may be due to external load system or temperature change or lack of fit.

MODULE 2 **CHAPTER 13**

SESSION 21

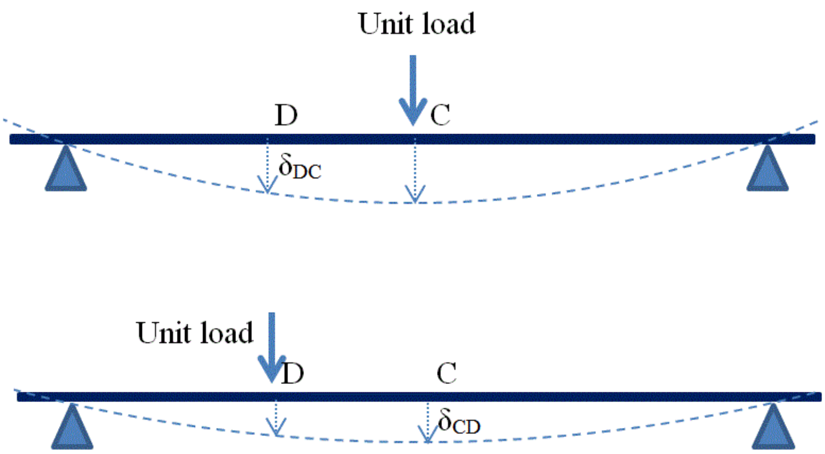
Learning objectives

Students will be able to understand

- *Maxwell's (Reciprocal) Theorem.*

13.1 MAXWELL'S THEOREM

"In any Elastic structure, the displacements at point 'D' due to an unit load at 'C', is equal to deflection at 'C' due to unit load at D."



[Fig. 13.1

δ_{XY} is the deflection at point X due to unit load at point Y. δ_{DC} is shown.

The unit load is transferred to point D. Maxwell's theorem equates δ_{CD} to δ_{DC}.

i.e., $\delta_{CD} = \delta_{DC}$

Maxwell's law is valid both in prismatic and non-prismatic structure.

Maxwell's law is independent of cross-sections i.e., sectional properties.

In the following fig [13.2]

As applied to beam deflections and rotations, Maxwell's theorem of reciprocal deflectionshas the following three versions:

(1) The deflection at A due to unit force at B is equal to the deflection at B due to unitforce at A as shown in Figure 17.1 a

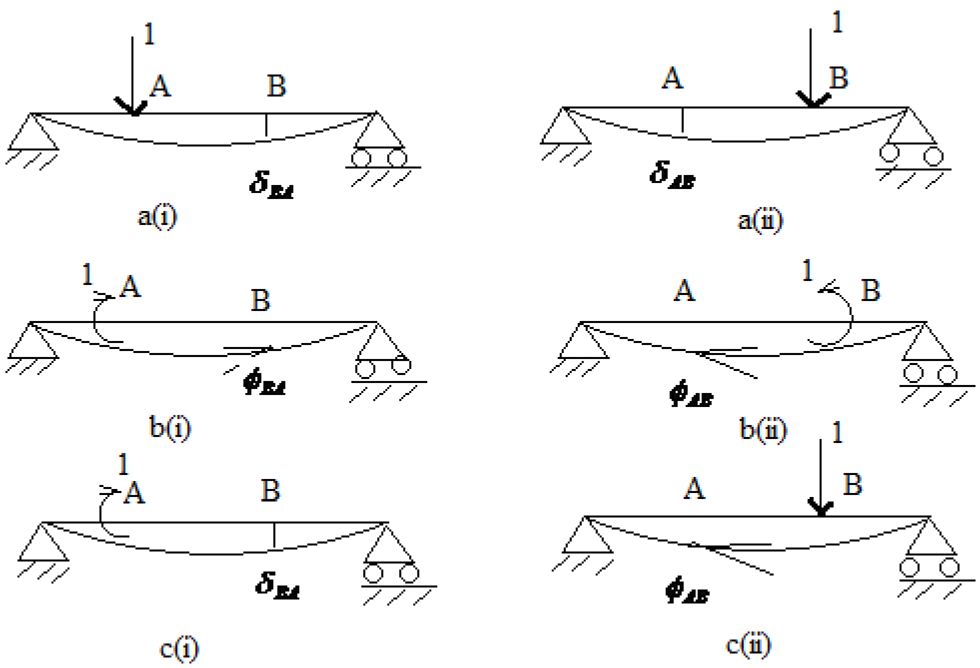
$\delta_{AB} = \delta_{BA}$

(2) The slope at A due to unit couple at B is equal to the slope at B due to unit couple atA as shown in Figure 17.1 b

$\phi_{AB} = \phi_{BA}$

(3) The slope at A due to unit load at B is equal to the deflection at B due to unit coupleat A as shown in Figure 17.1 c

$\phi_{AB'} = \delta_{BA'}$



[Fig. 13.2]

MODULE 2 **CHAPTER 13**

SESSION 22

Learning objectives

Students will be able to understand

- *Maxwell Betti's Theorem.*

13.2 MAXWELL BETTI'S THEOREM

If an elastic system is in equilibrium under one set of forces with their corresponding displacements and if the same system is also in equilibrium under second set of forces acting through the same points with their corresponding displacements, then the product of the first group of forces and the corresponding displacements caused by second group is equal to the product of the second group of forces and the corresponding displacements caused by the first group

$$P_A\Delta_{A'} + P_B\Delta_{B'} = P'_A\Delta_A + P'_B\Delta_B$$

Where P and Δ constitute first group of forces and their corresponding displacements and P' and Δ' constitute second group of forces and displacements.

That is the virtual work done by the first set of forces acting through the second set of displacements is equal to the virtual work done by the second set of forces acting through the first set of displacements.

In Betti's theorem, the symbols P and Δ can also denote couples and rotations respectively as well as forces and linear deflections i.e

$$M_A\theta_{A'} + M_B\theta_{B'} = M'_A\theta_A + M'_B\theta_B$$

Thus, according to Betti's law,

$$\sum P\Delta' + \sum M\theta' = \sum P'\Delta + \sum M'\theta$$

MODULE 2 **CHAPTER 14**

SESSION 23

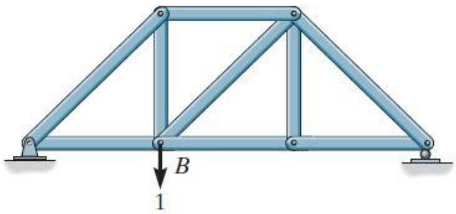
Learning objectives

Students will be able to understand

- *Application of Virtual work method (Trusses)*

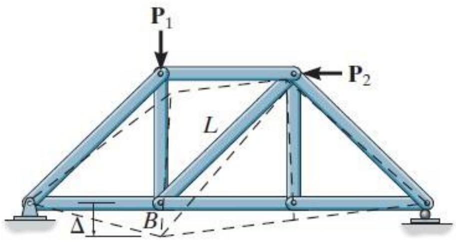
14.1 METHOD OF VIRTUAL WORK: TRUSSES

We can use the method of virtual work to determine the displacement of a truss joint when the truss is subjected to an external loading, temperature change, or fabrication errors. Eachof these situations will now be discussed.



Apply virtual unit load to *B*

(a)



Apply real loads $\mathbf{P}_1$, $\mathbf{P}_2$

(b)

[Fig. 14.1]

External Loading. For the purpose of explanation let us consider the vertical displacement Δ of joint B of the truss in Fig 14.1(a). Here a typical element of the truss would be one of itsmembers having a length L, Fig.14.1(b)

If the applied loadings P_1 and P_2 and cause a linear elastic material response, then this element deforms an amount

$$\Delta = \frac{NL}{AE}$$

$\Delta = \sum udL$, the virtual-work expression for the truss is therefore

$$\Delta = \frac{\sum nNL}{AE}$$

Where,

1= external virtual unit load acting on the truss joint in the stated direction of Δ
n= internal virtual normal force in a truss member caused by the external virtual unit load.
Δ = external joint displacement caused by the real loads on the truss.
N= internal normal force in a truss member caused by the real loads.
L= length of a member.
A= cross sectional area of a member.
E = modulus of elasticity of a member.

Here the external virtual unit load creates internal virtual forces n in each of the truss members. The real loads then cause the truss joint to be displaced Δ in the same direction as the virtual unit load, and each member is displaced nL/AE in the same direction as its respective n force. Consequently, the external virtual work Δ equals the internal virtual work or the internal (virtual) strain energy stored in all the truss members, that is, $\sum nL/AE$

Temperature. In some cases, truss members may change their length due to temperature. If α is the coefficient of thermal expansion for a member and ΔT is the change in its temperature, the change in length of a member is $dL = \alpha \Delta TL$ Hence, we can determine the displacement of a selected truss joint due to this temperature change is written as

$1..\Delta = \sum n\alpha\Delta TL$

where
1= external virtual unit load acting on the truss joint in the stated direction of Δ

n= internal virtual normal force in a truss member caused by the external virtual unit load.

Δ = external joint displacement caused by the temperature change.

α = coefficient of thermal expansion of member.

ΔT = change in temperature of member.

L= length of member.

MODULE 3 **CHAPTER 15**

SESSION 24

Learning objectives

Students will be able to understand

- *Introduction to trusses.*
- *Assumptions & rules in truss analysis.*

15.1 INTRODUCTION

A truss is a structure composed of straight, slender members connected at their ends by frictionless pins or hinges. A truss can be categorized as simple, compound, or complex. A simple truss is one constructed by first arranging three slender members to form a base triangular cell.

The conditions of determinacy, indeterminacy, and instability of trusses can be stated as follows:

$m+r<2j$ structure is statically unstable

$m+r=2j$ structure is determinate

$m+r>2j$ structure is indeterminate

where,

m= number of members.

r= number of support reactions.

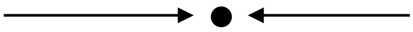
j= number of joints.

15.2 BASIC ASSUMPTIONS

- Members of the truss are primarily subjected to axial force only. Shear force and Bending moment are neglected.
- The self weight of the members are neglected.
- Loads will be act at the joints only.
- Joints are frictionless hinges. (No rotational resistance)
- All the members are lying in one plane (called middle plane of truss)

CG of the joints and CG of the members coincide in order to avoid bending.

SIGN CONVENTION

Towards the joint- axial compression 

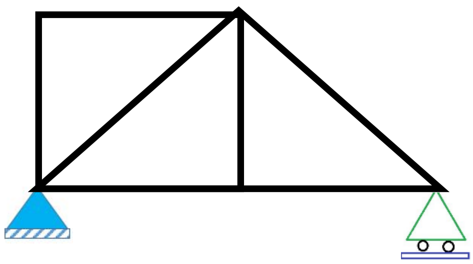
Away from the joint- axial tension 

15.3 BASIC RULES

- A single force cannot exist in nature; if at all it exists, it must be zero.
- If two forces act at a joint & if they're not be in the same line, then each force must be zero.
- If three forces act at a joint & if two of them are in same line then 3rd force must be zero.
- Horizontal force can't have vertical component and vice versa.

SOLVE THE FOLLOWING

Qn. -



Which members will be zero members in the above fig.?

MODULE 3 **CHAPTER 15**

SESSION 25

Learning objectives

Students will be able to understand

- *Methods of truss analysis.*
- *Methods of joints.*

15.4 METHODS OF TRUSS ANALYSIS

The analysis of truss consists of two steps:

i. Computation of reactions.
ii. Computation of axial forces.

Methods:

1. Analytical
   a. Methods of joints.
   b. Methods of sections.
2. Graphical
   a. Maxwell's method.
   b. Culmann's method.

15.4.1 METHODS OF JOINTS:

This method is based on the principle that if a structural system constitutes a body in equilibrium, then any joint in that system is also in equilibrium and, thus, can be isolated from the entire system and analyzed using the conditions of equilibrium. The method of joint involves successively isolating each joint in a truss system and determining the axial forces in the members meeting at the joint by applying the equations of equilibrium. The detailed procedure for analysis by this method is stated below.

Procedure for Analysis

•Verify the stability and determinacy of the structure. If the truss is stable and determinate, then proceed to the next step.

•Determine the support reactions in the truss.

•Identify the zero-force members in the system. This will immeasurably reduce the computational efforts involved in the analysis.

•Select a joint to analyze. At no instance should there be more than two unknown member forces in the analyzed joint.

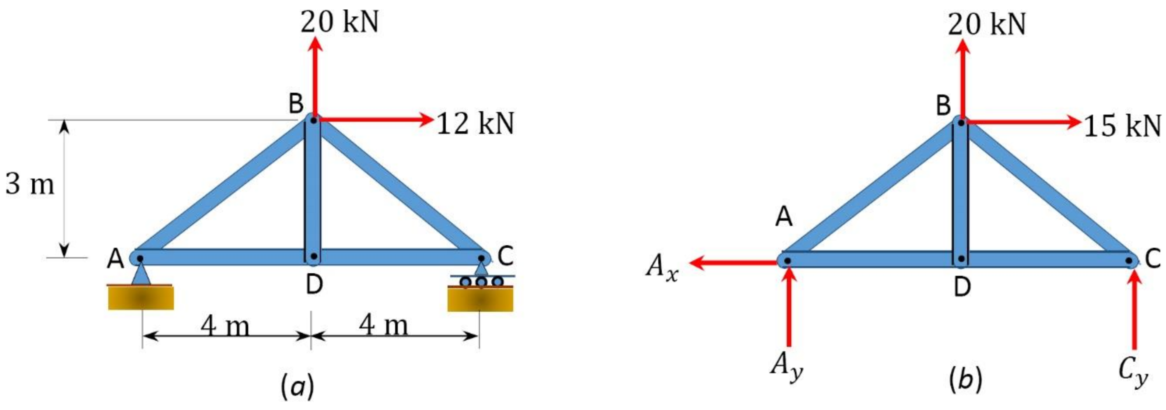
•Draw the isolated free-body diagram of the selected joint, and indicate the axial forces in all members meeting at the joint as tensile (i.e. as pulling away from the joint). If this initial assumption is wrong, the determined member axial force will be negative in the analysis, meaning that the member is in compression and not in tension.

•Apply the two equations $\Sigma FX=0\Sigma FX=0$ and $\Sigma FY=0\Sigma FY=0$ to determine the member axial forces.

•Continue the analysis by proceeding to the next joint with two or fewer unknown member forces.

Example 1

Using the method of joint, determine the axial force in each member of the truss shown in Figure a.



Solution

Support reactions. By applying the equations of static equilibrium to the free-body diagram shown in Figure b, the support reactions can be determined as follows:

$+\curvearrowleft \sum M_A=0 \qquad 20(4)-12(3)+(8)C_y=0 \qquad C_y=-5.5kN$

+↑ ∑Fy=0 Ay−5.5+20=0 Ay=−14.5kN

+→ ∑Fx=0 −Ax+12=0 Ax=12kN Cy=5.5kN

↓ Ay=14.5kN

↓ Ax=12kN

←+↶ ∑MA=0 20(4)−12(3)+(8)Cy=0 Cy=−5.5kN

↓+↑ ∑Fy=0 Ay−5.5+20=0 Ay=−14.5kN

↓+→ ∑Fx=0 −Ax+12=0 Ax=12kN

Analysis of joints. The analysis begins with selecting a joint that has two or fewer unknown member forces. The free-body diagram of the truss will show that joints A and B satisfy this requirement. To determine the axial forces in members meeting at joint AA, first isolate the joint from the truss and indicate the axial forces of members as FAB and FAD, as shown in Figure c. The two unknown forces are initially assumed to be tensile (i.e., pulling away from the joint). If this initial assumption is incorrect, the computed values of the axial forces will be negative, signifying compression.

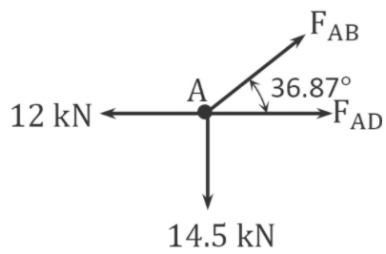
Analysis of joint AA.

+↑ ∑Fy=0 FABsin36.87∘−14.5=0 FAB=24.17

+→∑Fx=0 −12+FAD+FABcos36.87∘=0 FAD=12−24.17cos36.87∘=−7.34kN

+↑∑Fy=0 FABsin36.87∘−14.5=0 FAB=24.17

+→∑Fx=0 −12+FAD+FABcos36.87∘=0
FAD=12−24.17cos36.87∘=−7.34kN



(c)

After completing the analysis of joint AA, joint BB or DD can be analyzed, as there are only two unknown forces.

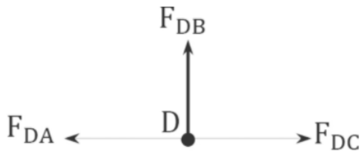
Analysis of joint DD.

+↑ $\sum Fy=0$ FDB=0

+→ $\sum Fx=0$ −FDA+FDC=0 FDC=FDA=−7.34kN

+↑ $\sum Fy=0$ FDB=0

+→$\sum Fx=0$ −FDA+FDC=0 FDC=FDA=−7.34kN



(d)

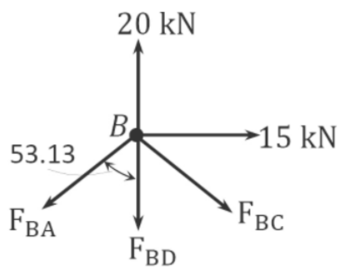
Analysis of joint BB.

+→ $\sum Fx=0$ −FBAsin53.13+FBCsin53.13+15=0

FBCsin53.13=−15+24.17sin53.13 FBC=5.42kN

+→ $\sum Fx=0$ −FBAsin53.13+FBCsin53.13+15=0

FBCsin53.13=−15+24.17sin53.13 FBC=5.42kN



(e)

MODULE 3 **CHAPTER 15**

SESSION 26

Learning objectives

Students will be able to understand

- *Methods of sections in truss analysis.*

15.4.2 METHODS OF SECTIONS

Analysis of Trusses by Method of Section-

Sometimes, determining the axial force in specific members of a truss system by the method of joint can be very involving and cumbersome, especially when the system consists of several members. In such instances, using the method of section can be timesaving and, thus, preferable. This method involves passing an imaginary section through the truss so that it divides the system into two parts and cuts through members whose axial forces are desired. Member axial forces are then determined using the conditions of equilibrium. The detailed procedure for analysis by this method is presented below.

Procedure for Analysis of Trusses by Method of Section

•Check the stability and determinacy of the structure. If the truss is stable and determinate, then proceed to the next step.

•Determine the support reactions in the truss.

•Make an imaginary cut through the structure so that it includes the members whose axial forces are desired. The imaginary cut divides the truss into two parts.

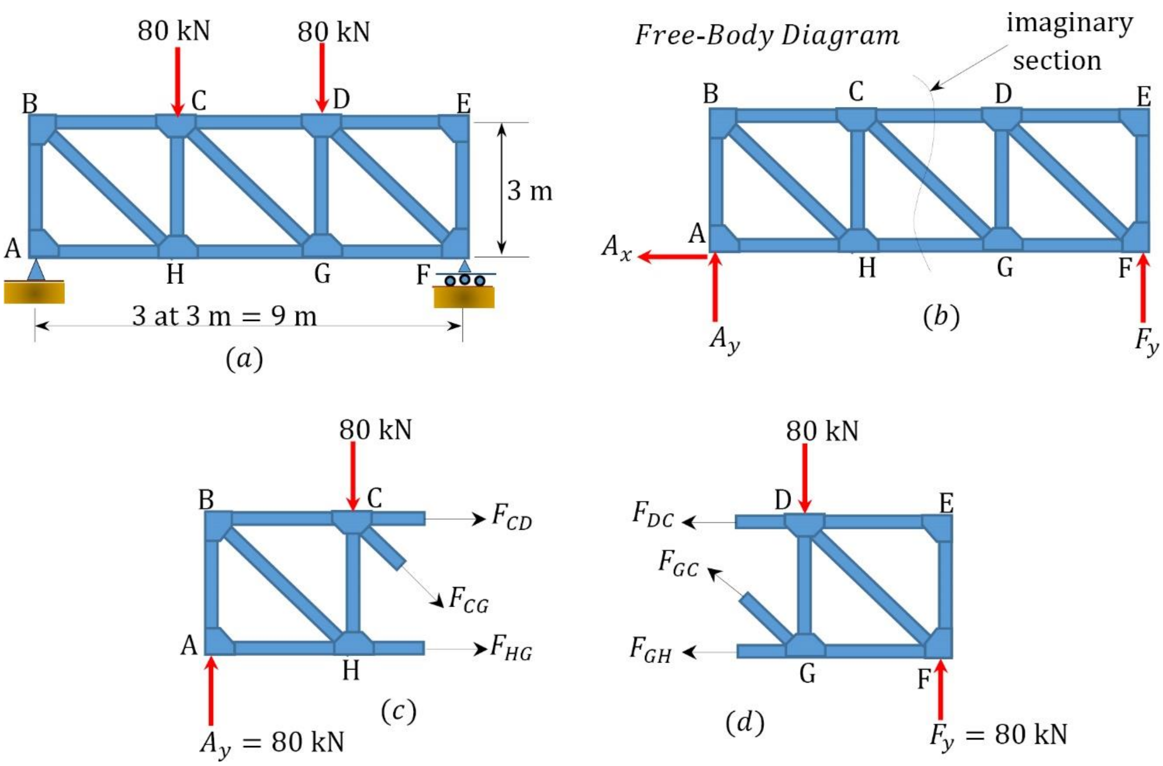
•Apply forces to each part of the truss to keep it in equilibrium.

•Select either part of the truss for the determination of member forces.

•Apply the conditions of equilibrium to determine the member axial forces.

Example 2

Using the method of section, determine the axial forces in members CD, CG, and HG of the truss shown in Figure a.



Solution

Support reactions.

By applying the equations of static equilibrium to the free-body diagram in Figure b, the support reactions can be determined as follows:

Ay=Fy=160/2=80Kn

+⟶ ΣFx=0 Ax=0 Ay=Fy=160/2=80kN

+⟶ ΣFx=0 Ax=0

Analysis by method of section. First, an imaginary section is passed through the truss so that it cuts through members CD, CG, and HG and divides the truss into two parts, as shown in Figure c and Figure d. Member forces are all indicated as tensile forces (i.e., pulling away from the joint). If this initial assumption is wrong, the calculated member forces will be negative, showing that they are in compression. Either of the two parts can be used for the analysis. The left-hand part will be used for determining the member forces in this example. By applying the equation of equilibrium to the left-hand segment of the truss, the axial forces in members can be determined as follows:

Axial force in member CD.

To determine the axial force in member CD, find a moment about a joint in the truss where only CD will have a moment about that joint and all other cut members will have no moment. A close examination will show that the joint that meets this requirement is joint G. Thus, taking the moment about G suggests the following:

$+\circlearrowleft \quad \sum M_G=0 \quad -80(6)+80(3)-F_{CD}(3)=0 \quad F_{CD}=-80 kN (C)$

Axial force in member HG.

$+\circlearrowleft \quad \sum M_C=0 \quad -80(3)+F_{HG}(3)=0 \quad F_{HG}=80 kN (T)$

Axial force in member CG. The axial force in member CG is determined by considering the vertical equilibrium of the left-hand part. Thus,

$+\uparrow \quad \sum F_y=0 \quad 80-80-F_{CG}\cos 45^\circ=0 \quad F_{CG}=0$

MODULE 3 **CHAPTER 16**

SESSION 27

Learning objectives

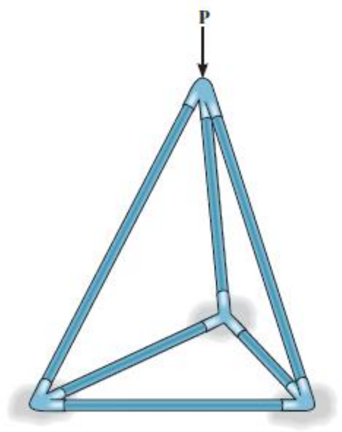
Students will be able to understand

- *What is space truss.?*

16.1 ROOF TRUSS

A *space truss* consists of members joined together at their ends to form a stable three-dimensional structure. It was shown that the simplest form of a stable two-dimensional truss consists of the me

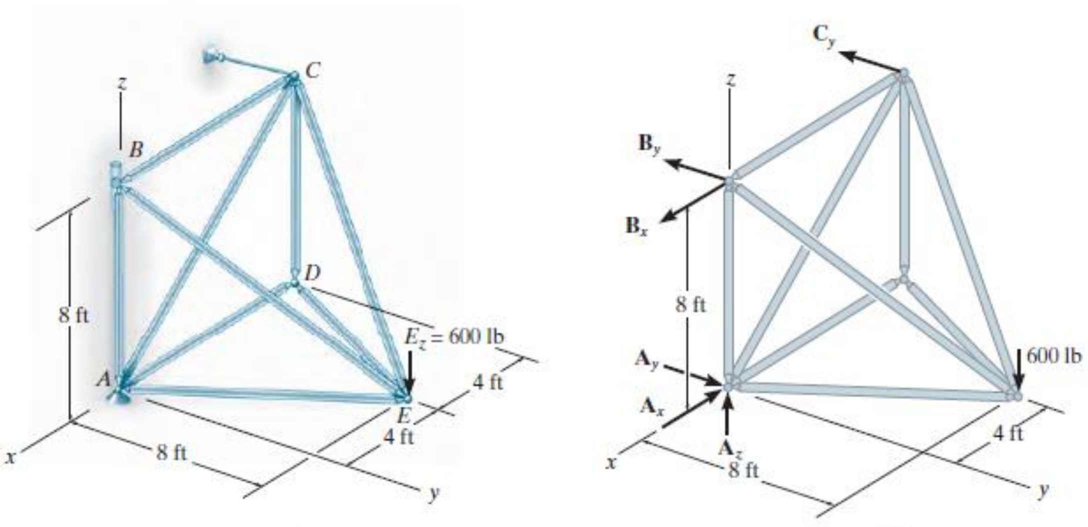
mbers arranged in the form of a triangle. We then built up the simple plane truss from this basic triangular element by adding two members at a time to form further elements. In a similar manner, the simplest element of a stable space truss is a *tetrahedron*, formed by connecting six members together with four joints as shown in following Fig. Any additional members added to this basic element would be redundant in supporting the force **P**. A simple space truss can be built from this basic tetrahedral element by adding three additional members and another joint forming multiconnected tetrahedrons.



The *external stability* of the space truss requires that the support reactions keep the truss in force and moment equilibrium about any and all axes. This can sometimes be checked by inspection, although if the truss is unstable a solution of the equilibrium equations will give inconsistent results. *Internal stability* can sometimes be checked by careful inspection of the member arrangement. Provided each joint is held fixed by its supports or connecting members, so that it cannot move with respect to the other joints, the truss can be classified as internally stable. Also, if we do a force analysis of the truss and obtain inconsistent results, then the truss configuration will be unstable or have a "critical form."

Example 1.

Determine the force in each member of the space truss shown in Fig. *a*. The truss is supported by a ball-and-socket joint at *A*, a slotted roller joint at *B*, and a cable at *C*.



(a) (b)

SOLUTION

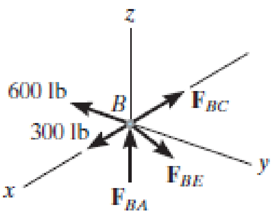
The truss is statically determinate since or Fig. *b*.

Support Reactions. We can obtain the support reactions from the free-body diagram of the entire truss, Fig. *b*, as follows:

The truss is statically determinate since b+r=3j or 9+6 = 3(5)

Support Reactions. We can obtain the support reactions from the free-body diagram of the entire truss, Fig. 3–38*b*, as follows:

$$\Sigma M_y = 0; \quad -600(4) + B_x(8) = 0 \quad B_x = 300 \text{ lb}$$
$$\Sigma M_z = 0; \quad C_y = 0$$
$$\Sigma M_x = 0; \quad B_y(8) - 600(8) = 0 \quad B_y = 600 \text{ lb}$$
$$\Sigma F_x = 0; \quad 300 - A_x = 0 \quad A_x = 300 \text{ lb}$$
$$\Sigma F_y = 0; \quad A_y - 600 = 0 \quad A_y = 600 \text{ lb}$$
$$\Sigma F_z = 0; \quad A_z - 600 = 0 \quad A_z = 600 \text{ lb}$$



(c)

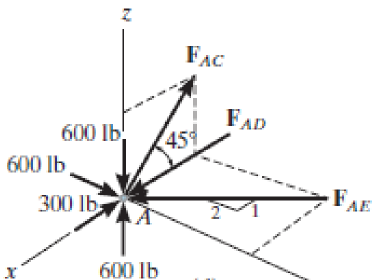
Joint *B*. We can begin the method of joints at B since there are three unknown member forces at this joint, Fig. 3–38*c*. The components of $\mathbf{F}_{BE}$ can be determined by proportion to the length of member BE, as indicated by Eqs. 3–5. We have

$$\Sigma F_y = 0; \qquad -600 + F_{BE}\left(\tfrac{8}{12}\right) = 0 \qquad F_{BE} = 900 \text{ lb (T)} \qquad \textit{Ans.}$$

$$\Sigma F_x = 0; \qquad 300 - F_{BC} - 900\left(\tfrac{4}{12}\right) = 0 \qquad F_{BC} = 0 \qquad \textit{Ans.}$$

$$\Sigma F_z = 0; \qquad F_{BA} - 900\left(\tfrac{8}{12}\right) = 0 \qquad F_{BA} = 600 \text{ lb (C)} \qquad \textit{Ans.}$$

Joint *A*. Using the result for $F_{BA} = 600$ lb (C), the free-body diagram of joint A is shown in Fig. 3–38*d*. We have



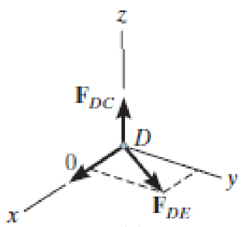
(d)

$$\Sigma F_z = 0; \qquad 600 - 600 + F_{AC} \sin 45^\circ = 0$$
$$F_{AC} = 0 \qquad \textit{Ans.}$$

$$\Sigma F_y = 0; \qquad -F_{AE}\left(\tfrac{2}{\sqrt{5}}\right) + 600 = 0$$
$$F_{AE} = 670.8 \text{ lb (C)} \qquad \textit{Ans.}$$

$$\Sigma F_x = 0; \qquad -300 + F_{AD} + 670.8\left(\tfrac{1}{\sqrt{5}}\right) = 0$$
$$F_{AD} = 0 \qquad \textit{Ans.}$$

Joint *D*. By inspection the members at joint D, Fig. 3–38*a*, support zero force, since the arrangement of the members is similar to either of the two cases discussed in reference to Figs. 3–36 and 3–37. Also, from Fig. 3–38*e*,

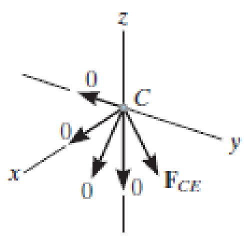


(e)

$$\Sigma F_x = 0; \qquad F_{DE} = 0 \qquad \textit{Ans.}$$

$$\Sigma F_z = 0; \qquad F_{DC} = 0 \qquad \textit{Ans.}$$

Joint *C*. By observation of the free-body diagram, Fig. 3–38*f*,



(f)

$$F_{CE} = 0 \qquad \textit{Ans.}$$

MODULE 3 CHAPTER 16

SESSION 28

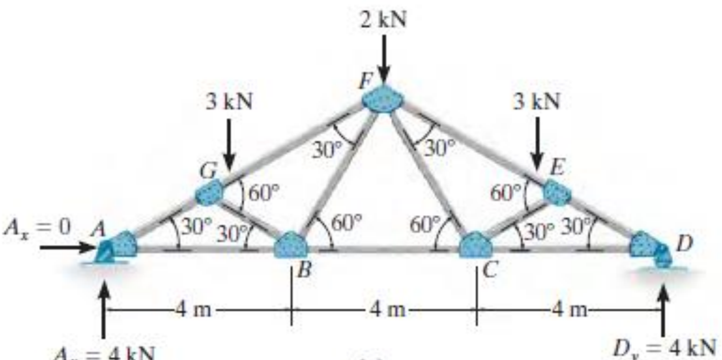
Learning objectives

Students will be able to understand

- *Problems of Trusses by methods of joints.*

PROBLEMS

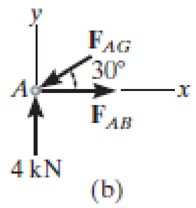
Qn. 1: Determine the force in each member of the roof truss shown in the Fig.. The dimensions and loadings are shown in Fig. *a*. State whether the members are in tension or compression.



(a)

SOLUTION

Only the forces in half the members have to be determined, since the truss is symmetric with respect to *both* loading and geometry.

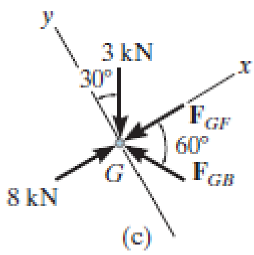


(b)

Joint A, Fig. b. We can start the analysis at joint *A*.

The free-body diagram is shown in Fig. *b*.

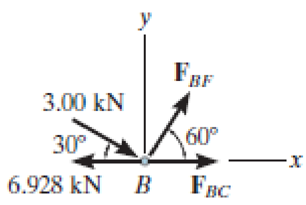
$$+\uparrow \sum F_Y = 0\,; 4 - F_{AG} Sin\ 30^o = 0 \;\; F_{AG} = 80kN\ (C)$$



(c)

$$+\rightarrow \sum F_x = 0\,;\; F_{AB} - 8\ Cos\ 30° = 0 \;\; F_{AB} = 6.928\ kN\ (T)$$

Joint G, Fig. c. In this case note how the orientation of the *x*, *y* axes avoid simultaneous solution of equations.



(d)

$$+\nwarrow \sum F_Y = 0\ ;\ FGB\ sin\ 60° - 3\ cos\ 30° = 0 \quad FGB = 3.00\ kN\ (C)$$

$$+\nearrow \sum F_x = 0\ ;\ 8 - 3\sin 30° - 3.00\cos 60° - FGF = 0\ FGF = 5.00\ kN\ (C)$$

Joint B, Fig. d.

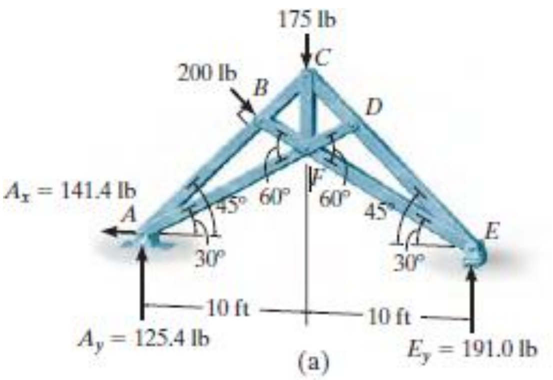
$$+\uparrow \sum F_Y = 0\ ;\ FBF \sin 60° - 3.00 \sin 30° = 0$$

$$FBF = 1.73\ kN\ (T)$$

$$+\rightarrow \sum F_x = 0\ ;\ FBC + 1.73\cos 60° + 3.00\cos 30° - 6.928 = 0$$

$$FBC = 3.46\ kN\ (T)$$

Qn. 2: Determine the force in each member of the scissors truss shown in Fig.*a*. State whether the members are in tension or compression. The reactions at the supports are given.



(a)

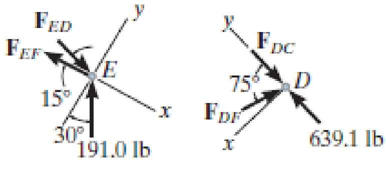
SOLUTION

The truss will be analyzed in the following sequence:

Joint E, Fig. b. Note that simultaneous solution of equations is

avoided by the *x*, *y* axes orientation.

$$+\nearrow \sum F_Y = 0\ ;\ 191.0\cos 30° - FED \sin 15° = 0$$



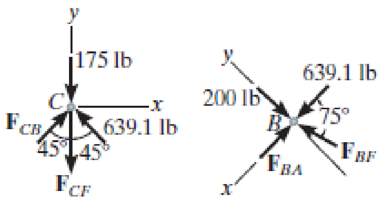
(b) (c)

$$FED = 639.1\, lb\, (C)$$

$$+\searrow \sum F_x = 0\,;\; 639.1 \cos 15° - FEF - 191.0 \sin 30° = 0$$

$$FEF = 521.8\, lb\, (T)$$

Joint D, Fig. c.

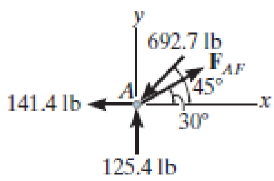


(d) (e)

$$\sum F_x = 0\,;\; -FDF \sin 75° = 0$$

$$FDF = 0$$

$$\sum F_Y = 0\,;\; -FDC + 639.1 = 0\; FDC = 639.1\, lb\, (C)$$



(f)

Joint C, Fig. d.

$$\sum F_x = 0\,;\; FCB \sin 45° - 639.1 \sin 45° = 0$$

$$FCB = 639.1\, lb\, (C)$$

$$\sum F_Y = 0\,;\; -FCF - 175 + 21639.12 \cos 45° = 0$$

$$FCF = 728.8\, lb\, (T)$$

Joint B, Fig. e.

$$\sum F_Y = 0\,;\; FBF \sin 75° - 200 = 0$$

$$FBF = 207.1\, lb\, (C)$$

$$\sum F_x = 0\,;\; 639.1 + 207.1 \cos 75° - FBA = 0$$

$$FBA = 692.7\, lb\, (C)$$

Joint A, Fig. f.

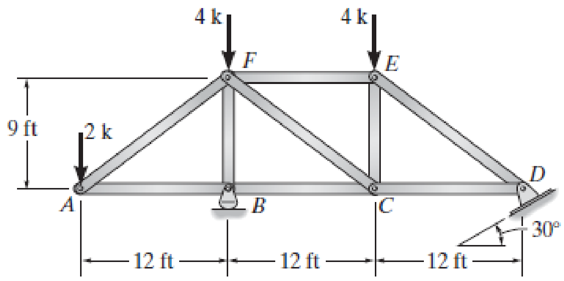
$$\sum F_x = 0\,;\; FAF \cos 30° - 692.7 \cos 45° - 141.4 = 0$$

$$FAF = 728.9\ lb\ (T)$$

$$\sum F_Y = 0\ ;\ \ 125.4 - 692.7\ sin\ 45° + 728.9\ sin\ 30° = 0$$

Solve:

Determine the force in each member of the truss. State if the members are in tension or compression.



MODULE 3 CHAPTER 16

SESSION 28

Learning objectives

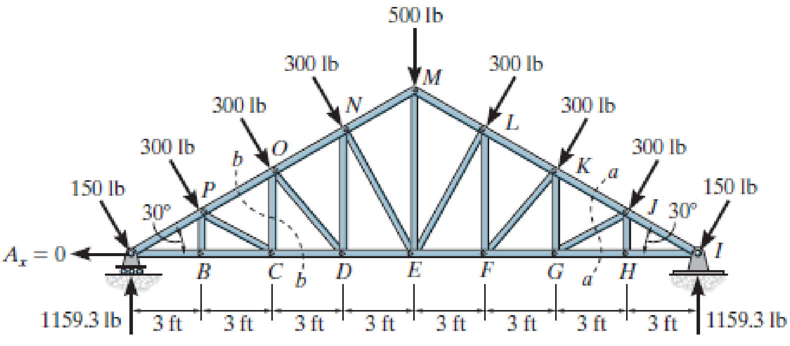
Students will be able to understand

- *Problems of Trusses by methods of sections.*

PROBLEMS

Qn. 1 : Determine the force in members *GJ* and *CO* of the roof truss shown in the photo. The dimensions and loadings are shown in Fig. *a*. State whether the members are in tension or compression. The reactions at the supports have been calculated.

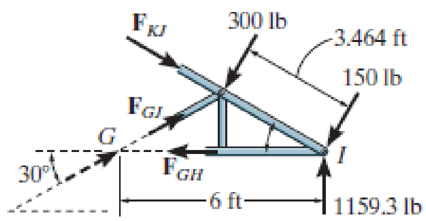
SOLUTION



(a)

Member CF.

Free-Body Diagram. The force in member *GJ* can be obtained by considering the section *aa* in Fig.*a*.The free-body diagram of the right part of this section is shown in Fig.*b*.



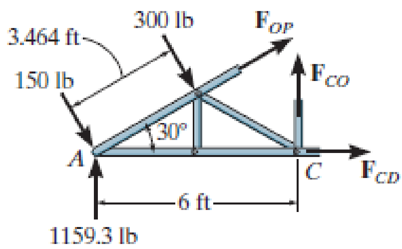
(b)

A direct solution for can be obtained by applying

$$\sum M_I = 0$$

$$-F_{GJ}\sin 30°(6) + 300(3.464) = 0$$

$$F_{GJ} = 346\ lb\ (C)$$



(c)

Member GC.

Free-Body Diagram. The force in CO can be obtained by using section bb in Fig. a. The free-body diagram of the left portion of the section is shown in Fig. c.

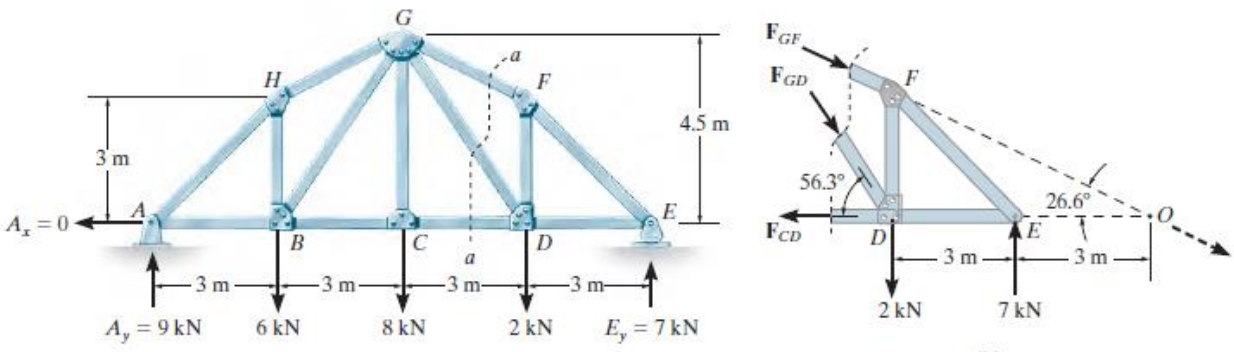
Equations of Equilibrium. Moments will be summed about point A in order to eliminate the unknowns F_{OP} and F_{CD}

$$\sum M_A = 0$$

-300(3.464) + FCO(6) = 0

FCO = 173 lb (T)

Qn. 2 : Determine the force in members *GF* and *GD* of the truss shown in Fig. a. State whether the members are in tension or compression. The reactions at the supports have been calculated**.**



(a) (b)

SOLUTION

Free-Body Diagram. Section aa in Fig.a will be considered. The free-body diagram to the right of this section is shown in Fig.b. The distance EO can be determined by proportional triangles or realizing that member GF drops vertically 4.5 - 3 = 1.5 m in 3 m, Fig. a Hence to drop 4.5 m from G the distance from C to O must be 9 m.Also, the angles that **F**GD and **F**GF make with the horizontal are $\tan^{-1}\frac{4.5}{3} = 56.3^{\circ}$ and $\tan^{-1}\frac{4.5}{9} = 26.6^{\circ}$ respectively.

Equations of Equilibrium. The force in *GF* can be determined directly by applying $\sum M_D = 0$ For the calculation use the principle of transmissibility and slide F_{GF} to point *O*.Thus

$$\sum M_D = 0$$

$$-FGF \sin 26.6\,°(6) + 7(3) = 0$$

FGF = 7.83 kN

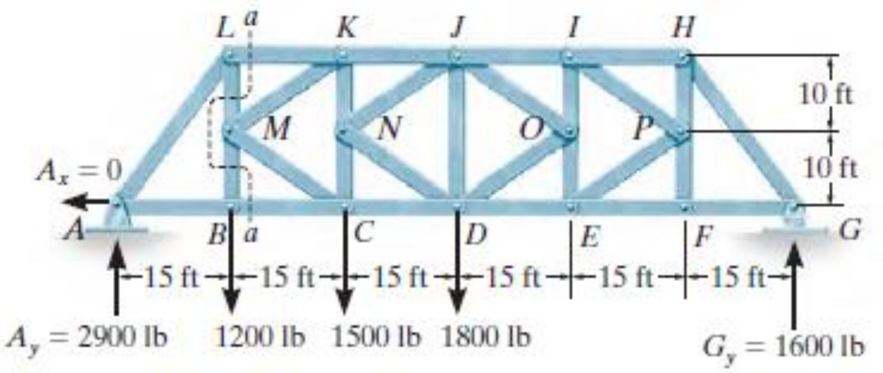
The force in *GD* is determined directly by applying $\sum M_O = 0$, For simplicity use the principle of transmissibility and slide $\mathbf{F}_{GD}$ to *D*.

Hence, -7(3) + 2(6) + FGD sin 56.3°(6) = 0

FGD = 1.80 kN (C)

Solve

Determine the force in members *BC* and *MC* of the K-truss shown in Fig. *a*. State whether the members are in tension or compression. The reactions at the supports have been calculated.



(a)

MODULE 4 **CHAPTER 17**

SESSION 29

Learning objectives

Students will be able to understand

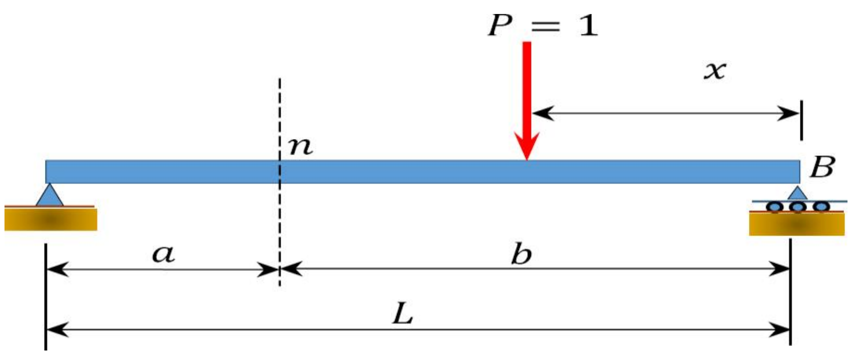
- What is Influence line diagram.
- What are its uses.

17.1 Influence Lines

Structures such as bridges and overhead cranes must be designed to resist moving loads as well as their own weight. Since structures are designed for the critical loads that may occur in them, influence lines are used to obtain the position on a structure where a moving load will cause the largest stress. Influence lines can be defined as a graph whose ordinates show the variation of the magnitude of a certain response function of a structure as a unit load traverses across the structure.

17.1.1 *Influence Lines for Beams by Static Equilibrium Method*

To grasp the basic concept of influence lines, consider the simple beam shown in Figure below. Statics help to determine the magnitude of the reactions at supports *A* and *B*, and the shearing force and bending moment at a section *n*, as a unit load of arbitrary unit, moves from right to left.



Beam Reactions;

Taking the moment about *B* as the unit load moves a distance *x* from the right-hand end suggests the following:

$$+\sum M_B = 0$$
$$-R_A L + Px = 0$$
$$R_A = \frac{Px}{L}$$

Setting P = 1 suggests the following:

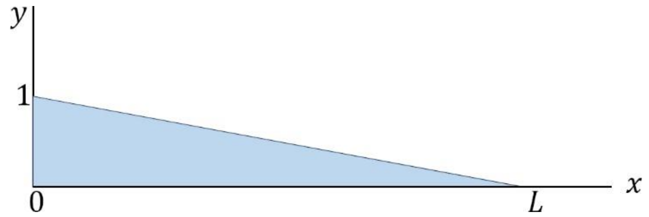
$$R_A = \frac{x}{L}$$

Equation is the expression for the computation of the influence line for the left-end reaction of a simply supported beam. The influence line for R_A can be represented graphically by putting some values of x into the equation. Since the equation is linear, two points should be enough.

When $x = 0$, $R_A = 0$

When $x = L$, $R_A = 1$

The graphical representation of the influence line for R_A is shown in Figure below, and the ordinate of the diagram corresponding to any value of x gives the magnitude of R_A at that point.



Similarly, the expression for the influence line for the reaction R_B is found by taking the moment about A.

$$\sum M_A = 0$$

$$R_B L - P(L - x) = 0$$

$$R_B = \frac{P(L-x)}{L}$$

Setting P = 1 into equation above suggests the following:

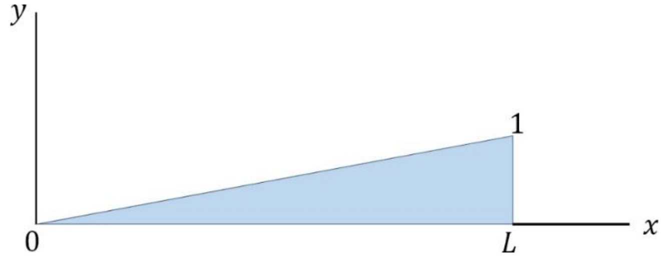
$$R_B = \frac{(L-x)}{L}$$

This equation is the expression for the computation of the influence line for the right-end reaction of a simply supported beam. Substituting some values for x into the equation helps to construct the influence line diagram for R_B.

When $x = 0$, $R_B = 1$

When $x = L$, $R_B = 0$

The graphical representation of the influence line for R_B is shown in figure below.



Shearing Force at Section *n*

When the unit load is on the right side of the section, the shear force at the section can be computed considering the transverse forces on the left side of the section, as follows:

Shearing force, $V = R_A = \frac{x}{L}$

When $x = 0, V = 0$

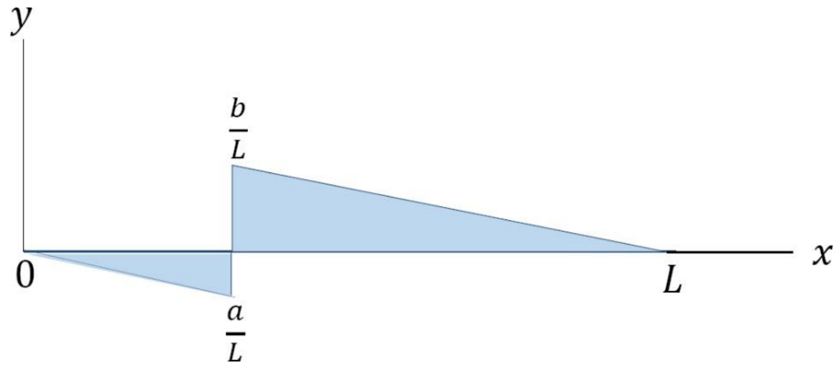
When $x = b, V = \frac{b}{L}$

When the unit load is on the left side of the section, it is easier to compute the shear force in the section by considering the forces on the right side of section, as follows:

$V = -R_B = -\frac{(L-x)}{L}$

When $x = b, V = -\frac{(L-b)}{L} = -\frac{a}{L}$

When $x = L, V = 0$



Bending Moment at a Section n

When the unit load is on the right side of the section, the bending moment at the section can be computed as follows:

$M = R_A(L-x) = \frac{x}{L}(L-x)$

When $x = 0, M = 0$

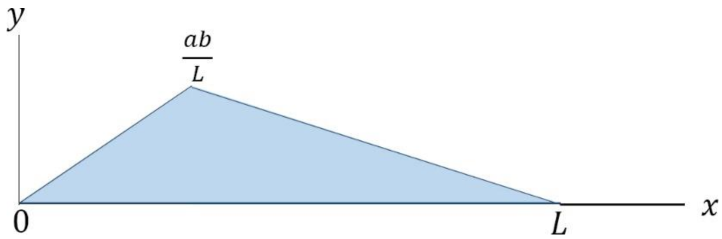
When $x = b, M = \frac{ab}{L}$

When the unit load is on the left side of section, the bending moment at the section can be computed as follows:

$M = R_B x = \frac{(L-x)}{L}x$

When $x = 0, M = 0$

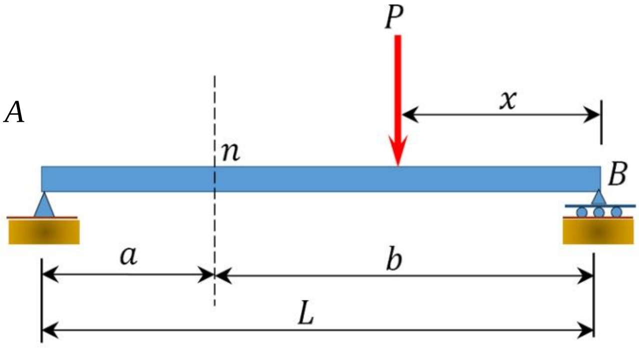
When $x = b, M = \frac{ab}{L}$



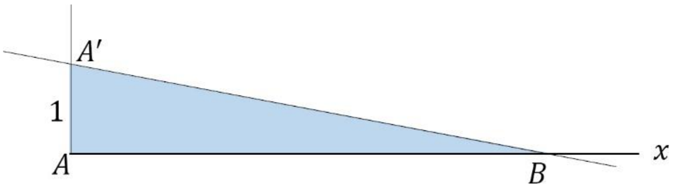
MODULE 4 CHAPTER 18

SESSION 30

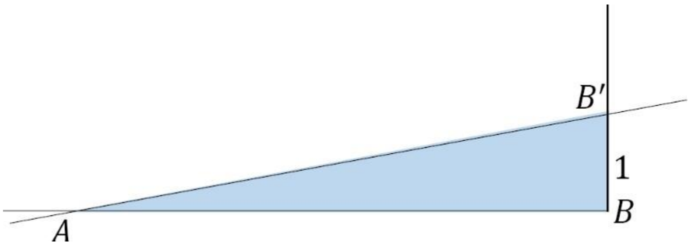
18.1 Influence Line Diagram for simply supported beam



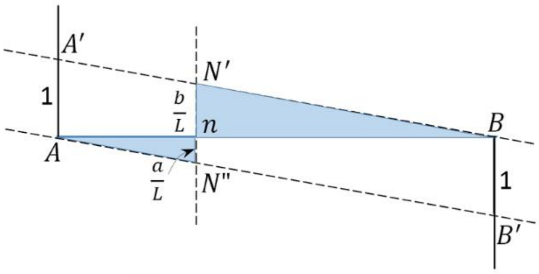
Influence Line for Left End Support Reaction R_A



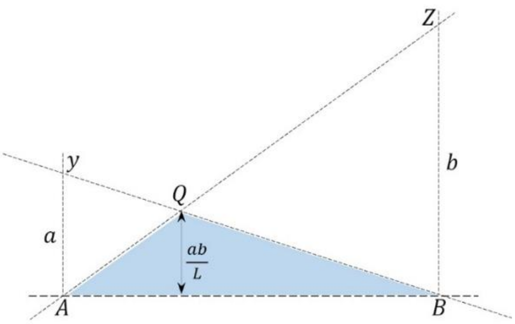
Influence Line for Right End Support Reaction R_B



Influence Line for Shearing Force at Section n



Influence Line for Bending Moment at Section n



Q: - A simply supported beam of span 10m carries a udl of 20 kN/m over its central 4m length. With the help of influence line diagram, find the shear force at 3m from the left support.

$$\frac{x}{l} = \frac{3}{10} = 0.3$$

$$\frac{(l-x)}{l} = \frac{7}{10} = 0.7$$

Shear force at X = intensity of udl x area of udl below the udl

$$= 20 * \frac{(0.7+0.3)}{2} * 4 = 40 \ kN$$

Q: - A single rolling load of 100 kN moves on a girder of span 20m.

(a) Construct the influence lines for (i) shear force and (ii) bending moment for a section 5m from the left support.

(b) Construct the influence lines for points at which the maximum shears and maximum bending moment develop. Determine these values.

Solution:

(a) To find maximum shear force and bending moment at 5m from the left support:

(i) **Maximum positive shear force**

By inspection of the ILD for shear force, it is evident that maximum positive shear force occurs when the load is placed just to the right of D.

Maximum positive shear force = load * ordinate = 100 * 7.5

At D, SFmax + = 75 kN.

(ii) **Maximum negative shear force**

Maximum negative shear force occurs when the load is placed just to the left D.

Maximum negative shear force = load * ordinate = 100 * 0.25 At D, SFmax = -25 kN.

(iii) **Maximum bending moment**

Maximum bending moment occurs when the load is placed on the section D itself.

Maximum bending moment = load * ordinate = 100 * 3.75 = 375 kNm

(b) Maximum positive shear force will occur at A. Maximum negative shear force will occur at B. Maximum bending moment will occur at mid span. The ILs are sketched in fig.

(i) **Positive shear force**

Maximum positive shear force occurs when the load is placed at A. Maximum positive shear force = load * ordinate = 100*1

SFmaxmax + = 100 kN

(ii) **Negative shear force**

Maximum negative shear force occurs when the load is placed at B. Maximum negative shear force = load * ordinate = 100 * (-1)

SFmaxmax = - 100 kN

(iii) **Maximum bending moment**

Maximum bending moment occurs when the load is at mid span Maximum bending moment = load * ordinate = 100 * 5 = 500 kNm

Q: - Draw the ILD for shear force and bending moment for a section at 5m from the left hand support of a simply supported beam, 20m long. Hence, calculate the maximum bending moment and shear force at the section, due to a uniformly distributed rolling load of length 8m and intensity 10 kN/m run.

Solution:

(a) Maximum bending moment:

Maximum bending moment at a D due to a udl shorter than the span occurs when the section divides the load in the same ratio as it divides the span.

In the above fig. $\frac{A_1D}{B_1D} = \frac{AD}{BD} = 0.25, \quad A_1D = 2M, B_1D = 6M$

Ordinates:

Ordinate under A_1 = (3.75/5)*3 = 2.25

Ordinate under B_1 = (3.75/15)*9 = 2.25

Maximum bending moment = Intensity of load * Area of ILd under the load

$$= 10 * \frac{(3.75 + 2.25) * 8}{2}$$

At D, M_{max} = 240 kNm

(b) **Maximum positive shear force**

Maximum positive shear force occurs when the tail of the UDL is at D as it traverses from left to right.

Maximum positive shear force occurs when the tail of the UDL is at D as it traverses from left to right.

Ordinate under $B_1 = \frac{0.75}{15} * (15 - 8) = 0.35$

Maximum positive shear force = Intensity of load * Area of ILD under load

$$= 10 * \frac{(0.75 + 0.35) * 8}{2}$$

$$SF_{max} = + 44 \text{ kNm}$$

(c) **Maximum negative shear force**

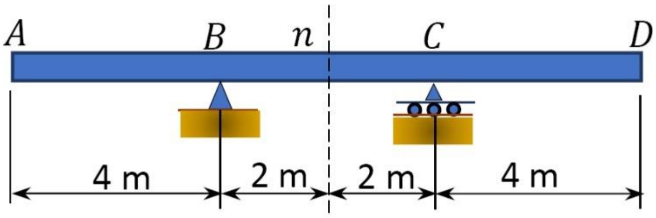
Maximum negative shear force occurs when the head of the UDL is at D as it traverses from left to right.

Maximum negative shear force = Intensity of load * Area of ILD under the load = 10(1/2*0.25*5)

Negative SFmax = 6.25 kN.

19.1 Influence Line Diagram for doubly Overhanging beams

Q: - For the double overhanging beam shown in Figure (a) below, construct the influence lines for the support reactions at *B* and *C* and the shearing force and the bending moment at section *n*.



(a)

Solution

I.L. for B_y.

Step 1. At the position of support *B* (point *B*), plot an ordinate +1.

Step 2. Draw a straight line connecting the plotted point (+1) to the zero ordinate at the position of support *C*.

Step 3. Continue the straight line in step 2 until the end of the overhangs at both ends of the beam. The influence line for B_y is shown in Figure b.

Step 4. Determine the ordinates of the influence line at the overhanging ends using a similar triangle, as follows:

Ordinate at *A*:

$$\frac{1}{BC} = \frac{x}{AC}; \Rightarrow x = \frac{AC}{BC} = \frac{8}{4} = 2 \text{ m}$$

Ordinate at *D*:

$$\frac{1}{BC} = \frac{x}{CD}; \Rightarrow x = \frac{CD}{BC} = \frac{4}{4} = 1 \text{ m}$$

I.L. for C_y.

Step 1. At the position of support *C* (point *C*), plot an ordinate +1.

Step 2. Draw a straight line connecting the plotted point (+1) to the zero ordinate at the position of support *B*.

Step 3. Continue the straight line in step 2 until the end of the overhangs at both ends of the beam. The influence line for B_y is shown in Figure c.

Step 4. Determine the ordinates of the influence line at the overhanging ends using a similar triangle, as follows:

Ordinate at *D*:

$$\frac{1}{BC} = \frac{x}{BD}; \Rightarrow x = \frac{BD}{BC} = \frac{8}{4} = 2 \text{ m}$$

Ordinate at *A*:

$$\frac{x}{AB} = \frac{1}{BC}; \Rightarrow x = \frac{AB}{BC} = \frac{4}{4} = 1 \text{ m}$$

I.L. for shear Vn.

Step 1. At the position of support *B* (point B), plot an ordinate +1.

Step 2. Draw a straight line connecting the plotted point (+1) to the zero ordinate at the position of support *C*. Continue the straight line at *C* until the end of the overhang at end *D*.

Step 3. At the position of support *C* (point *C*), plot an ordinate –1.

Step 4. Draw a straight line connecting the plotted point (–1) to the zero ordinate at the position of support *B*. Continue the straight line at *B* until the end of the overhang at end *A*.

Step 5. Draw a vertical passing through the section whose shear is required to intersect the lines in step 2 and step 3.

Step 6. Connect the intersections to obtain the influence line, as shown in Figure d.

Step 7. Determine the ordinates of the influence lines at other points by using similar triangles, as previously demonstrated.

I.L. for Moment M_n.

Step 1. At point *B*, plot the ordinate equal +2 m.

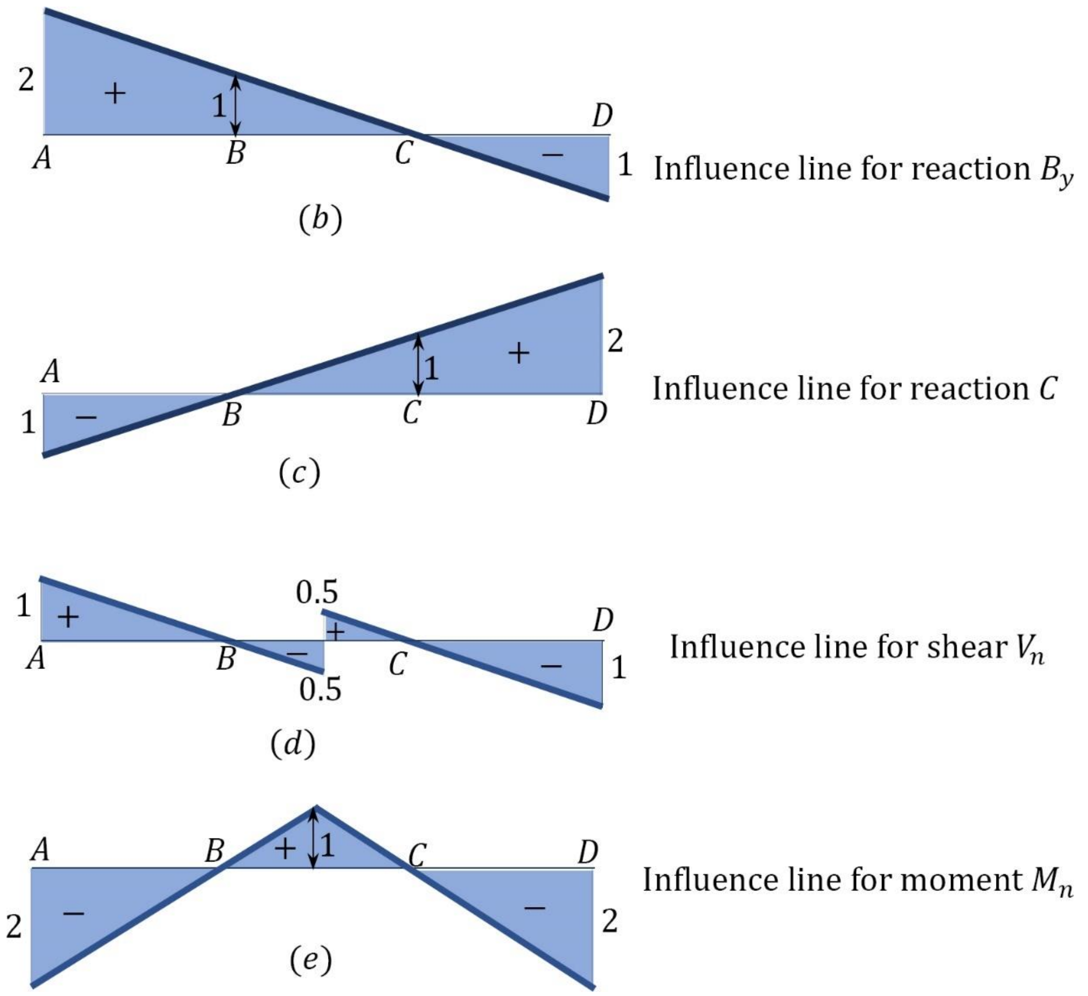
Step 2. Draw a straight line connecting the plotted ordinate in step 1 to the zero ordinate in support *C*.

Step 3. At point *C*, plot the ordinate equal +2 m.

Step 4. Draw a straight line connecting the plotted ordinate in step 3 to the zero ordinate at support *B*.

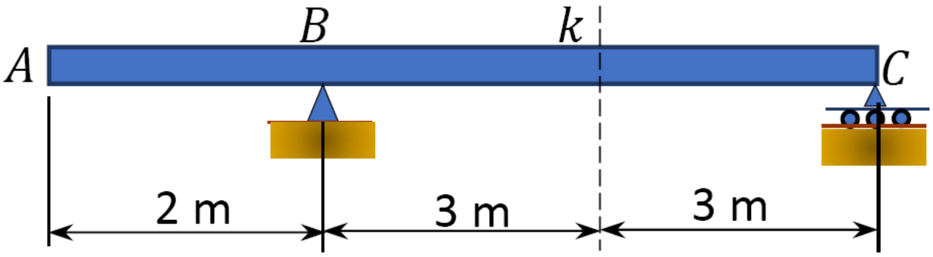
Step 5. Continue the straight lines from the intersection of the lines drawn in steps 2 and 4 through the supports to the overhanging ends, as shown in Figure e.

Step 6. Determine the values of the influence lines at other points using similar triangles, as previously demonstrated.



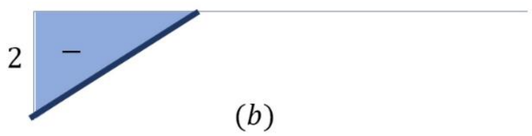
19.2 Influence Line Diagram for singly Overhanging beams

Q: - For the beam with one end overhanging support B, as shown in Figure(a), construct the influence lines for the bending moment at support *B*, the shear force at support *B*, the support reactions at *B* and *C*, and the shearing force and the bending moment at a section "*k*."



(a)

Solution:-

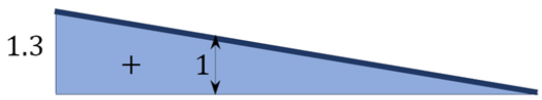


(b)

Influence line for moment M_B

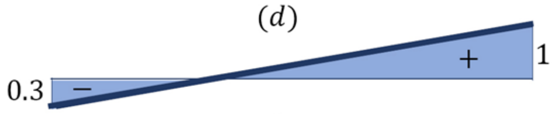
(c)

Influence line for shear V_B



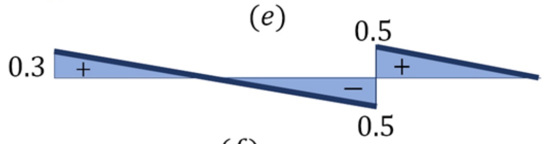
(d)

Influence line for reaction B_y



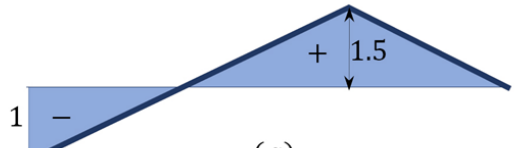
(e)

Influence line for reaction C



(f)

Influence line for shear V_k



(g)

Influence line for shear M_k

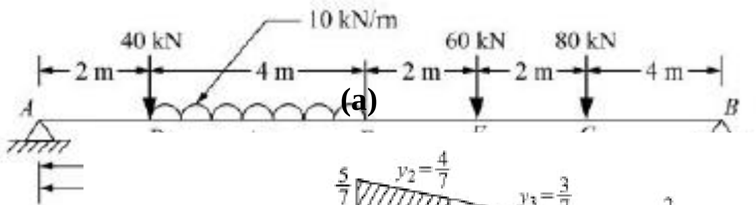
Q: Using influence line diagrams determine the shear force and bending moment at section C in the simply supported beam shown in Figure (a)

Solution:

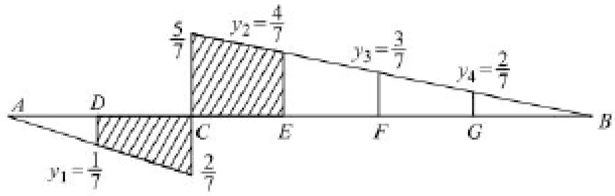
S.F. at C: Influence line diagram for shear force at C is as shown in Figure (b).

$$F_C = -40 \times \frac{1}{7} - 10 \times \frac{1}{2}\left[\frac{1}{7}+\frac{2}{7}\right] \times 2 + \frac{10\times1}{2}\left[\frac{5}{7}+\frac{4}{7}\right] 2 + 60 \times \frac{3}{7} + 80\times \frac{2}{7}$$

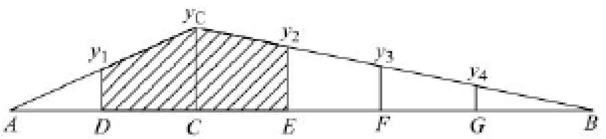
$$= 51.43 \text{ kN}$$



(a)



(b)



(c)

ILD for bending moment at C is as shown in Figure 5.11(c)

Maximum ordinate, $$y_C = \frac{4(14-4)}{14} = \frac{20}{7}$$

$$\therefore \quad y_1 = \frac{10}{7},\ y_2 = \frac{8}{10}\times\frac{20}{7} = \frac{16}{7}$$

$$y_3 = \frac{6}{10}\times\frac{20}{7} = \frac{12}{7}$$

$$y_4 = \frac{4}{10}\times\frac{20}{7} = \frac{8}{7}$$

$$\therefore \quad M_C = 40\times\frac{10}{7}+\frac{10\times 1}{2}\left[\frac{10}{7}+\frac{20}{7}+\frac{20}{7}+\frac{16}{7}\right]2 + 60\times\frac{12}{7} + 80\times\frac{8}{7}$$

$$= 345.71 \text{ kN-m}$$

MODULE 4 CHAPTER 20

SESSION 33

20.1 Uniformly Distributed Load Longer Than the Span:

Let a uniformly distributed load of intensity w move from left to right.

(a) Maximum S.F. and B.M. at given sections Load intensity times the area of ILD over loaded length gives the value of stress resultant (SF/BM).

Negative S.F. is maximum, when the load covers portion AC only.

Maximum negative $F_C = w \times$ Area of ILD for F_C in length AC

$$= w\left(\frac{1}{2}\right)z\left(\frac{z}{L}\right) = \frac{wz^2}{2L}$$

Positive S.F. is maximum when the uniformly distributed load occupies the portion CB only and

Maximum positive $F_C = w \times$ Area of ILD for F_C in length CB

$$= w\left(\frac{1}{2}\right)(L-z)\left(\frac{L-z}{L}\right)$$

$$= \frac{w(L-Z)^2}{2L}$$

From Figure 5.12(c), it is clear that maximum moment at C will be, when the udl covers entire span,

$$M_{FC,\max} = w \times \text{Area of ILD for } M_C$$

$$= w \times \frac{1}{2}L\frac{z(L-z)}{L}$$

$$= \frac{wz(L-z)}{2}$$

(b) Absolute maximum values any where in the beam Negative S.F. is maximum when $z = L$, i.e., at B when the load occupies entire span AB

$$\text{Absolute maximum S.F.} = w \times \frac{1}{2} \times 1 \times L = \frac{1}{2}wL$$

Similarly, maximum positive S.F. occurs when z = 0; i.e., at A when the load occupies entire span AB

$$\text{Maximum positive S.F.} = w \times \frac{1}{2} \times 1 \times L = \frac{wL}{2}$$

Maximum moment at any section

$$= \frac{1}{2} \times \frac{wz(L-z)}{L} \times L = \frac{1}{2} \times wz(L-z)$$

This is maximum, when $z = \frac{L}{2}$ i.e., at mid-span

$\therefore$ Absolute maximum moment $= \frac{1}{2} \times w \times \frac{L}{2} \times (L - L/2) = \frac{wL^2}{8}$, at mid-span

20.2 Uniformly Distributed Load Smaller Than the Span:

Let the length of uniformly distributed load w/unit length be d. Let it move from left to right over beam AB of span L. We are considering the case when, $d < L$. Now, position of this load for maximum shear force and bending moment at section C (Refer Figure 5.13) are to be determined.

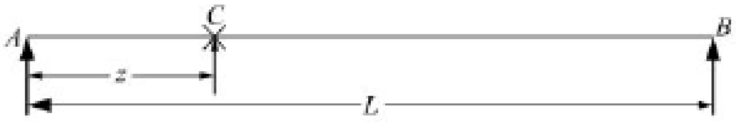


Figure. 5.13(a) Simply supported beam with given point C

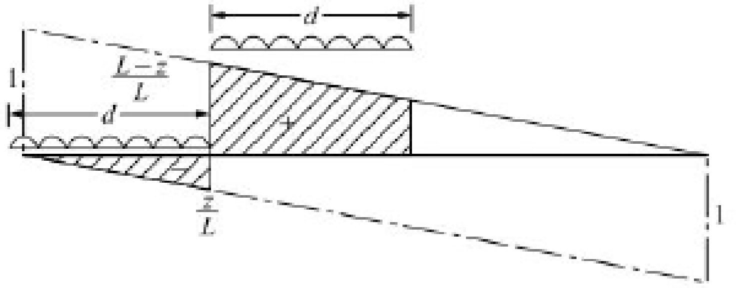


Figure. 5.13(b) Position of load for maximum +ve and –ve SF

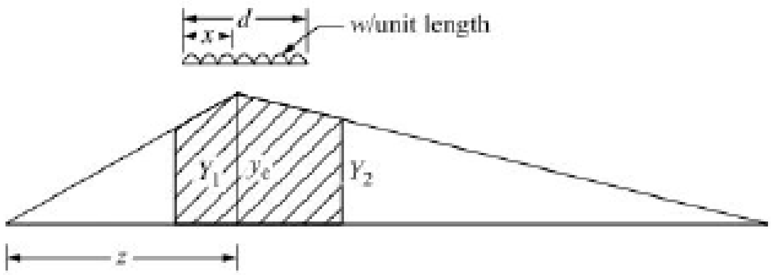


Figure. 5.13(c) Position of load for maximum moment

From ILD for shear force at C, it is clear that maximum shear force will develop when the head of the load reaches the section.

For maximum positive shear force the tail of the udl should reach the section.

Maximum bending moment will develop at C when the load is partly to the left of the section and partly to the right of section. Let the position of the section be as shown in Figure 5.13(c). Referring to this figure.

$$M_C = w \times \frac{x(y_1 + y_c)}{2} + w(d - x)\frac{(y_c + y_3)}{2}$$

For M_C to be maximum,

$$\frac{dM_C}{dx} = 0 = \frac{w(y_1 + y_c)}{2} - \frac{w(y_c + y_2)}{2}$$

i.e., $y_1 = y_2$

Thus, moment at C will be maximum when the ordinates of ILD for M_C at head and tail of the udl are equal.

Now, $y_1 = y_2$

i.e., $$\frac{(z-x)}{z} y_C = \frac{(L-z)-(d-x)}{L-z} y_C$$

$\therefore$ $$(z-x)(L-z) = z(L-z-d+x)$$

$$Lz - z^2 - Lx + xz = Lz - z^2 - dz + xz$$

i.e., $$Lx = dz$$

or $$\frac{x}{d} = \frac{z}{L}$$

i.e., Bending moment at a section is maximum when the load is so placed that the section divides the load in the same ratio as it divides the span.

Once the position of moving load is identified for maximum values the required values can be easily found.

Position for absolute maximum moment Obviously for this y_C should be maximum

Now, $$y_C = \frac{z(L-z)}{L}$$

For y_C to be maximum,

$$\frac{dy_C}{dz} = 0 = L - 2z$$

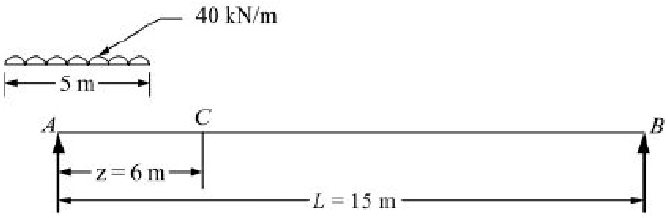
or $$z = \frac{L}{2}$$

i.e., Absolute maximum moment occurs at mid-span. The position of the load is to be such that the section divides the load in the same ratio as it divides the span which means that absolute maximum moment C.G. of the load will be at the mid-span.

MODULE 4 **CHAPTER 20**

SESSION 34

Q: A simply supported beam has a span of 15 m. Uniformly distributed load of 40 kN/m and 5 m long crosses the girder from left to right. Draw the influence line diagram for shear force and bending moment at a section 6 m from left end. Use these diagrams to calculate the maximum shear force and bending moment at this section.

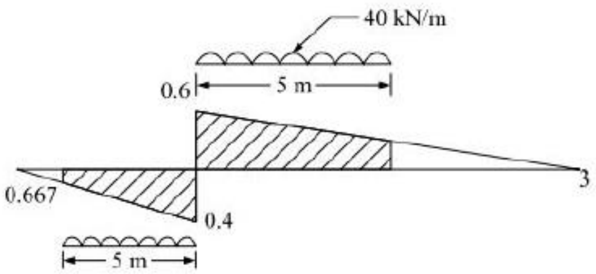


(a)

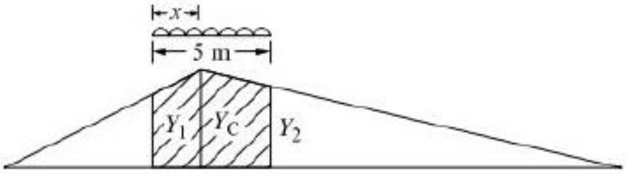
The beam is shown in Figure (a). For point C, which is at 6 m from 4, ILD for shear force F and bending moment M are to be found.

ILD for F: ILD ordinate at just to the left of C is

$$= -\frac{z}{L} = -\frac{6}{15} = -0.4$$



) Load positions for maximum +ve and (–ve) F_C



Load position for maximum M_C

ILD ordinate at just to right of C

$$= \frac{L-z}{L} = \frac{15-6}{15} = 0.6$$

ILD for F is as shown in Figure 5.14(b).

At C, negative S.F. is maximum when the head of load touches C. At this time, tail of the udl is at a distance of 1 m from 4 as shown in Figure. Ordinate under tail end of load is

$$= \frac{1}{6} \times 0.4 = 0.0667$$

$\therefore$ Negative maximum $F_C = 40 \times \left[\frac{0.0667+0.4}{2}\right] \times 5$

$$= 46.667 \text{ kN}$$

For maximum positive S.F. at C, tail of the load should be at C as shown in Figure 5.14(b). Ordinate under head to the load

$$= \frac{4}{9} \times 0.6 = 0.267$$

$\therefore$ Maximum positive SF = 40 × Area of ILD under the load

$$= 40 \times \frac{0.6+0.267}{2} \times 5$$

$$= 86.67 \text{ kN}$$

ILD for moment at C is as shown in Figure 5.14(c) in which

$$y_C = \frac{z(L-z)}{L} = \frac{6 \times 9}{15} = 3.6$$

For maximum moment, load position should be such that the section divides the load in the same ratio as it divides the span. Referring to Figure.

$$\frac{x}{5-x} = \frac{6}{9}$$

or $$9x = 30 - 6x$$

or $$x = 2\text{m}$$

$$y_1 = \left(\frac{6-2}{6}\right) y_C = \frac{4}{6} \times 3.6 = 2.4$$

y_2 will be same as y_1

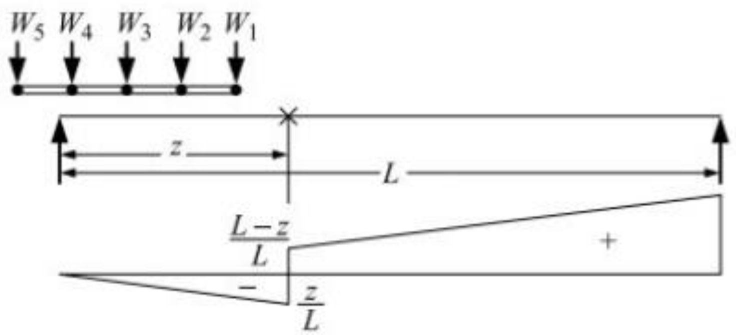
Maximum moment $= w \times$ Area of ILD for M_C under the loaded length

$$= 40\left[\left(\frac{2.4+3.6}{2}\right)\times 2+\left(\frac{3.6\times 2.4}{2}\right)\times 3\right]$$

$$= 600\,\text{kNm}$$

21.1 A Train of Concentrated Loads

A train of concentrated loads moving over a simply supported beam from left to right is shown in Fig.



It is required to find

(a) Maximum shear force at C

(b) Maximum bending moment at C

(c) Absolute maximum shear force in the beam

(d) Absolute maximum bending moment in the beam.

(a) *Maximum shear force at C:* Influence line diagram for shear force at C is shown in Figure. As soon as W_1 enters the span negative shear force develops at C. It increases as the load moves on. Some more loads may enter the span and hence, the rate of increase in S.F. goes up. This will continue till the load W_1, reaches the section C. As soon as W_1 crosses section C it contributes to positive shear, thus, reducing the negative shear. Hence, there will be a drop in shear force value. Further movement causes more increase in shear force till the second load reaches C. There is a second peak value and a sudden drop, when the second load crosses. Thus, shear force will have a peak value whenever a load is on the section. Highest value among these peak values is to be selected. By two or three trial values, it is possible to get maximum negative shear force value. It is to be noted that for maximum negative shear force, most of the loads are to the left of the section.

Similarly, for maximum positive shear force, there are peak values whenever a load comes on the section and the maximum value is obtained when most of the loads are to the right of the section.

(b) Maximum bending moment: Let R, be the resultant of the loads on the left of the section and R, be resultant of the loads on the right of the section.

Distance between R, and R, be d and R, be at a distance x from C.

Let ordinate of ILD for moment at Chey, under R, and y, under R, and maximum ordinate at C be y_c.

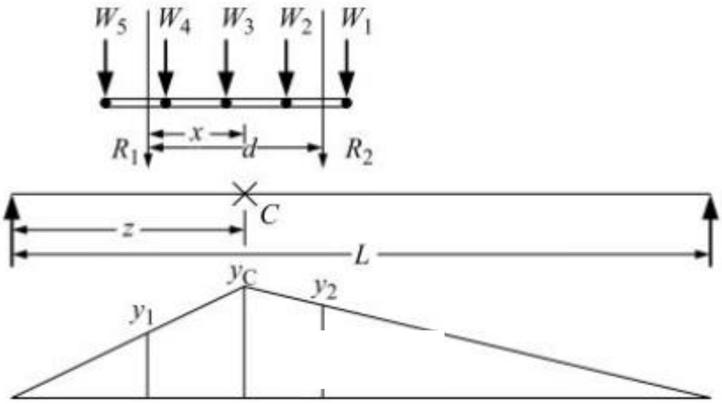


Figure 5.16 ILD for M_C

$$M_c = R_1 y_1 + R_2 y_2$$

$$= R_1 \left(\frac{z-x}{z}\right) y_c + R_2 \times \frac{(L-z)-(d-x)}{L-z} \times y_c$$

For M_c to be maximum

$$\frac{dM_c}{dx} = -\frac{R_1 y_c}{z} + R_2 \left(\frac{y_c}{L-z}\right) = 0$$

$$\frac{R_1}{z} = \frac{R_2}{L-z}$$

i.e., the average load on the left-side portion of the beam is same as the average load on the right- side portion of the beam. But seldom we get exactly equal average load on both sides of the section. For example, when load W, is to the left of the section, the average load on left side may be heavier. When it just rolls over the section, the average load on right-hand side may become heavier. Hence, the above condition for maximum bending moment can be interpreted as the bending moment is maximum when that load is on the section.

Thus, due to a train of moving loads on a simply supported beam, maximum moment at the given section develops when the load W, is on the section where the load W, is such that as it rolls on the section and comes to the other side, heavier portion of the beam becomes lighter and lighter portion becomes heavier.

In case of some load entering and some leaving the span, the change of portion heavier becoming lighter and lighter portion becoming heavier may happen under more than one particular load. All such cases are to be considered to identify which position gives maximum moment at the section.

(*c*) *Absolute maximum shear force* At any section, influence line ordinate for negative shear is $\left(\frac{z}{L}\right)$ and for positive shear it is $\left(\frac{L-z}{L}\right)$. Hence, when $z = 0$, i.e,. at support A, ILD ordinate for positive shear force is maximum (= 1) and when $z = L$, i.e., at support B ILD ordinate for negative shear force is maximum (= 1) ILD. For shear force at support sections A and B are as shown in Figure 5.17.

Figure. 5.17(a) Simply supported beam subjected to a train of load

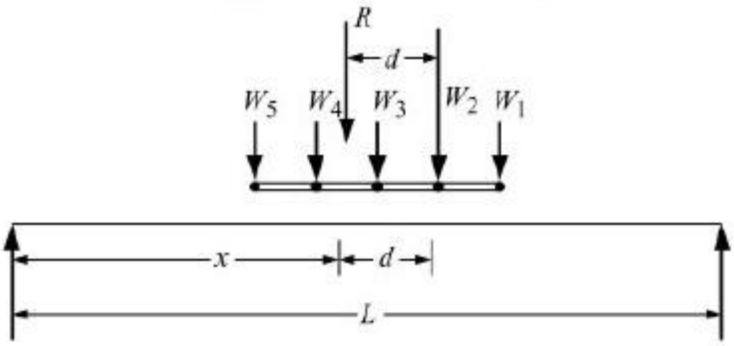
Figure. 5.17(b) ILD for F_A

Figure. 5.17(c) ILD for F_B

Obviously, maximum shear force occurs when one of the load is on support A. When the load starts moving from left to right, contribution of leading loads to shear force at A decreases but more number of loads may come on the beam and they will contribute to additional shear. However, no general conclusions can be drawn to say whether increase due to additional load is more or decrease due to the reduced contribution from leading loads is more. It needs a few trials to arrive at conclusions. However, it can be definitely said that one of the loads should be on the support A to get absolute maximum positive SF.

Similarly, to get absolute maximum negative shear force, one of the loads should be on support B (just to the left of the section) and a few trials may be required to get absolute maximum negative shear force which occurs at support B.

(*d*) ***Maximum moment under a load*** Let a train of concentrated loads W_1, W_2, W_3...move on a simply supported beam AB from left to right as shown in Figure 5.18. Now, the condition for moment to be maximum under wheel load W_2 is required. Let R be the resultant of all loads. Let its distance from W_2 be d and from support A be x (Refer Figure 5.18)



Simply supported beam subjected to a train of loads

Now, $$R_A = \frac{R(L-x)}{L}$$

Therefore, moment under load W_2

$$M = R_A\,(x + d) - Rd$$

$$= R\left[\frac{L-x}{L}\right](x + d) - Rd$$

$$= \frac{R}{L}\,[Lx - x^2 + Ld - xd] - Rd$$

For the moment to be maximum

$$\frac{dM}{dx} = 0 = \frac{R}{L}\,[L - 2x - d]$$

or $$x = \frac{L}{2} - \frac{d}{2}$$

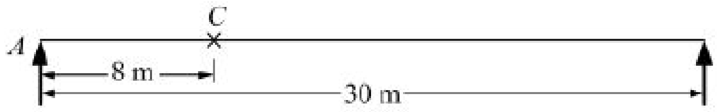
Distance of W_2 from A

$$= x + d = \frac{L}{2} - \frac{d}{2} + d$$

$$= \frac{L}{2} + \frac{d}{2}$$

(e) Absolute maximum bending moment Influence line diagram ordinate for bending moment is maximum at the center of span. Hence, bending moment will be maximum near the center of the span when heavier loads are near to the center. Since, the maximum moment always occurs under a wheel load, it can be concluded that absolute maximum moment occurs under one of the loads when the resultant of all the loads and the load under consideration are equidistant from the center of the beam. The maximum moment under possible loads can be evaluated and the maximum of these selected as absolute maximum.

Q: Four-point loads, 8, 15, 15 and 10 kN have centre to centre spacing of 2 m between consecutive loads and they traverse a girder of 30 m span from left to right with 10 kN load lending. Calculate the maximum bending moment and shear force at 8 m from the left support.



Solution:

The beam is shown in Figure 5.19. ILD for shear force at 8 m from left support is shown in Figure along with possible load position for maximum negative shear force. Maximum negative SF at C.

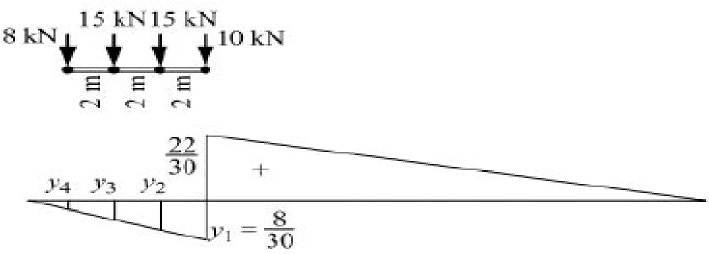


Figure 5.19(b) Load position for maximum –ve F_C

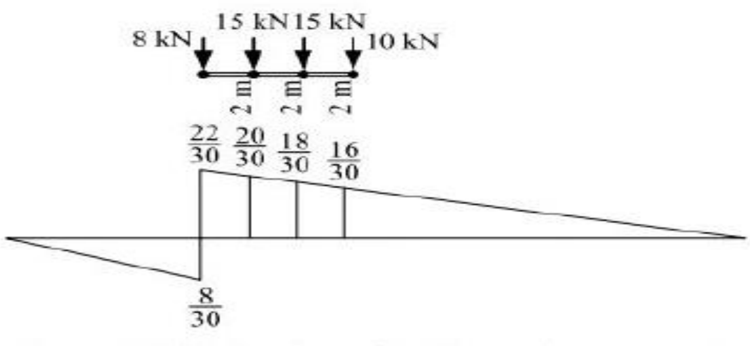
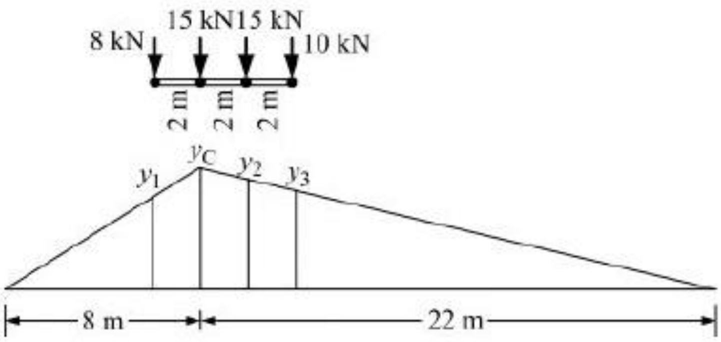


Figure 5.19(c) Load position for maximum +ve F_C



$$= 10\,y_1 + 15\,y_2 + 15\,y_3 + 8\,y_4$$

$$= 10 \times \frac{8}{30} + 15 \times \frac{6}{30} + 15 \times \frac{4}{30} + 8 \times \frac{2}{30}$$

$$= 8.2 \text{ kN}$$

For maximum positive SF at C, load position is as shown in Figure 5.19(c). S.F. at C

$$= 10 \times \frac{16}{30} + 15 \times \frac{18}{30} + 15 \times \frac{20}{30} + 8 \times \frac{22}{30}$$

$$= 30.2 \text{ kN}$$

Check for another position, i.e., when W_3 = 15 kN load is on the section.

$$\text{S.F. at } C = 10 \times \frac{18}{30} + 15 \times \frac{20}{30} + 15 \times \frac{22}{30} - 8 \times \frac{6}{30}$$

$$= 25.4 \text{ kN}$$

∴ Maximum positive shear force is = 30.2 kN

ILD for bending moment at C is as shown in Figure 5.19(d). The maximum ordinate.

$$y_c = \frac{z(L-z)}{L} = \frac{8(30-8)}{30}$$

$$= 5.867$$

To find the load position for maximum moment, average load on portion AC and CB are to be found as loads crosses section C one after another.

Table 5.1 Calculations to find load position for maximum M_C

Load crossing	*Average load* AC W_{1av}	*Average load* BC W_{2av}	*Remarks*
10 kN	$\frac{38}{8}$	$\frac{10}{22}$	$W_{1av} > W_{2av}$
15 kN	$\frac{23}{8}$	$\frac{25}{22}$	$W_{1av} > W_{2av}$
15 kN	$\frac{8}{8}$	$\frac{40}{22}$	$W_{1av} < W_{2av}$

Hence, load position for maximum moment at C is when second 15 kN load is on C. Referring to Figure 5.19(d),

Maximum $$M_c = 8\,y_1 + 15\,y_c + 15\,y_2 + 10\,y_3$$

$$= 8\left(\frac{6}{8}\right)y_c + 15\,y_c + 15\left(\frac{20}{22}\right)y_c + 10\left(\frac{18}{22}\right)y_c$$

$$= 251.21 \text{ kNm, Since } y_c = 5.867$$

MODULE 5 **CHAPTER 22**

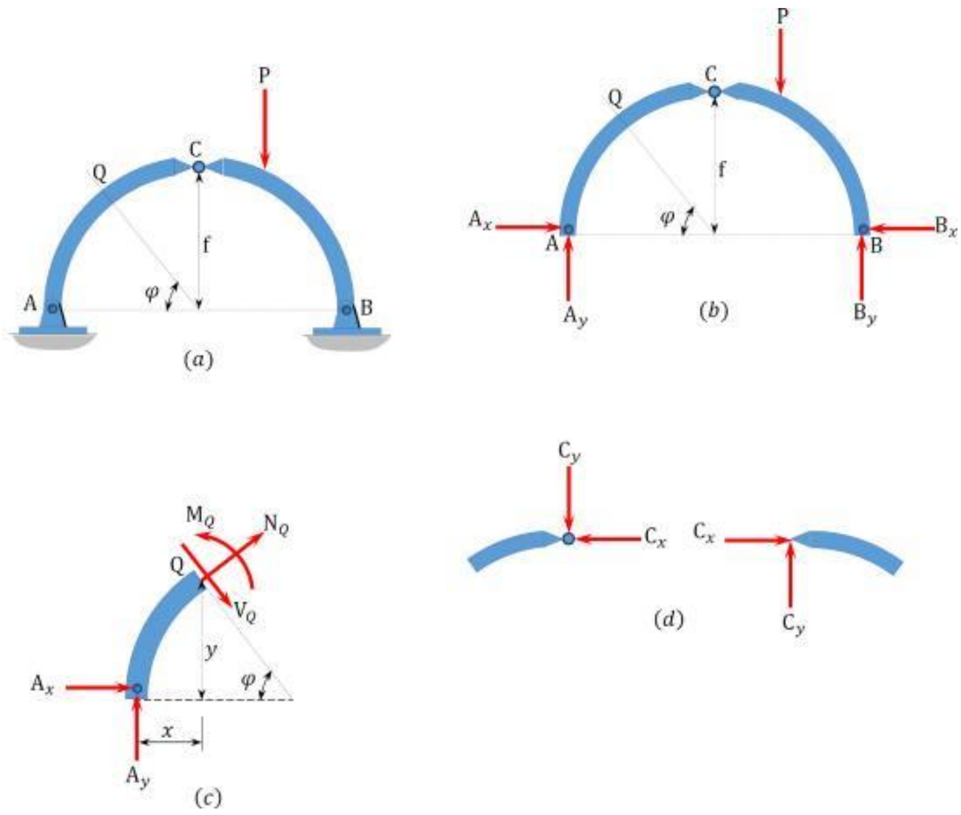
SESSION 37

22.1 Three-Hinged Arch:

A three-hinged arch is a geometrically stable and statically determinate structure. It consists of two curved members connected by an internal hinge at the crown and is supported by two hinges at its base. Sometimes, a tie is provided at the support level or at an elevated position in the arch to increase the stability of the structure.

1 Derivation of Equations for the Determination of Internal Forces in a Three-Hinged Arch

Consider the section Q in the three-hinged arch shown in Figure 6.2a. The three internal forces at the section are the axial force, N_Q, the radial shear force, V_Q, and the bending moment, M_Q. The derivation of the equations for the determination of these forces with respect to the angle φ are as follows:



Bending moment at point Q.

$$M_\varphi = A_y x - A_x y = M^b_{(x)} - A_x y$$

Bending moment at point Q.

$$M_\varphi = A_y x - A_x y = M^b_{(x)} - A_x y$$

where

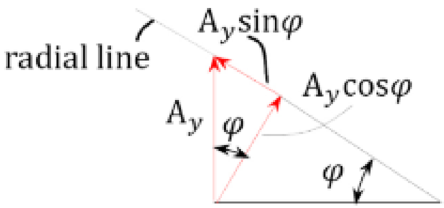
- $M^b_{(x)} =$ moment of a beam of the same span as the arch.
- $y =$ ordinate of any point along the central line of the arch.

$$\text{For a parabolic arch, } y = \frac{4fx}{L^2}(L - x)$$

$$\text{For a circular arch, } y = \sqrt{\mathrm{R}^2 - \left(\frac{\mathrm{L}}{2} - x\right)^2}\,\mathrm{R} + f$$

- $f =$ rise of arch. This is the vertical distance from the centerline to the arch's crown.
- $x =$ horizontal distance from the support to the section being considered.
- $L =$ span of arch.
- $R =$ radius of the arch's curvature.

Radial shear force at point Q.



(e)

$$V_\varphi = A_y \sin\varphi - A_x \cos\varphi = V^b \sin\varphi - A_x \cos\varphi$$

where V^b is the shear of a beam of the same span as the arch.

Axial force at a point Q.

$$N_\varphi = -A_y \cos\varphi - A_x \sin\varphi = -V^b \cos\varphi - A_x \sin\varphi$$

MODULE 5 CHAPTER 22

SESSION 38

22.1 Analysis for Static Loads:

Consider a three-hinged arch subjected to loads as shown in Figure. Since, the ends are hinged there will be two reaction components at each end namely vertical and horizontal. Hence, totally there are four reaction components namely, V_A. H_A, V_B and H_B.

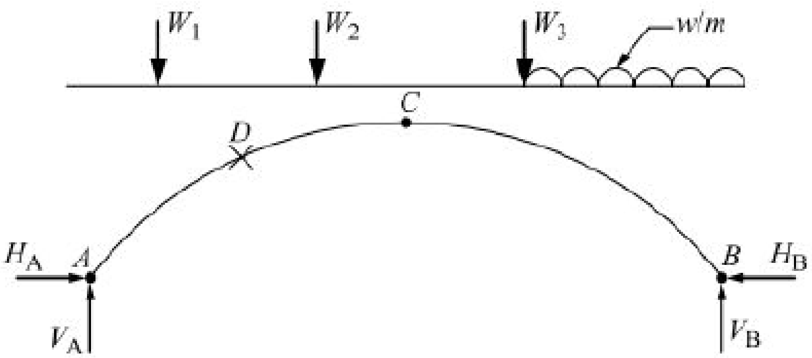


Figure 7.8 Three hinged arch subjected to loads

For any plane structure there are three independent equations of equilibrium which can be used conveniently.

$$\left.\begin{array}{l}\sum F_H = 0 \\ \sum F_v = 0 \\ M_A \text{ or } M_B = 0\end{array}\right\}$$

In this case, the fourth equation is also available, i.e.

Mc =0, Since C is a hinge

If no horizontal load is acting, which is the usual case, Equation gives H_A=H_B say H. In such case, the following three equations are used.

$$\left.\begin{array}{ll}\sum F_v = 0 & \ldots(a) \\ M_A \text{ or } M_B = 0 & \ldots(b) \\ M_C = 0 & \ldots(c)\end{array}\right\}$$

Since, the loads tend to spread the arch, the horizontal thrust is in the inward direction as shown in the figure.

Now, consider a section at D,

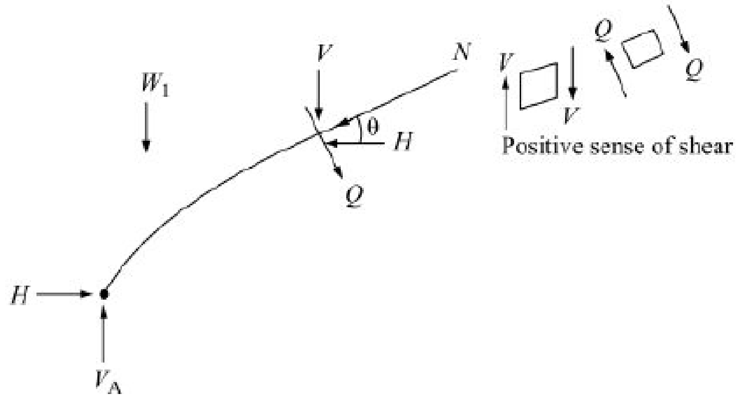
Let V, be the vertical shear

Q, the radial shear

and

N, the normal thrust.

All these forces are shown in their positive senses in Figure. Let the normal to the section make an angle θ with the horizontal.



Internal forces at *D*

Then, $N = V \sin\theta + H \cos\theta$

and $Q = V \sin\theta - H \cos\theta$

The moment at D can be obtained by considering all the forces including the reaction on any one part of the arch. Sagging moment M is taken as positive moment here.

Example 1 A three-hinged circular arch hinged at the springing and crown points has a span of 40 m and a central rise of 8 m. It carries a uniformly distributed load 20 kN/m over the left-half of the span together with a concentrated load of 100 kN at the right quarter span point. Find the reactions at the supports, normal thrust and shear at a section 10 m from left support.

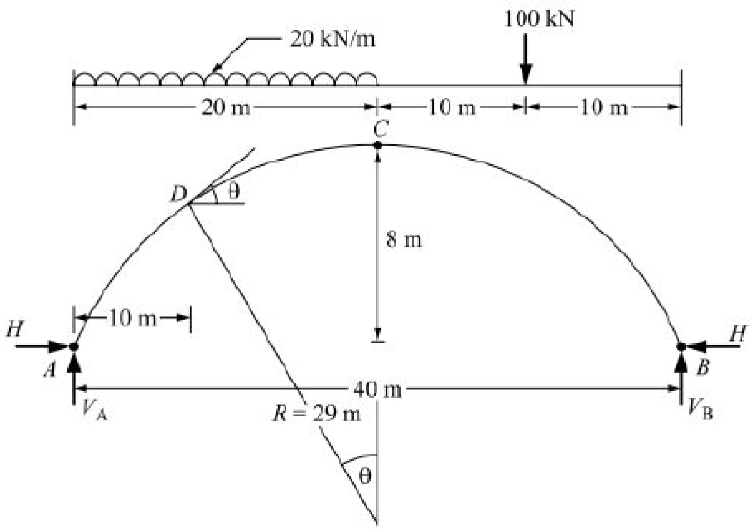


Figure 7.10 Example 7.1

Solution The arch is shown in Figure 7.10

$$\Sigma M_B = 0, \text{ gives}$$

$$V_A \times 40 - 20 \times 20 \times 30 - 100 \times 10 = 0$$

$$V_A = 325 \text{ kN}$$

$$\Sigma F_v = 0, \text{ gives}$$

$$V_A + V_B = 20 \times 20 + 100$$

$\therefore$ $$V_B = 500 - V_A = 500 - 325$$

$$= 175 \text{ kN}$$

Since, C is hinged,

$$M_C = 0, \text{ gives}$$

$$V_B \times 20 - 100 \times 10 - H \times 8 = 0$$

$$175 \times 20 - 100 \times 10 - H \times 8 = 0$$

or $$H = 312.5 \text{ kN}$$

Let D be the point 10 m from the left support where the normal thrust and shear are to be found. Now, from the property of circles.

$$h\ (2R - h) = \frac{L}{2} \times \frac{L}{2}$$

$$8\ (2R - 8) = \frac{40}{2} \times \frac{40}{2} = 400$$

$\therefore$ $$R = 29 \text{ m}$$

$\therefore$ $$\text{Slope at } D = \theta = \sin^{-1}\left(\frac{10}{R}\right) = \sin^{-1}\left(\frac{10}{29}\right)$$

$\therefore$ $$\theta = 20.171°$$

Vertical shear at D, $$V = V_A - 20 \times 10$$

$$= 325 - 200 = 125 \text{ kN}$$

$\therefore$ $$N = V \sin\theta + H \cos\theta$$

$$= 125 \sin 29.171° + 312.5 \cos 20.171°$$

$$= 336.437 \text{ kN}$$

$$Q = V \cos\theta - H \sin\theta$$

$$= 125 \cos 20.171° - 312.5 \sin 20.171°$$

$$= 9.575 \text{ kN}$$

MODULE 5 **CHAPTER 23**

SESSION 39

23.1 Bending Moment Diagrams

In the arch, at any section D (x, y), the bending moment may be looked as a sum of the moment in an equivalent beam minus the ordinate time the horizontal thrust. Thus,

M = Beam moment – Hy

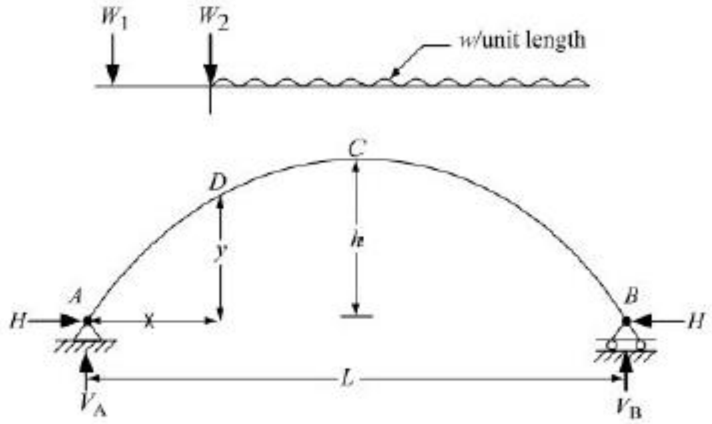


Figure (a) A typical arch

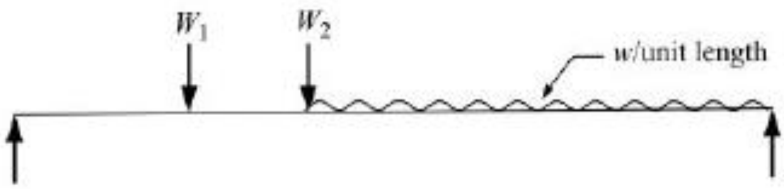


Figure (b) Equivalent beam

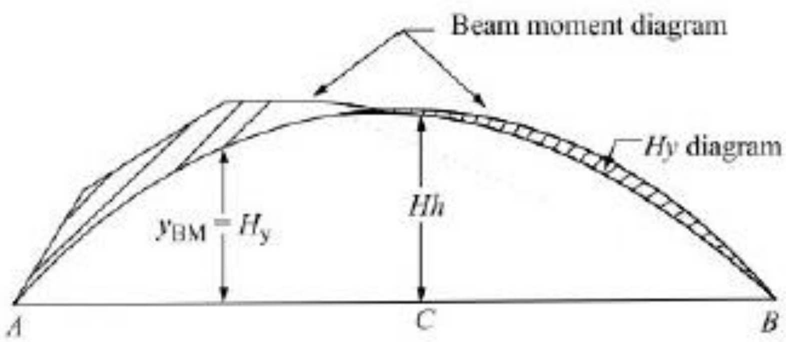


Figure (c) Bending Moment Diagram

Example A symmetric three-hinged parabolic arch of span 36 m and rise 6 m is subjected to a concentrated load of 120 kN at a point 12 m from left support. Draw the bending moment diagram for the arch.

Solution The arch is shown in Figure 7.18(*a*)

$$\sum M_B = 0, \text{ gives}$$

$$V_A \times 36 - 120\,(36 - 12) = 0$$

$$V_A = 80 \text{ kN}$$

$$\therefore \quad V_B = 120 - 80 = 40 \text{ kN}$$

Beam moment diagram is a triangle with maximum ordinate at the load point, its ordinate being

$$= \frac{120 \times 12(36-12)}{36} = 960 \text{ kNm}$$

This is drawn first (Figure 7.18(*b*)). Now, at mid-span the net bending moment is zero. The ordinate of the beam moment diagram at mid-span is

$$= \frac{960 \times 18}{24} = 720 \text{ kNm}$$

Since, $M_c = 0$ in the arch, $Hh = 720$

or

$$H = \frac{720}{h} = \frac{720}{6} \text{ kN} = 120 \text{ kN}$$

A parabola is drawn with its central ordinate equal to 720 kNm as shown in Figure 7.18(*b*). The equation of this parabola is

$$y_{BM} = Hy = H \times \frac{4hx(L-x)}{L^2}$$

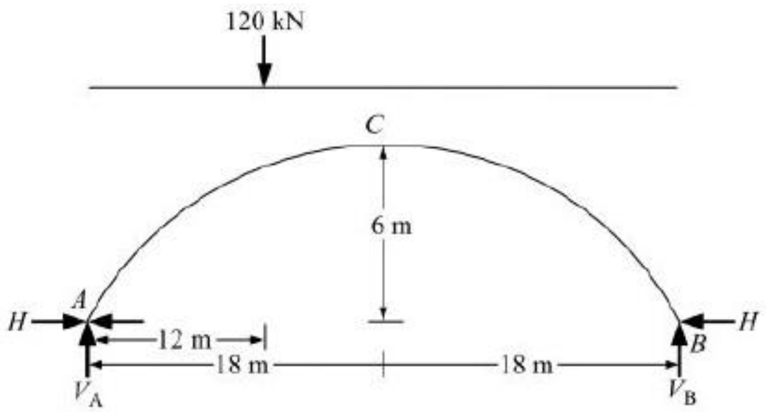


Figure (a)

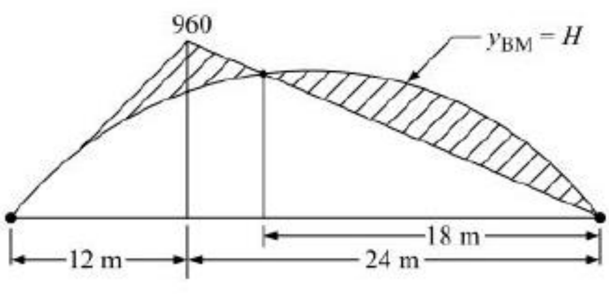


Figure (b) Bending moment diagram

MODULE 5 CHAPTER 23

SESSION 40

23.1 Influence Line Diagrams

Consider the three-hinged arch of span L and rise a shown in Figure 7.20. The influence line diagrams for following are discussed in this section.

1. For horizontal thrust H
2. Moment at section D
3. Normal thrust at D
4. Radial shear at D

where D is the point at distance from the left support A.

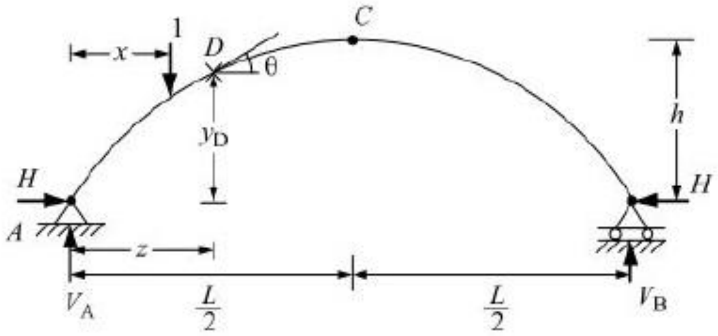


Figure (a) A typical three-hinged arch subject to unit load

Let x be the distance of the unit load from the support A.

ILD for *H*

Taking the moment about A, we get

$$V_B = \frac{x}{L}$$

$$\therefore \qquad V_A = \frac{L-x}{L}$$

When the load is in portion AC taking the moment about hinge C, we get

$$Hh = V_B \times \frac{L}{2} = \frac{x}{L} \times \frac{L}{2}$$

or $$H = \frac{x}{2h}, \text{ linear variation}$$

When $x = 0; \qquad H = 0$

When $\quad x = \dfrac{L}{2}; \quad H = \dfrac{L}{4h}$

When the unit load is in portion *CB*, considering the left-half-portion and taking moment about *B*, we get

$$Hh = V_A \times \frac{L}{2} = \frac{L-x}{L} \times \frac{L}{2} = \frac{L-x}{2}$$

$\therefore \quad H = \dfrac{L-x}{2h}$, linear variation

When $\quad x = \dfrac{L}{2}, \quad H = \dfrac{L}{4h}$

When $\quad x = L, \quad H = 0$

Hence, ILD for H is a triangle with its maximum ordinate equal to $\dfrac{L}{4h}$ at hinge C as shown in Figure 7.20(b) .

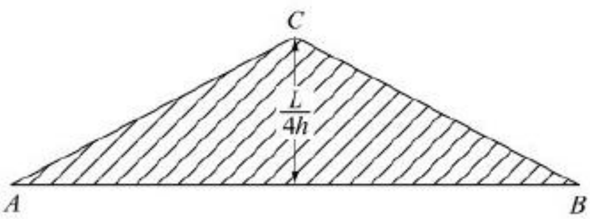


Figure 7.20(b) ILD for H

ILD for moment at *D*

Bending moment at any given section in the arch = Beam moment – Hy

where, beam moment means, the moment in an equivalent beam. Thus,

$$M_D = \text{Beam moment at } D - Hy_D$$

Hence, ILD for M_D in the arch will be drawn as the difference diagram of beam moment diagram and the Hy_D moment diagram. We know that ILD for beam moment at D is a triangle with maximum ordinate $\dfrac{z(L-z)}{L}$ at D as shown in Figure 7.21(c). Since, ILD for H is a triangle with maximum ordinate of $\left(\dfrac{L}{4h}\right)$ at hinge C, Hy_D diagram is a triangle with $\left(\dfrac{L}{4h} \times y_D\right)$ as the maximum ordinate at C. This is to be subtracted from the beam moment diagram. Hence, this triangle is drawn on the same side as beam moment diagram (Refer Figure 7.21(c)) and the difference diagram is marked as the ILD for the bending moment at D.

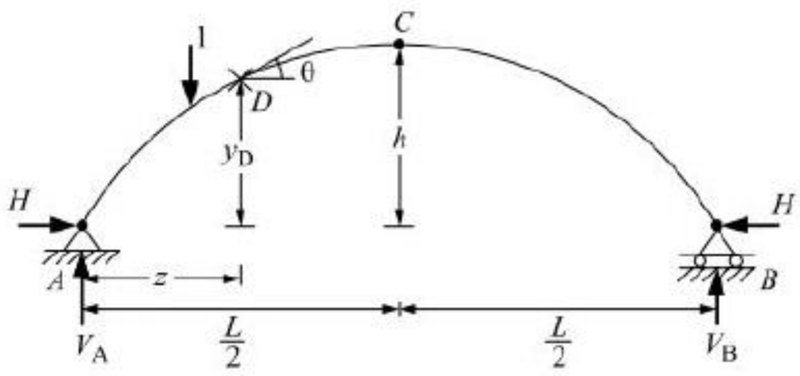


Figure 7.21(**a**) A typical three-hinged arch with unit load

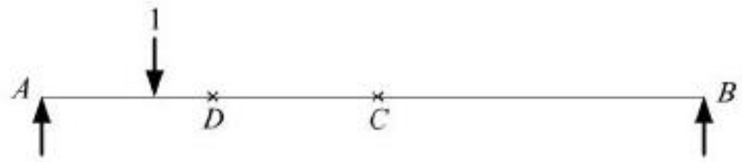


Figure 7.21(b) Equivalent beam

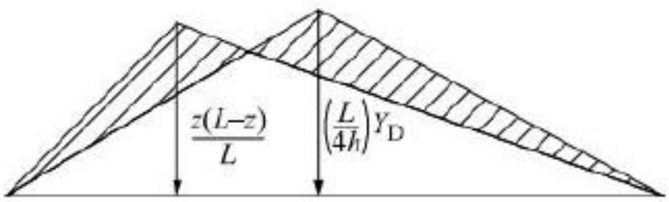


Figure 7.21(c) ILD for M_D

In case, of a parabolic arch, we know

$$y_D = \frac{4h\,(L-z)}{L^2}$$

Hence, the maximum ordinate for Hy_D term

$$= \frac{L}{4h} \times \frac{4h\,(L-z)}{L^2}$$

$$= \frac{z(L-z)}{L}$$

Which is same as that of the beam moment ordinate at D.

ILD for normal thrust at D (N_D)

As discussed in section 7.3, the normal thrust at section D is given by

$$N_D = V \sin\theta + H \cos\theta$$

where θ is the slope of the arch with the horizontal (Refer Figure 7.22) and V is the vertical shear. Now, ILD for $V \sin\theta$ and $H \cos\theta$ will be drawn so as to get a diagram for the normal thrust.

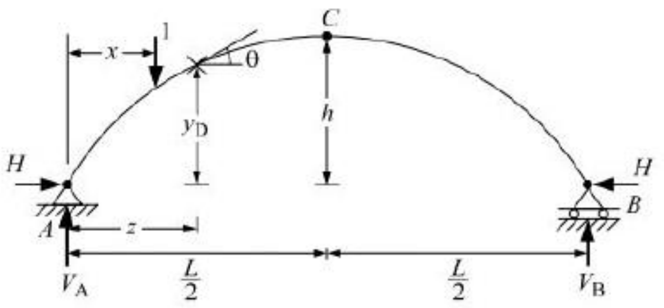


Figure 7.22(a) A typical three-hinged arch with unit load

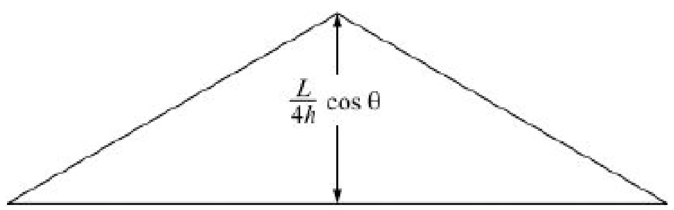


Figure 7.22(b) ILD for H cos θ

$H \cos\theta$ diagram is a triangle similar to ILD for the horizontal thrust but multiplied by $\cos\theta$ as shown in Figure 7.22(b). Since, V is the vertical shear at D, which is obviously the same as that in an equivalent beam, $V \sin\theta$ diagram is shown in Figure 7.22(c). Now, to get ILD for N_D, the diagrams are drawn in such a way that the addition is obtained by drawing ILD for $M \cos\theta$ on one side and drawing ILD for $V \sin\theta$ on the opposite side as shown in Figure 7.22(d)

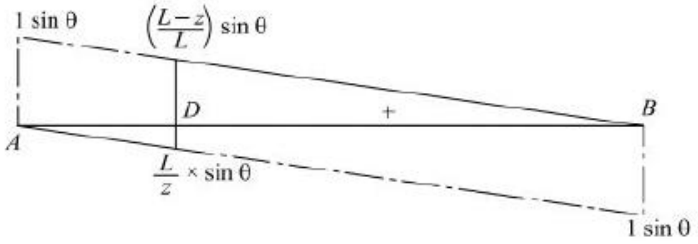


Figure 7.22(c) ILD for H sin θ

$$Q_D = V\ \cos - H\ \sin\ \theta$$

First, the $V \cos\theta$ diagram is drawn. This is similar to the ILD for SF in a beam but multiplied by the constant $\cos\theta$ (Refer Figure 7.23(*c*)). Then, $H\sin\theta$ diagram is drawn so that the difference diagram is available. This difference diagram is the ILD for Q_D and is shown hatched in the Figure 7.23(*d*).

ILD for radial shear (Q_D)

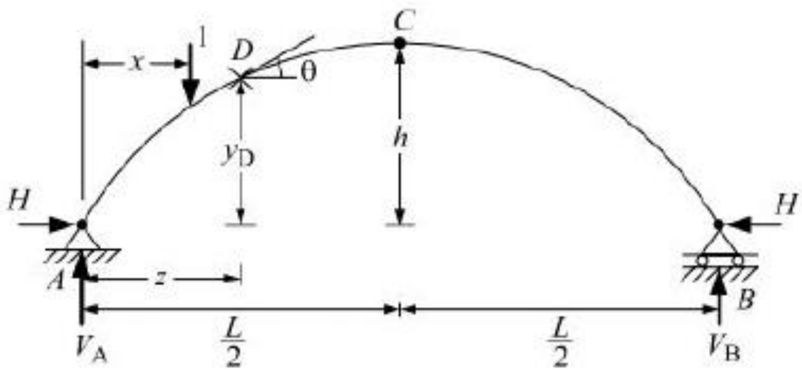


Figure 7.23(a) A typical three-hinged arch with unit load

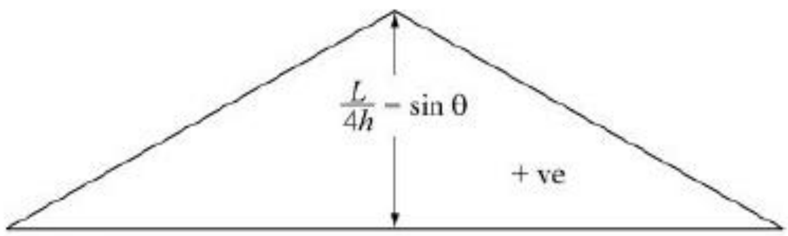


Figure 7.23(b) ILD for $H \sin\theta$

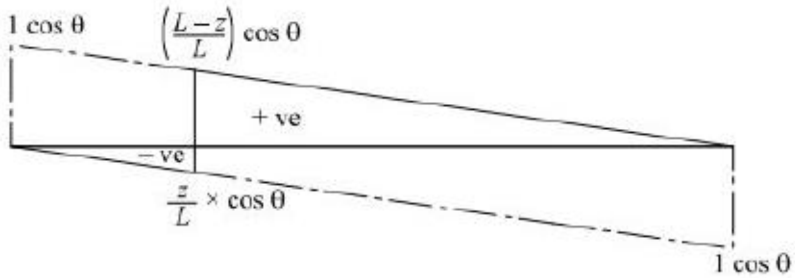


Figure 7.23(c) ILD for $V \sin\theta$

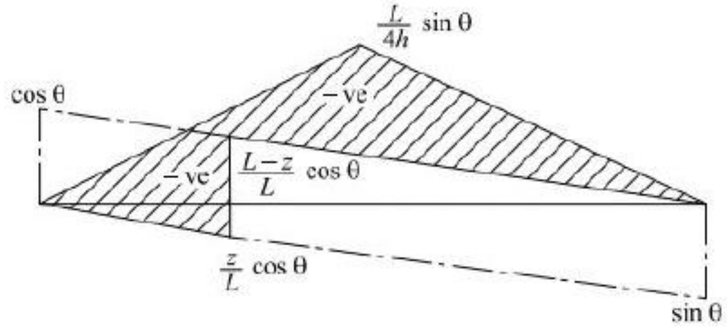


Figure 7.23(d) ILD for Q_D

As seen in section 7.3,

$$Q_D = V\cos - H\sin\theta$$

First, the $V\cos\theta$ diagram is drawn. This is similar to the ILD for SF in a beam but multiplied by the constant $\cos\theta$ (Refer Figure 7.23(*c*)). Then, $H\sin\theta$ diagram is drawn so that the difference diagram is available. This difference diagram is the ILD for Q_D and is shown hatched in the Figure 7.23(*d*).